A Revisit to the Annuity Role of Estate Tax

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A Revisit to the Annuity Role of Estate Tax

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Abstract

This paper finds that previous conclusions that a uniform lump-sum estate tax could provide annuity income were reached by not including the bequest income that households receive. We argue that since agents leave behind bequest, they should also receive bequest income from their parents. Moreover, the differential timing and sizes of bequest income will generate unequal wealth effects even with the actuarially fair annuity markets. To restore the first best allocations, the government has to adopt an estate tax regime that is no longer uniform: the estate would depend on both the timing and sizes of bequests that households receive. Thus the uniform estate tax no longer bears the annuity role. The paper reemphasizes the importance of accounting for bequest income received by households in any model that discusses intergenerational transfers and related policies.

Key words: Estate Tax; Annuity; Social Security.

JEL code: D15, E62, H21

1 Introduction

This paper revisits the annuity role of the estate tax by endogenizing bequest income in the model. The conventional wisdom has established that the first best allocations can be implemented by imposing uniform estate taxes when the annuity markets are missing and households have strong bequest motive. We found that this result was established by not accounting for bequest income received into the model. As a contrast, this paper demonstrates that estate tax regime can only implement the first best allocations by imposing taxes that are contingent on the level and timing

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of bequest income received by households. The key difference between this paper and the previous literature is that the bequest income is considered into the model. Actually, when agents in the model have strong bequest motives, they will leave behind bequest, and thus they should also receive bequest income from their parents. The timing and the size of bequest income will generate unequal wealth effects even where there are actuarially fair annuity markets. We show that to restore the first best allocations, defined as those when annuities are available, the government has to adopt an estate tax regime that is no longer uniform. Instead, the estate taxes should be contingent on the timing and the level of the bequest income received by each household. This trait is not similar to any current estate regimes, and we are not advocating this scheme either. We only point out that once we allow the bequest income determined in the model, the uniform estate tax regime no longer provides annuity role.

The proper taxation of inherited wealth is a highly debated issue. There is a recent surge of interest towards this subject to acquire better understanding and to form better policies. From children’s perspective, inheritances are pure luck since they cannot choose their parents and hence, estate taxes play an important redistribution role. As to parents, some may claim that the estate tax is not fair since it penalizes parents who save for their children. There is no doubt that redistribution role of the estate tax is important and the optimal estate tax literature takes this seriously in various model settings.

Another and often overlooked role of the estate tax is its role as an annuity especially when private actuarially-fair annuity markets are thin. The actuarially-fair annuities provide insurance against longevity risks. By pooling premiums and only paying the survivors, an annuity regime provides those who survive higher than market returns and improve ex-ante welfare. Kopczuk [2003] studies the annuity role of estate taxation in a delicate model and makes the following claim: If there are no actuarially fair annuity markets, it might be a good idea to raise all or part of tax revenue in the form of an estate tax. Estate taxation can bring about a transfer from the short-lived to the other individuals. A risk neutral government, then, can transfer resources between different states of the world at actuarially fair rates without loss in revenue. Hence, estate taxation can substitute for private annuity markets, and, even social security.

Recently, Caliendo et al. [2014] show that conventional wisdom regarding the insurance role of

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1For instance, Farhi and Werning [2010] present a model with altruistic parents and heterogeneous productivity and show that estate tax should be progressive and marginal estate tax should be negative (saving subsidy). Piketty and Saez [2013] derive optimal inheritance tax formulas that capture the key equality-efficiency trade-off. In contrast to Farhi and Werning [2010], they find that the optimal tax rate is positive and quantitatively large if the elasticity of bequests to the tax rate is low, the concentration of bequest is high, and society cares those who receive little inheritance relatively more. Cremer and Pestieau [2001] reach a different conclusion in a two period economy finding marginal tax rates may be regressive and positive under some circumstances. There is also literature that conducts a positive analysis of estate taxation in large scale quantitative models. For instance, Cagetti and Nardi [2009] study the effects of abolishing estate taxation in a quantitative model that generates the observed wealth inequality along with other characteristics of the US economy. They find that abolishing estate tax would not generate a large increase in inequality. All these studies investigate the distributional features of inheritance tax while ignoring the possible annuity benefits provided by estate taxation.
a fully funded social security system is not correct. When the fundamental cost - social security crowds out the bequests that households leave (and receive) in general equilibrium - is taken into account, any welfare gains from participating in a public annuity pool with an above-market return will be canceled out. Hence, they conclude that social security is not a substitute for annuity markets. Unlike a private annuity market actuarially fair fully funded social security fails to provide any welfare gains at all even when households have no other way to insure against longevity risks. Our paper is closer to Caliendo et al. [2014] regarding methodology but focuses on a different type of tax-transfer instrument, which is the estate tax.

We conduct our analysis by using a simple two-period overlapping generations model. Each agent is subject to survival risk, the only risk in the model, and can live up to two periods. Agents work and receive labor income in the first period, and they retire and live on savings in the second period. The agents have bequests motive and will choose the amount of bequests left behind in both periods. To focus on the annuity role of the estate tax, we assume no population growth and zero net interest rate. In order to make the comparison easier, we first generate the results in Kopczuk [2003] ignoring the bequest income and show that the estate taxes can generate the first best solution. Afterwards, we proceed to account for the bequest income that a household is supposed to receive. This simple but realistic extension leads to a conclusion that lump sum estate taxation does not have an annuity role anymore. To generate a bridge between two distinct sets of literature, we extend the model with a fully funded social security. We show that the social security program has only wealth effect and cannot affect the inter-temporal choice. The only way to rectify the inter-temporal choice is to use the estate tax. Yet, the estate tax still fails to play the annuity role: the only form of the lump-sum and estate tax regime to restore the Laissez Faire allocations is the amounts of lump-sum and estate taxes are sensitive to the timing and the size of bequest income. These characterizations are different from any existing tax regime in the real world.

The rest of the paper is structured as follows. Section 2 introduces main model. Section 3 extends the model by including some extensions such as consumption tax and exogenous labor supply. Section 4 concludes.

2 The Model

2.1 The Benchmark Model

In this section we first reproduce the conviction from Kopczuk [2003] by not including the bequest income. Then we proceed to account for the bequest income that a household is supposed to receive. Thanks to the bequest income, the key result that a lump sum tax can implement the first best allocation when the annuity market is missing is no longer valid.

Suppose an agent can live up to two periods. The survival probability to the second period
is $p \in (0, 1)$ where as for the first period it is unity. The government requires a per capital tax revenue $R$. Throughout the paper we assume that there are actuarially fair annuities, however, in Section 2.1.1, we show that our results also hold when annuities are actuarially not-fair. The agent receives labor income $y$ when young, pays a lump sum tax $T$, and solves the following maximization problem over the life time:

$$\max_{c_1, c_2, B_1, B_2, a, k} u(c_1) + (1 - p)v(B_1) + pu(c_2) + pv(B_2),$$

subject to,

$$c_1 + a + k = y - T,$$

$$B_1 = k,$$

$$c_2 + B_2 = \frac{a}{p} + k.$$

The utility function $u$ and $v$ are both strictly increasing and strictly concave. The constraint (2) implies that the agent can choose both annuity ($a$) and storage ($k$), with the latter for the purpose of bequest. The net return on storage is assumed to be zero. $B_1$ and $B_2$ represent the bequests to be left by the agents.

Rewriting the life time budget constraint gives us the following

$$c_1 + (1 - p)B_1 + pc_2 + pB_2 = y - T.$$  

It’s obvious that the government wants to implement tax $T = R$ in order to satisfy the revenue requirement, and that the optimal allocations satisfy: $u'(c_1) = u'(c_2) = v'(B_1) = v'(B_2)$. We denote the optimal allocations by $(c_1^*, c_2^*, B_1^*, B_2^*)$. Given $u$ and $v$ are strictly concave, we can characterize: $c_1^* = c_2^* = c^*$, $B_1 = B_2 = B^*$ and $(1 + p)c^* + B^* = y - R$. This summarizes the characterizations of the first best allocations in the economy when actuarially fair annuity markets exist.

Now consider the situation where the annuity markets are completely missing. Instead, the agent only has an access to a storage technology. Without government intervention, it’s known that the first best allocations won’t be achieved. The following tax regime is supposed to implement the first best: the agent pays a lump sum tax $T$ when young, and estate tax $E$ if she dies in the first period. The estate tax is equal to zero in the second period. We still assume away the bequest income.
In this case, the budget constraints that the agent faces include:

\[ c_1 + k = y - T, \]  
\[ B_1 = k - E, \]  
\[ c_2 + B_2 = k. \]  
\[ (6) \]
\[ (7) \]
\[ (8) \]

To implement the first best allocation, we need to set \(^2\)

\[ E = c^*, \]  
\[ T = y - 2c^* - B^*. \]  
\[ (9) \]
\[ (10) \]

In order to demonstrate how this tax scheme works, we need to show that the first best allocations are not only feasible, but also satisfy the first order conditions under such tax arrangement. Inserting the tax \( T \) and \( E \) to the budget constraints, we can have the following equations:

\[ c_1 = y - (y - 2c^* - B^*) - k, \quad B_1 = k - c^* \text{, and} \quad c_2 = k - B_2. \]

It’s straightforward to show that as long as \( k = c^* + B^* \), the first best allocations \( (c_1 = c_2 = c^* \text{ and } B_1 = B_2 = B^*) \) satisfy the budget.

On the other hand, the first order conditions required for the following optimization problem

\[ u(y - T - k) + (1 - p)v(k - E) + pu(k - B_2) + pv(B_2) \]

are:

\[ -u'(c_1) + (1 - p)v'(B_1) + pu'(c_2) = 0, \]  
\[ -pu'(c_2) + pv'(B_2) = 0. \]  
\[ (11) \]
\[ (12) \]

As in above, the first best allocations again satisfy them.

To further verify the taxes, we notice that the tax revenue needs to be satisfied:

\[ T + (1 - p)E = R. \]

By inserting the amount of taxes, we get \( y - R = (1 + p)c^* + B^* \), which is the same as the one shown in the first best allocations. In this economy when annuity markets are missing, the allocations demonstrate not only smoothness across time but also equality across agents thanks to the lump sum estate taxation. Thus the estate tax has the role of providing annuities. We have to emphasize that this trait, however, is achieved when bequest income is not considered.

Departing from the above analysis where bequest income that an agent can receive over his lifetime is not accounted for, the following analysis will show that once the bequest income is included,

\(^2\)We only consider the interior solution here.
the timing and size of the bequest income received by a household will have wealth effect. Since different timing and size of bequest income generate wealth heterogeneity in the model, there will be inequalities in both consumption and bequest that are left behind among households.

Since households do leave behind bequests to their offsprings, the bequest income should be included instead of being ignored in the life time budget constraint. In order to establish the benchmark case, we restart with the economy where there are actuarially fair annuities. Now, there will be at least two types of agents\(^3\) in the economy: those who receive bequest when young and otherwise. For the first type who receive bequests when they are young (type I), the budget constraints are:

\[
\begin{align*}
c_1 + a + k &= y - T + Be_1, \\
B_1 &= k, \\
c_2 + B_2 &= \frac{a}{p} + k.
\end{align*}
\]

(13) \hspace{2cm} (14) \hspace{2cm} (15)

For the second type who receive bequest in the second period (type II), the budget constraints are:

\[
\begin{align*}
c_1 + a + k &= y - T, \\
B_1 &= k, \\
c_2 + B_2 &= \frac{a}{p} + k + Be_2.
\end{align*}
\]

(16) \hspace{2cm} (17) \hspace{2cm} (18)

In these constraints, \(Be_1\) or \(Be_2\) denotes the bequest income received in either period one (in this case parents die early) or period two. The life time budget constraints for type I and II agents therefore are

\[
\begin{align*}
c_1 + pc_2 + (1 - p)B_1 + pB_2 &= y - T + Be_1, \\
c_1 + pc_2 + (1 - p)B_1 + pB_2 &= y - T + pBe_2,
\end{align*}
\]

(19) \hspace{2cm} (20)

respectively.

We can now show the first main result of this paper: once the bequest income is accounted for instead of being ignored, we can’t generate the allocations where \(c_1 = c_2 = c^*\) and \(B_1 = B_2 = B^*\) that hold for different types of agents even when there are actuarially fair annuities. The proof is simple. By contradiction, suppose instead we do have \(c_1 = c_2 = c^*\) and \(B_1 = B_2 = B^*\), and there is no inequality across all agents. Then we must also have \(Be_1 = Be_2 = B^*\) as well. However, this contradicts the fact both budget constraints in (19) and (20) have to hold at the same time. This proposition is summarized as follows.

\(^3\)As the analysis shown later, there will actually be infinitely many different types. But to prove our point that there will be inequality among agents, we only need two different types here.
Proposition 1. With actuarially fair annuities and with bequest motive, each household chooses \( c_1 = c_2 \) and \( B_1 = B_2 \). However, the differential timing and the amount of bequest income received affects the life time wealth. This wealth effect hence renders inequalities in consumption and bequest in the Laissez Faire economy. Thus once the bequest income is determined by a uniform estate tax, it no longer bears the annuity role.

To analytically further illustrate the inequality, we assume that \( u(.) = v(.) \). With this specification, it’s straightforward to show that \( c_1 = c_2 = B_1 = B_2 \) at the optimum. Thus, there exists smooth consumption and bequest. The level and the timing of the bequest income agents receive, however, will be different. Let’s sort the bequest incomes from the lowest and denote them by \( Be(1), Be(2), \ldots \).

Suppose the household who received \( Be(1) \) when she is old will also leave behind \( Be(1) \). Her budget constraint is:

\[
    c_1 + (1 - p)B_1 + pc_2 + pB_2 = y - R + Be(1),
\]

Combined with the first order conditions that \( c_1 = c_2 = B_1 = B_2 \), and denoting the allocations of this type by 1, we thus have \( c_1(1) = c_2(1) = B_1(1) = B_2(1) = Be(1) = \frac{y-R}{2} \). Those households who receive \( Be(1) \) when young will be better off than the type II households, and their life time budget constraint and allocations are:

\[
    c_1 + (1 - p)B_1 + pc_2 + pB_2 = y - T + pBe(1),
\]

\[
    c_1 = c_2 = B_1 = B_2 = \frac{3}{2(2 + p)}(y - R).
\]

To continue, those who receive bequest income \( \frac{3(y-R)}{2(2+p)} \) when old have the following life time budget constraint and allocations:

\[
    c_1 + (1 - p)B_1 + pc_2 + pB_2 = y - R + p\frac{3(y-R)}{2(2 + p)},
\]

\[
    c_1 = c_2 = B_1 = B_2 = \frac{4 + 5p}{2(2 + p)^2}(y - R).
\]

Those who receive bequest income \( \frac{3(y-R)}{2(2+p)} \) when young have:

\[
    c_1 + (1 - p)B_1 + pc_2 + pB_2 = y - T + p\frac{3(y-R)}{2(2 + p)},
\]

\[
    c_1 = c_2 = B_1 = B_2 = \frac{7 + 2p}{2(2 + p)^2}(y - R).
\]

Since our main goal is to illustrate the existence of inequality created by bequest income received, the above example is enough.\(^4\)

\(^4\)A detail exercise showing the path of this when time \( t \to \infty \) has been put into the Appendix.
Now we show that the actual distribution of bequest income, and hence the induced level of consumption and bequest left behind, is indeed not stationary in an economy with actuarially fair annuities and without any government intervention, where the only uncertainty is survival risks. We want to demonstrate that, the distribution tends to be fanning out over time, and that on top of survival risks, the unequal bequest incomes that parents leave behind for their children generate inequality behind the veil of ignorance. It’s noticeable that even with the same level of bequest income, the wealth effect is different depending on the timing of bequest income. In addition, the bequest income does not alter inter-temporal choices (Euler equations). Actually, if we assume that both $u$ and $v$ are strictly increasing and strictly concave functions, then we have $c_1 = c_2$ and $B_1 = B_2$ for all agents.

In this Laissez Faire economy, the distribution of bequest income is not stationary, hence the distributions on the levels of consumption and bequests that’s left behind are also non-stationary. The proof is by contradiction. Suppose there is a stationary distribution after $n$ generations. Assume that the lowest level of bequest is $Be_1$, and the mass of agents leaving behind $Be_1$ is $b_1$. The mass of agents that leaving $Be_1$ early is $(1 - S)b_1$, while those who leave $Be_1$ late has mass $Sb_1$. Suppose $Be_1$ is also the lowest level of bequest left behind among generation $n+1$. In order to have a stationary distribution, the mass of agents who leave $Be_1$ must be the same as the previous generation. However, note that the $Sb_1$ households who receive $Be_1$ late will have less lifetime wealth than $(1 - S)b_1$ households who receive them early. Hence, the only possible level of mass to leave $Be_1$ is $Sb_1$ which is strictly less than $b_1$, the mass of agents leaving $Be_1$ in the previous generation. Hence the proof.

Next, we argue that after many generations, the lowest level of bequest income is strictly above 0. This is also easy to prove by using contradiction. With strong bequest motive, even those who receive 0 bequest income will leave some bequest behind. By combining the above arguments, we can show that the distribution of bequest income is non-stationary. Neither are the distribution of level of consumption. The above discussions have been summarized in the following proposition:

**Proposition 2.** Due to the differential timing of the bequest received, the distribution of bequest income is non-stationary in nature with the lowest level of bequest strictly positive.

We then study the economy where the private annuity markets are completely missing. In this environment, the estate tax is believed to be able to provide the annuity role. We will examine the possibility that estate tax regime can reproduce the distribution of allocations, including the inequality, in the economy with annuities. We find out that in order to implement the first best allocations, where inequality exists, the estate tax needs to be contingent on the timing and size of the bequest income that received by households. For instance, suppose at the steady state, a household receives bequest income $Be(n)$ when young, and another household receives the same
level of bequest income when old. Their budget conditions are:

\[ c_1 + k = y - T + Be(n), \]  
\[ B_1 = k - E_1, \]  
\[ c_2 + B_2 = k; \]  

and

\[ c_1 + k = y - T, \]  
\[ B_1 = k, \]  
\[ c_2 + B_2 = k - E_2 + Be(n), \]

respectively. Instead of a single tax \( E \), we need to bring two different estate taxes for two different types. More specifically, \( E_1 \) is the estate tax recommended for type I agent and similarly \( E_2 \) for type II. To implement the first best allocations, denote the first type by \( n_1 \), and the second by \( n_2 \), along with the allocations, we want:

\[ E_1(n_1) = c^*(n_1), \]  
\[ T(n_1) = y - 2c^*(n_1) - B^*(n_1) + Be(n), \]  
\[ E_2(n_2) = c^*(n_2), \]  
\[ T(n_2) = y - 2c^*(n_2) - B^*(n_2) + pBe(n). \]

The levels of estate and lump sum taxes are both dependent on the size and the timing of the bequest income received by the household. We want to emphasize that although this implementation can restore the first best allocations, it’s not similar to any estate tax regimes prevailing in the world. Hence the following proposition.

**Proposition 3.** Uniform estate tax and lump sum tax can not generate the first best allocations, where inequality exists. Instead, both the estate tax and the lump sum tax need to be contingent on timing and size of the bequest income received by households.

### 2.1.1 With Actuarially-Not-Fair-Annuity

Assume that the annuity guarantees higher return \((R_a)\) than the market return \((R)\). Following Lockwood [2012], we assume that \( R_a = (1 - \lambda) \frac{R}{p} \) where \( \lambda \geq 0 \) is the load, that is, the percentage by which premiums exceed expected discounted benefits. Further, \( \lambda = 0 \) represents the actuarially fair case. Both the groups want to maximize

\[ \max_{c_1, c_2, B_1, B_2, a, k} u(c_1) + (1 - p)v(B_1) + pu(c_2) + pv(B_2), \]
where the budget constraint of type I is

\[ c_1 + a + k = y - T + Be_1, \quad (38) \]
\[ B_1 = k, \quad (39) \]
\[ c_2 + B_2 = a (1 - \lambda) \frac{R}{p} + k. \quad (40) \]

The budget constraint of type II is

\[ c_1 + a + k = y - T, \quad (41) \]
\[ B_1 = k, \quad (42) \]
\[ c_2 + B_2 = a (1 - \lambda) \frac{R}{p} + k + Be_2. \quad (43) \]

If we rewrite the above budget constraints, we get

\[ R (1 - \lambda) c_1 + pc_2 + [R (1 - \lambda) - p] B_1 + pB_2 = R (1 - \lambda) (y - T + Be_1), \quad (44) \]
\[ R (1 - \lambda) c_1 + pc_2 + [R (1 - \lambda) - p] B_1 + pB_2 = R (1 - \lambda) (y - T) + pBe_2, \quad (45) \]

for type I and II agents respectively. First of all, to have the coefficient of \( B_1 \) strictly positive, we need a condition \( R (1 - \lambda) - p > 0 \) which imposes a further restriction on the inequality \( R_a > R \). Note that \( R = 1 \) implies that the loading should be less than the probability of death.

Now we show that a flat tax rate does not work for an economy with annuity that is not actually fair. Note that if we want to have \( c_1 = c_2 = c^* \) and \( B_1 = B_2 = B^* \), and there is no inequality across all agents, we must have \( Be_1 = Be_2 = B^* \) as well. It can be verified from the above two budget constraints that it is possible only when \( R_a = 1 \). This can be shown easily by comparing the above two budget constraints. The LHSs are the same. If we equate the RHSs, we get \( Be_2/Be_1 = R (1 - \lambda)/p \) which, in fact, is exactly equal to \( R_a \) by construction. Since we need \( Be_1 = Be_2 = B^* \) at the steady state, the requirement \( 1 = B^*/B^* = R (1 - \lambda)/p = R_a \) should hold. However this contradicts the fact that \( R_a = (1 - \lambda) \frac{R}{p} > 1 \) for any load \( \lambda \) so that we can have a strictly positive bequest at the equilibrium. Therefore, there does not exist any load \( \lambda \) for which a flat tax rate can be justified. Combining this with the above proposition we can have the following:

**Proposition 4.** For both actuarially fair as well as not fair annuities and with bequest motive, because of differential longevity and hence receiving bequest at different time point in life, endogenous inequality is created in consumption and bequest in the Laissez Faire economy.
2.2 Model with Social Security

The reasons that social security is included in this discussion are as follows: 1) social security benefits are like annuity income and 2) estate tax liabilities can implicitly provide annuity income, hence “estate tax is social security for the rich.” Caliendo et al. [2014] show that social security has no meaningful annuity role once the bequest income is endogenized in the model. In the previous section, we show that if bequest income is accounted for in the model, the estate tax does not have annuity role as well. This section reinforces the previous two results by including the social security into our model. We show that in an environment where private annuity markets are missing and social security is introduced, neither social security nor estate tax can provide meaningful annuities.

Suppose the government still aims at imposing per capital tax revenue $R$. On top of that, the government is running a balanced budget social security program: agent pays social security tax at the rate $\tau$ when young and receives social security benefits $b$ when old. Also, agents pay a lump sum tax $T$. We again start with ignoring the bequest income and assuming that annuity is present. The agent now solves the following maximization problem:

$$
\max_{c_1,c_2,B_1,B_2,a,k} u(c_1) + (1 - p)v(B_1) + pu(c_2) + pv(B_2),
$$

subject to,

$$
c_1 + a + k = y(1 - \tau) - T, \quad (47)
$$

$$
B_1 = k, \quad (48)
$$

$$
c_2 + B_2 = \frac{a}{p} + k + b. \quad (49)
$$

The self-balanced social security program has a budget constraint

$$
y\tau = pb,
$$

or simply $b = y\tau/p$. By inserting social security benefits $b$ into the budget, the life time budget constraint of the agent can be written as:

$$
c_1 + (1 - p)B_1 + pc_2 + pB_2 = y - T.
$$

This budget constraint is exactly the same as when there is no social security. Hence we confirm the well-known result that when private annuity markets exist, social security does not improve welfare. Next we assume away the annuity markets, where one may think that social security and estate tax can work together to restore the first best allocations by providing meaningful annuities.
With the same objective function, the household’s budget conditions are:

\[ c_1 + k = y(1 - \tau) - T, \]  
\[ B_1 = k - E, \]  
\[ c_2 + B_2 = k + b. \]  

(50)  
(51)  
(52)

As mentioned above, the size of the social security program is \( b = y\tau/p \). In order to restore the first order allocations, we can implement the following tax regime:

\[ E = c^* - b, \]  
\[ T = y - 2c^* - B^* - \tau y + b. \]  

(53)  
(54)

Note that with the social security, the estate tax decreases since the private saving, \( k \) is reduced to \( B^* + c^* - b \). In the absence of social security, \( k = B^* + c^* \). On the other hand, the lump sum tax increases to maintain the tax revenue and the budget i.e. \( b - \tau y = \frac{y\tau}{p} - \tau y > 0 \). The total revenue can thus still meet the requirement, \( R \):

\[ T + (1 - p)E = y - 2c^* - B^* - \tau y + b + (1 - p)(c^* - b) \]
\[ = y - 2c^* - B^* + (1 - p)c^* = y - (1 + p)c^* - B^* = R. \]

The conclusion is that with the combination of social security, estate tax, and lump sum tax, the first best allocations can be implemented when the private annuity markets are missing. The results are not surprising in that with one more government tax and transfer program in place (the social security), the government capabilities of arranging allocations can only grow.

However, once we incorporate bequest income into the model, we demonstrate the following points: 1) with both actuarially fair annuity markets and social security program, there will be inequality in bequest income received by households and hence inequalities in consumption and bequest left behind; 2) with both actuarially fair annuity markets and social security program, a uniform lump sum tax and estate scheme can’t generate equal allocations; and 3) without private annuities, the social security program combined with uniform estate tax program can not deliver the first best allocations.

To show the first point, we only need to demonstrate that a constant level of bequest income received at different timing can have different wealth effects. The differential wealth effects will generate (at least) two different types of agents with different optimal consumption and bequest
choices. The budgets constraints of agents who receive bequest income when young are

\[ c_1 + a + k = y(1 - \tau) + Be - T, \quad (55) \]

\[ B_1 = k - E, \quad (56) \]

\[ c_2 + B_2 = \frac{a}{p} + k + b. \quad (57) \]

The budget constraints of a household that receives bequest income when old are:

\[ c_1 + a + k = y(1 - \tau) - T, \quad (58) \]

\[ B_1 = k - E, \quad (59) \]

\[ c_2 + B_2 = \frac{a}{p} + k + Be + b. \quad (60) \]

It’s obvious that these two types of agents have different life time budget constraints. Hence they will have different levels of consumption and bequest left behind. Similar to what we established in Proposition 1, there will be inequality in the economy when there are public social security program and private annuity markets.

We then show that a uniform lump sum tax and estate tax scheme can’t generate equal allocations. Suppose otherwise that we do have equal allocations, where \( c_1 = c_2 \) and \( B_1 = B_2 \) across all agents. Hence, the bequest income: \( Be_1 = Be_2 \) for these households and that these allocations are achieved by uniform lump sum tax, \( T \), and estate tax, \( E \) across all agents. Such assumptions, however, would render a contradiction. Since the consumption and bequest allocations are the same across time and across agents, so will be the bequest income. Slightly abusing the notation, assume \( c_1 = c_2 = c^* \) and \( B_1 = B_2 = Be_1 = Be_2 = B^* \) for all agents. Denote the savings, \( a \) and \( k \) for types 1 and 2 as \( a(1), k(1) \) and \( a(2), k(2) \), respectively. With the same taxes, we have the following based on the first period budget:

\[ a(1) + k(1) = B^* + a(2) + k(2). \]

From the condition for bequest, \( B_1 \), we should have:

\[ k(1) = k(2). \]

Last, from the second period budget constraint across those two types of agents:

\[ \frac{a(1)}{p} + k(1) = \frac{a(2)}{p} + k(2) + B^*. \]

Hence there will be contradiction. The above result is summarized in the following proposition.

**Proposition 5.** Despite the existence of the actuarially fair annuity markets, once bequest income
is accounted for and determined in the model, uniform social security and estate tax program cannot deliver equal allocations across agents. Instead, agents’ bequest incomes differ because of the size and timing they are received. The households will in turn leave behind different bequest income.

Last we turn to the environment where actuarially fair annuity markets are completely missing. Bequest income is accounted for and determined in the model, with social security program and estate tax programs are in place. Without the ability to buy annuities, whose returns are higher than storage technology, one would think that social security and estate tax may act like annuities for different reasons. Social security pays benefits as long as the retirees survive. Compared to payroll taxes, the social security benefits actually deliver the same return as annuities. As Caliendo et al. (2014) show that once the social security is introduced, private saving would be dampened implying lower bequest income. This wealth effect neutralizes the gain from social security’s provision of public annuities. Hence the social security does not provide meaningful annuities. Kopczuk (2003) shows that combined with lump sum taxes, the estate tax can defer the tax liabilities to the end of one’s life-span. By being in favor of the survivors, this delayed tax regime can act like annuities. Therefore, the estate tax can restore the first best allocations. However, we have demonstrated that even the “first best allocations” now include inequalities across agents due to taken bequest income into account. The root cause of the problem is that out of strong bequest motive, households chooses to leave behind equal amount of bequest no matter how long they live. Even though the amount of bequest income a household leaves behind is the same, the timing of bequest income received matters from the off-springs’ perspective. Earlier bequest income might be preferred to the later bequest income since the agent might not live long enough to receive that bequest income. The differential timing of the bequest income received thus creates differential wealth effects. These heterogeneous life time wealth in turn imply that households consume at different levels and leave different levels of bequest income. The differential timing and size of bequest income hence create inequalities despite the same labor income among all agents.

If the government’s goal is to neutralize the differential wealth effects and implement equal allocations among all households, then social security program alone can’t accomplish the task. After all, the social security program is egalitarian by taxing households equally and doling out benefits conditional on survival. The only instrument for equal allocations is thus lump sum and estate taxes. However, it will be the redistribution role, not the annuity role, that can restore equal allocations. Notice that the redistribution role is not the scope of this study.

If the government’s goal is to implement the Laissez Faire allocations where inequality exists, then similar to our previous study where social security was not introduced, the government should impose a lump sum and estate tax scheme that depends on the size and the timing of the bequest received by the household. As we have done previously, we denote the level of possible bequest income, sorted from the lowest, by $Be(1), Be(2), \cdots$. The household that receives bequest income,
Be(n), when young has the following budget conditions:

\[
\begin{align*}
    c_1 + k &= y(1 - \tau) + Be(n) - T(n_1), \\
    B_1 &= k - E(n_1), \\
    c_2 + B_2 &= k + b.
\end{align*}
\]  

(61)  
(62)  
(63)

The budget constraints of a household that receives bequest income, Be(n), when old are:

\[
\begin{align*}
    c_1 + k &= y(1 - \tau) + T(n_2), \\
    B_1 &= k - E(n_2), \\
    c_2 + B_2 &= k + b + Be(n).
\end{align*}
\]  

(64)  
(65)  
(66)

The design of the lump-sum and estate tax is:

\[
\begin{align*}
    E(n_1) &= c^*(n_1) - b, \\
    T(n_1) &= y - 2c^*(n_1) - B^*(n_1) + Be(n) + b - \tau y, \\
    E(n_2) &= c^*(n_2) - b, \\
    T(n_2) &= y - 2c^*(n_2) - B^*(n_2) + pBe(n) + b - \tau y.
\end{align*}
\]  

(67)  
(68)  
(69)  
(70)

Since the social security program has only wealth effect and cannot affect the inter-temporal choice, or Euler equation, the only way to rectify the inter-temporal choice is through the estate tax. However, this tax scheme has its limitation: though consumption and bequests are smooth across time for each individual household, the tax burden is not. Actually, both the lump sum and estate tax depend on the timing and the size of bequest income received by each household. Hence unlike uniform social security regime, the estate tax regime is not egalitarian. This system is not reminiscent to any current estate system, nor advocated by the authors. This result is similar to Proposition 3.

### 3 Further Discussions

In this section we extend the model to consider 1) consumption tax and 2) labor supply decision. We demonstrate that even combined with another tax instrument, it’s hard to argue that estate tax can provide meaningful annuities. Further, we show that incorporating labor supply decision does not change our main result.
3.1 Consumption Tax

In this section, we ask whether estate tax can provide meaningful annuities when government collects both consumption and estate taxes. In reality, both estate and consumption taxes are likely to co-exist. One may ask can the combination of these two programs provide annuities. The answer is again, no. We only need to show that if there are just two types of agent regarding bequest income they receive, a uniform estate tax program combined with consumption tax can’t generate the first best allocations. We assume that the government imposes consumption taxes ($t_1$ and $t_2$ in two periods, respectively) and then see whether a lump sum estate tax, in the presence of these consumption taxes, can restore the first best allocations. When $t_1$ and $t_2$ are assumed to be lump-sum, the budget constraints for type I agents are:

$$c_1 + t_1 + a + k = y + Be_1,$$  \hspace{1cm} (71)

$$B_1 = k,$$ \hspace{1cm} (72)

$$c_2 + t_2 + B_2 = \frac{a}{p} + k.$$ \hspace{1cm} (73)

The type II agent receives bequest in the second period. His budget constraints are given as follows:

$$c_1 + t_1 + a + k = y,$$ \hspace{1cm} (74)

$$B_1 = k,$$ \hspace{1cm} (75)

$$c_2 + t_2 + B_2 = \frac{a}{p} + k + Be_2.$$ \hspace{1cm} (76)

The life time budget constraints for type I and II agents therefore are:

$$c_1 + pc_2 + (1 - p)B_1 + pB_2 = y + Be_1 - (t_1 + pt_2),$$ \hspace{1cm} (77)

$$c_1 + pc_2 + (1 - p)B_1 + pB_2 = y + pBe_2 - (t_1 + pt_2).$$ \hspace{1cm} (78)

Given the budget constraints differ for two different types of agents, we reach the same conclusion. It can also be verified that instead of a lump-sum tax, if we have a proportional tax, there is no change in conclusion. If that proportional tax rate on consumption be $t$ for both periods it is straightforward to check that the budget constraints for type I and type II agents are

$$(1 + t)(c_1 + pc_2) + (1 - p)B_1 + pB_2 = y + Be_1,$$ \hspace{1cm} (79)

$$(1 + t)(c_1 + pc_2) + (1 - p)B_1 + pB_2 = y + pBe_2,$$ \hspace{1cm} (80)

respectively and by using the same logic, we can conclude the same.
3.2 Endogenous Labor Supply

In this section, we extend our model by adding labor supply decision. Even when there was no heterogeneity in labor income, inequalities in consumption and bequest can be generated. Since there is a strong bequest motive, households choose bequest to leave for their offsprings. With the bequest income taken into account, there is wealth effect in the household’s life-time budget condition. The same level of bequest income means differently to those households who receive the income at different times. Those who receive them early receive de facto higher incomes than those who receive them late. Bequest income received late in life should be discounted by the survival probability because the receiver might not survive to get that income. Hence variations in timing of bequest income received generates further inequalities in the size of bequest as well as consumption allocations.

Here we want to emphasize two important channels. First, the differential wealth effects generated by the variations in timing of bequest income can influence labor decisions. We expect those who receive higher bequest income or receive bequest income early will work less. Second, the differential hours worked will render income effect, and in turn affect consumption allocations and bequest left for the children. We expect those who work less and thus receive less labor income tend to consume and bequeath less than otherwise. These two effects reinforce each other and can potentially reduce the level of inequality.

Assume household chooses hours worked and hence receive labor income in the first period. In the second period, the household will retire and stop working. The retirement decision is exogenous. The labor income when young is \( y = wl \), where \( w \) is the market wage rate and \( l \) is the unit of labor supplied. We assume that the market wage rate is the same for everyone. We follow the setup from the previous section, restrict ourselves to actuarially fair annuity markets only. The household chooses saving in annuity and storage technology, labor supply, as well as consumption and bequests in both periods in order to maximize the following life time utility:

\[
\max_{c_1, c_2, l, a, k, B_1, B_2} \quad u(c_1) + h(1 - l) + (1 - p)v(B_1) + pu(c_2) + pu(B_2),
\]

where \( h \) is a strictly increasing and strictly concave function. We normalize the time endowment as 1. If the agent receives bequest early, the constraints are

\[
\begin{align*}
c_1 + a + k &= wl(1 - \tau) + Be - T, \\
B_1 &= k, \\
c_2 + B_2 &= \frac{a}{p} + k + b,
\end{align*}
\]

and therefore the life time budget constraint is:

\[
c_1 + pc_2 + (1 - p)B_1 + pB_2 = wl(1 - \tau) + Be + pb - T.
\]
Suppose the social security is fully funded and hence \(wl\tau = pb\) for each agent. The social security then does not affect the lifetime budget constraint:

\[
c_1 + pc_2 + (1 - p)B_1 + pB_2 = wl + Be - T.
\]

Instead of receiving the bequests early, if it is received when he is old, the constraints are

\[
c_1 + a + k = wl(1 - \tau) - T, \quad B_1 = k, \quad c_2 + B_2 = \frac{a}{p} + k + b + Be.
\]

Given the same assumption of \(wl\tau = pb\) for each agent, the lifetime budget becomes

\[
c_1 + pc_2 + (1 - p)B_1 + pB_2 = wl(1 - \tau) + pBe + pb - T.
\]

The first order conditions with respect to consumption, bequest left and leisure levels are as follows

\[
u'(c_1) = u'(c_2) \quad \text{and} \quad v'(B_1) = v'(B_2) \\
u'(c_1)w(1 - \tau) = h'(1 - l).
\]

It is clear from the above that the same \(c^*\), \(B^*\) and \(l^*\) can’t be optimal for both optimizations.

To implement the first best allocations in the absence of annuity, we have the following budget constraints for type I and type II households respectively -

\[
c_1 + k = wl(1 - \tau) + Be, \quad B_1 = k - E, \quad c_2 + B_2 = k + b
\]

and

\[
c_1 + k = wl(1 - \tau), \quad B_1 = k - E, \quad c_2 + B_2 = k + b + Be.
\]
In order to implement the first best, we then need:

\[
E_1(n_1) = c^*(n_1) - \tau w l^*(n_1) = c^*(n_1) - b(n_1),
\]
(93)

\[
T(n_1) = y - 2c^*(n_1) - B^*(n_1) + B e(n),
\]
(94)

\[
E_2(n_2) = c^*(n_2) - \tau w l^*(n_2) = c^*(n_2) - b(n_2),
\]
(95)

\[
T(n_2) = y - 2c^*(n_2) - B^*(n_2) + pB e(n).
\]
(96)

As in the earlier cases, again we show that the level of estate and the lump sum taxes are both dependent on the size and the timing of the bequest income received by the household. Also a point to note here is that unlike the previous case where labor supply is exogenous, \( b \) is not the same for two different types of agents.

4 Conclusion

This paper revisits the annuity role of estate tax. We show that previous conclusions that a uniform lump-sum estate tax could implicitly provide annuity income were reached by not including the bequest income that household receives. However, while agents leave behind bequest, they should also receive bequest income from their parents. This differential timing and sizes of bequest income generate unequal wealth effects even with actuarially fair annuity markets. Moreover, the distributions of wealth and consumption are not stationary over time even in the first best allocations. To restore the first best allocations, the government has to adopt an estate tax regime that is no longer uniform. Thus once the bequest income is determined by a uniform estate tax, it no longer bears the annuity role. The result is robust to many different specifications of the model. Our paper once again manifests the importance of accounting for and tracing the bequest income received by households in any model that aims to discuss intergenerational transfers and related policies.
Appendix

Let us focus on all the cases that can appear in the exercise we have presented in Section 2.1. There we have started with the assumption that the agents receive bequests when they are old, that is, their parents survive for two periods. We start with the other possibility where agents receive bequests income when they are young. We then extend the derivations of the case that have been presented in Section 2.1.

Along the extreme path of no survival (NS) where all the agents survive for one period till infinity, we have \( c_1 = c_2 = B_1 = B_2 = (y - R) / (1 + p) \) when \( t \to \infty \). If \( Be(1) \) be the bequest income received when young, her budget constraint is:

\[
c_1 + (1 - p)B_1 + pc_2 + pB_2 = y - T + Be(1).
\]

We denote the allocations of the type \( i = I, II \) agent for generation \( -t \) by \( (G_t; i) \) who is taking the decision in period \( t \). Combined with the first order conditions that \( c_1 = c_2 = B_1 = B_2 \) we have

\[
c_1(G_t; I) = c_2(G_t; I) = B_1(G_t; I) = B_2(G_t; I) = Be(1) = a(y - R) \text{ where } a = (1 + p)^{-1}.
\]

Let us now check the allocations in period \( t + 1 \) for \( G_{t+1} \) generations. If she receives bequest income of \( (y - R) / (1 + p) \) when old, she will have the following life time budget and resource allocations:

\[
c_1 + (1 - p)B_1 + pc_2 + pB_2 = y - R + pa(y - R),
\]

\[
c_1(G_{t+1}; II) = c_2(G_{t+1}; II) = B_1(G_{t+1}; II) = B_2(G_{t+1}; II) = \frac{1 + ap}{2 + p}(y - R).
\]

Instead of that, if an agent in generation \( t + 1 \) receives the same bequest income \( (y - R) / (1 + p) \) when young she will have the following life time budget and resource allocations:

\[
c_1 + (1 - p)B_1 + pc_2 + pB_2 = y - T + a(y - R),
\]

\[
c_1(G_{t+1}; I) = c_2(G_{t+1}; I) = B_1(G_{t+1}; I) = B_2(G_{t+1}; I) = \frac{1 + ap}{2 + p}(y - R) = a(y - R).
\]

Therefore the allocations of type \( I \) agents for the generations \( t \) and \( t + 1 \) are the same. Let us continue this analysis for the generation \( t + 2 \). Note that if we again derive the case where parental generations leave \( a(y - R) \) early, we are back to the same tree. Thus let us show the case where \( (G_{t+1}; II) \) agents leave \( \frac{1 + ap}{2 + p}(y - R) \). If an agent in generation \( t + 2 \) receives this bequest when she is young, the allocations will not be different from above, that is,

\[
c_1(G_{t+2}; I) = c_2(G_{t+2}; I) = B_1(G_{t+2}; I) = B_2(G_{t+2}; I) = \frac{1 + ap}{2 + p}(y - R).
\]
However if bequest is received when she is old, her life time budget and allocations will be

\[ c_1 + (1 - p)B_1 + pc_2 + pB_2 = y - T + p \frac{1 + ap}{2 + p} (y - R), \]

\[ c_1(G_{t+1}; I) = c_2(G_{t+1}; I) = B_1(G_{t+1}; I) = B_2(G_{t+1}; I) = \left( 1 + \frac{p}{2 + p} + \left( \frac{p}{2 + p} \right)^2 \right) y - R \frac{1 + ap}{2 + p}. \]

We continue with this derivation for one more generation to derive the values when \( t \to \infty \). We can now safely write that if an agent in generation \( t + 3 \) receives \( \left( 1 + \frac{(1+ap)p}{2+p} \right) (y - R) \) when he is young, the allocations would be the same as their parental generations and therefore

\[ c_1(G_{t+3}; I) = c_2(G_{t+3}; I) = B_1(G_{t+3}; I) = B_2(G_{t+3}; I) = \left( 1 + \frac{(1 + ap)p}{2 + p} \right) \frac{y - R}{2 + p}. \]

However if bequest is received when she is old, we can show that her life time budget and allocations will be

\[ c_1(G_{t+3}; II) = c_2(G_{t+3}; II) = B_1(G_{t+3}; II) = B_2(G_{t+3}; II) = \left( 1 + \frac{p}{2 + p} + \left( \frac{p}{2 + p} \right)^2 + \left( \frac{p}{2 + p} \right)^3 \right) \frac{y - R}{2 + p}. \]

From here it is easy to see the allocations when \( t \to \infty \). The two extreme values of the distributions are given by

\[ c_1(G_\infty, S) = c_2(G_\infty, S) = B_1(G_\infty, S) = B_2(G_\infty, S) = \left( 1 + \frac{p}{2 + p} + \frac{p}{2 + p} \right) \frac{y - R}{2 + p} = \frac{y - R}{2} \]

and

\[ c_1(G_\infty; NS) = c_2(G_\infty; NS) = B_1(G_\infty; NS) = B_2(G_\infty; NS) = \frac{y - R}{1 + p} \]

respectively where \((G_\infty; S)\) and \((G_\infty; NS)\) notations have been used for the two extremes cases - survival (S) for two periods and survival for only one period (NS) of all the generations. It is easy to check that given \( 0 < p < 1 \), \( \frac{y - R}{1 + p} > \frac{y - R}{2} \). This particular path has been presented as the right path (starting with NS) in the diagram.

As we have stated at the beginning of the Appendix, we have started with the assumption that agents receive bequests early in their life, that is, their parents have died early. The left hand path
in the diagram (starting with S) represents the case where the first generation receives the bequest when the agents are old. This is precisely the case that has been presented in the main body of the paper in Section 2.1. First few periods have been calculated in the main text. If we follow the same path that we have just shown above, it can be verified that when $t \to \infty$, the households who have survived two periods (S) throughout till infinity, their allocations converge to $(y - R)/2$ and the path which has no survival (NS) throughout right after S (the left hand path in the diagram) will converge to the allocation $(y - R)/(2 + p)(1 - p)$. This completes our derivations for all the possible cases that can arise in this model. We present the diagram below:
References


