Emotions in Civil Litigation

by

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Emotions in Civil Litigation

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Abstract

In a civil-litigation game, monetary and emotional variables motivate a plaintiff and a defendant simultaneously to exert costly efforts; the emotional variables capture their relational emotions toward each other, and a non-monetary joy of winning. Based on the litigants’ efforts and exogenous relative advantages, a generally-formulated success function gives their probabilities of success. A cost-shifting rule shifts a proportion of the winner’s costs to the loser. In equilibrium, negative relational emotions (but not positive joy of winning) amplify the effects of cost shifting. Negative relational emotions increase the equilibrium relative effort and probability of success of the more advantageous litigant.

Keywords: relative payoffs, non-monetary joy of winning, interdependent preferences, litigation, contest theory

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1 Introduction

A civil lawsuit typically involves a plaintiff who seeks judicial remedies at the expense of a defendant. Civil lawsuits generate private and social benefits, such as enforcing the substantive law, guiding future conduct and deterring future injuries (Shavell 1997). Civil lawsuits also impose enormous costs on the litigants and on the public, such as the costs of hiring lawyers, discovering evidence, and providing judges. Moreover, some litigants incur legal costs that well exceed the monetary value of the subject of the dispute.

Economic analysis of litigation efforts and costs typically employs a contest model with rational and self-interested contestants. However, contest experiments consistently suggest that, rather than being purely self-interested, contestants tend to consider relative and non-monetary payoffs. Litigation models may give rise to misleading predictions and policy recommendations if they neglect well-documented behavioral traits. This shortcoming may undermine our understanding of the civil justice system, which is fundamental to the functioning of a society governed by the rule of law.

A contest model of civil litigation typically specifies two players — a plaintiff and a defendant — who simultaneously exert costly efforts to maximize their respective payoffs. Based on the litigants’ efforts and an exogenous parameter reflecting their (prior) relative advantages, a success function gives their respective (posterior) probabilities of success. The defendant transfers a judgment sum to the plaintiff if and only the plaintiff wins. A cost-shifting rule specifies the extent to which the loser pays the winner’s litigation costs. To our best knowledge, Chen and Rodrigues-Neto (2017) offer the most general contest model of litigation. Further extending their model, we introduce non-monetary and emotional preferences, and reveal their profound implications on (equilibrium) litigation outcomes. In particular, we discover the surprising links between emotions and cost shifting.

In the present Emotional Litigation Game, each litigant acts to maximize an emotional payoff that represents her expectations regarding her monetary outcome, her non-monetary joy of winning and her negative or positive relational emotions toward the other litigant. The joy of winning arises from winning the lawsuit, while negative (respectively, positive) relational emotions arise from...
from harming (benefiting) the other litigant. To capture a great diversity of judicial systems, this game adopts general formulations of the success function, litigation cost function and cost-shifting rule. There exists a unique Nash equilibrium with positive effort levels.

The two different forms of non-monetary considerations — relational emotions and joy of winning — have different implications on the equilibrium properties of the Emotional Litigation Game. Intuitively, a greater joy of winning directly increases the marginal benefits of exerting costly efforts to increase the probability of success; more negative relational emotions generate similar direct effects because the litigant has a heightened desire to harm her adversary. However, unlike changes in the joy of winning, changes in relational emotions have indirect effects in cases where a cost-shifting rule operates to shift some or all of the winner’s costs to the loser. Such a cost-shifting rule creates externalities (in expectations) because, when she chooses her effort level, a litigant expects that with a positive probability some or all of her costs is borne by her adversary. More negative relational emotions indirectly amplify such externalities because a litigant derives a greater value from inflicting expected costs on her adversary. Moreover, more negative relational emotions (or more cost shifting) heighten incentives to exert efforts in an asymmetric manner; the litigant with stronger relative advantages experiences greater increases in incentives to exert efforts, because her expected reward from doing so is greater than the weaker litigant’s. Formalizing these observations, we prove that more negative relational emotions increase the equilibrium relative effort and probability of success of the relatively more advantageous litigant. Except in rare circumstances, more negative relational emotions also increase the litigants’ total litigation costs in equilibrium.

Drastically different normative implications arise from the subtle differences between emotional considerations that are outcome-dependent and those that are relational. Our equilibrium analysis suggests that the presence of relational emotions typically strengthens the cost-shifting rule, while the presence of joy of winning has no such effect. Hence, as section 5 will further elaborate, to understand and optimize cost shifting in civil litigation requires taking into account and responding differently to the two forms of emotions. While both outcome-dependent and relational emotions typically increase costs in litigated cases, only relational emotions interact with the cost-shifting rule.

Unlike most other contest models, the present model gives rise to conclusions that do not depend on the specific functional form of the success function, the degree of homogeneity of the cost function, or the extent of cost shifting. Building upon the axiomatization effort of Chen and Rodrigues-Neto (2017), the present model imposes general and reasonable assumptions on the success function without specifying its functional form. Subsuming oft-used functional forms, these assumptions accommodate judges with very different styles and ways of aggregating the litigants’ (prior) relative advantages and litigation efforts. These assumptions also capture uncertainty.
regarding the identity or decisionmaking style of the judge.\footnote{More precisely, if a finite number of success functions satisfy the present assumptions, then their convex combination also satisfies the present assumptions. Thus these assumptions capture the scenario in which each of the potential judges rules according to a different success function and the litigants have a common prior probability for each judge being assigned to their case. For a more detailed discussion, see Chen and Rodrigues-Neto (2017), subsection 8.1.}

The literature on the economics of civil litigation is vast, and Sanchirico (2012) contains recent surveys of the seminal contributions. Specifying extreme cost-shifting rules that either shift all or none of the winner’s costs to the loser, Braeutigam et al. (1984) and Katz (1987) were among the early proponents of applying contest models to study litigation efforts and costs. In reality, intermediate cost shifting is the norm (Hodges et al. 2010, p. 20), and authors that applied contest models to study intermediate cost shifting include Carbonara et al. (2015), Farmer and Pecorino (2016), Gong and McAfee (2000), Hause (1989), Hyde and Williams (2002), Luppi and Parisi (2012) and Plott (1987), while Baye et al. (2005) and Klemperer (2003) applied auction-theoretic models. More recently, Chen and Rodrigues-Neto (2017) axiomatized the contest model to study intermediate cost shifting under general formulations of the success function and cost function. While the success function in most models is exogenously given, the success functions in Skaperdas and Vaidya’s (2012) litigation model (with no cost shifting) are derived from the inference process of a Bayesian judge. To our best knowledge, the present paper is the first to extend the contest model of litigation to account for emotional preferences over non-monetary and relative outcomes.

We take the unorthodox step to incorporate emotional considerations into a litigation model in order to capture well-documented behavioral traits. As Millner and Pratt (1989) first observed and Dechenaux et al. (2015) (at pp. 614-616) recently surveyed, a variety of contest experiments consistently reveal that subjects typically exert significantly greater efforts than the equilibrium predictions of contest models based on pure self interest. An explanation is, in addition to the monetary outcome of winning, subjects tend to consider non-monetary and relative outcomes (see, for example, Mago et al. 2016, Price and Sheremeta 2011, Sheremeta 2010).\footnote{Alternative explanations for the over-extension of efforts include risk aversion as well as mistakes, judgmental biases, and problems with the experimental design (see the papers surveyed by Dechenaux et al. 2015 at p. 617).}

The present Emotional Litigation Game introduces the notions of joy of winning and relational emotions to capture, respectively, the non-monetary value of winning and concerns over relative outcomes. Moreover, the Tullock contest experiments conducted by Herrmann and Orzen (2008) and Fonseca (2009) suggest that the subjects’ spiteful preferences to harm their adversaries explain their over-exertion of efforts. For our purposes, their results are most relevant because the Tullock contest model is widely used to study civil litigation. In addition, may real-life litigants exhibit spiteful behaviors\footnote{In fact, an American judge observed that the adversarial litigation system — which is prevalent in common law jurisdictions — heightens antagonism and angst in divorce cases. See Duncan (2007), p. 11.} and in severe cases, courts have sanctioned vexatious litigants who repeatedly brought frivolous lawsuits to harass their adversaries.\footnote{For example, U.S. federal trial courts may prohibit vexatious litigants from filing lawsuits without prior permission.} Our equilibrium analysis thus pays particular
attention to those special cases involving litigants who have negative relational emotions to harm each other.

Interdependent preferences are also prevalent in non-contest situations, such as conspicuous consumption (Veblen 1899) and ultimatum and dictator games (see, for example, the recent survey in Dhami [2016] ch. 5. For instance, Cameron (1999) conducted high-stake experiments in Indonesia to confirm the frequently-obtained result that participants in ultimatum games tend to realize much fairer distribution of surplus than the equilibrium prediction based on rational and self-interested agents. Inequality aversion is one of the explanations that Fehr and Schmidt (1999) captured with their well-known formulation of utility functions. Using a variety of games, Charness and Rabin (2002) presented experimental results supporting alternative explanations based on concerns for joint surplus and reciprocity. Rabin (1993) constructed a complete-information model that endogenously generates intentions-based reciprocity. Bolton and Ockenfels (2000) offered an incomplete-information model that captures concerns for relative outcomes. Capturing preferences for reciprocity, Segal and Sobel (2007) presented a representation theorem for games with players who have preferences over strategies in addition to outcomes. Segal and Sobel (2008) used their model to introduce a condition for ensuring that a player is more likely to be kind to an opponent who treats him nicely. Pollak (1976) examined demand behaviors with a model in which interdependent preferences operate through past consumption.

Section 2 constructs the Emotional Litigation Game to introduce emotional variables into a general contest model that captures whole classes of success functions, cost functions and cost-shifting rules. Section 3 proves the existence and uniqueness of a Nash equilibrium with positive efforts, and reveals the different roles of relational emotions and joy of winning. Section 4 reveals how changes in relational emotions or joy of winning affect equilibrium outcomes. In particular, more negative relational emotions distort equilibrium outcomes in favor of the relatively more advantageous litigant, but typically increase the litigants’ total costs. Section 5 concludes with a discussion of normative implications and future research directions. Appendix A contains all proofs.

See the opinion of the U.S. Court of Appeals, Second Circuit in the case of Safir v. U.S. Lines, Inc. 792 F.2d 19 (2d Cir. 1986). Anglo-Australian courts also have a similar power. See, for example, section 8 of the Vexatious Proceedings Act 2008 (NSW) and the opinion of the English Court of Appeal in the case of Bhamjee v Forsdick & Ors (No 2) [2003] EWCA Civ 1113. A list of vexatious litigants in England is available at https://www.gov.uk/guidance/vexatious-litigants.

Earlier surveys of the literature on interdependent preferences include Camerer and Thaler (1995) and Sobel (2005).


2 The Emotional Litigation Game

The **Emotional Litigation Game** is a simultaneous-move game of complete information characterized by two risk-neutral players, Plaintiff and Defendant, their common set of actions $\mathbb{R}_+$, and their payoff functions $\tilde{u}_P, \tilde{u}_D : \mathbb{R}_+^2 \to \mathbb{R}$. Each payoff function has monetary and non-monetary components, including the joy of winning and the player’s emotions regarding the other player. The payoff functions and all exogenous parameters are common knowledge.

Plaintiff and Defendant simultaneously and respectively exert $e_P, e_D \geq 0$ levels of efforts. Giving each litigant’s monetary cost of exerting effort is a homogeneous cost function $C : \mathbb{R}_+ \to \mathbb{R}_+$ with an exogenous degree of homogeneity $k \geq 1$, where $k$ satisfies additional assumptions to be set out below. An exogenous parameter $0 < \mu < 1$ represents Plaintiff’s prior probability of success; Defendant’s prior probability of success is $1 - \mu$. Plaintiff (respectively, Defendant) is relatively more advantageous if $\mu > 0.5$. Given a prior parameter $\mu$ and a pair of efforts $(e_P, e_D)$, the judicial process with probability $\theta(e_P, e_D; \mu)$ requires Defendant to transfer an judgment sum $1$ to Plaintiff, where the success function $\theta : \mathbb{R}_+^2 \to [0, 1]$ satisfies additional assumptions to be set out below. Upon determination of the outcome of the case, a **cost-shifting rule** requires the loser to pay an exogenous $0 \leq \bar{\lambda} \leq 1$ proportion of the winner’s costs, where $\bar{\lambda}$ satisfies additional assumptions to be set out below. In particular, $\bar{\lambda} = 0$ characterizes the **American rule** that allows for no recovery, and $\bar{\lambda} = 1$ the **English rule** that allows for full recovery. Containing these monetary variables are Plaintiff and Defendant’s respective **monetary payoffs** $u_P, u_D : \mathbb{R}_+^2 \to \mathbb{R}$ given by

\[
u_P = \theta[1 - (1 - \bar{\lambda})C(e_P)] - (1 - \theta)[C(e_P) + \bar{\lambda}C(e_D)]
\]

\[
u_D = -\theta[1 + C(e_D) + \bar{\lambda}C(e_P)] - (1 - \theta)(1 - \bar{\lambda})C(e_D).
\]

Each litigant’s monetary payoff is her expected monetary outcome. Plaintiff’s monetary payoff $u_P$ is the weighted average of her monetary outcome in the event that she wins, $1 - (1 - \bar{\lambda})C(e_P)$, and her monetary outcome in the event that she loses, $-C(e_P) - \bar{\lambda}C(e_D)$. Weights $\theta$ and $1 - \theta$ are respectively her probabilities of winning and losing. Similarly, Defendant’s monetary payoff $u_D$ is the weighted average of her monetary outcome in the event that she loses, $-1 - C(e_D) - \bar{\lambda}C(e_P)$, and her monetary outcome in the event that she wins, $-(1 - \bar{\lambda})C(e_D)$.

Weights $\theta$ and $1 - \theta$ are respectively her probabilities of losing and winning.

Instead of acting solely to maximize her monetary payoff, each litigant acts to maximize an **emotional payoff** that includes the following non-monetary variables. In addition to any monetary

\[1^{10}\] Relative advantages reflect institutional factors that do not vary with litigation efforts but influence the outcome of the case. See Remark 1 in [Chen and Rodrigues-Neto (2017)] for a detailed discussion of relative advantages and litigation efforts.

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transfer, the winner derives an exogenous value \( v \geq 0 \), called her **joy of winning**\(^{11} \)
Moreover, each litigant derives value from her feelings about the other litigant’s outcome; an exogenous \( \xi < 1 \) captures such **relational emotions**, meaning that each litigant is indifferent between one unit of her own monetary payoff (or joy of winning) and \( \xi \) units of the other litigant’s. Containing these non-monetary variables and the monetary payoffs are Plaintiff and Defendant’s respective emotional payoffs \( \tilde{u}_P, \tilde{u}_D : \mathbb{R}_+^2 \rightarrow \mathbb{R} \) given by

\[
\tilde{u}_P = u_P + \theta v + \xi [u_D + (1 - \theta)v] \\
\tilde{u}_D = u_D + (1 - \theta)v + \xi [u_P + \theta v].
\]

Plaintiff’s emotional payoff \( \tilde{u}_P \) sums her monetary payoff \( u_P \), her expected joy of winning \( \theta v \) and her relational emotions regarding Defendant’s outcome, \( \xi [u_D + (1 - \theta)v] \). Similarly, Defendant’s emotional payoff \( \tilde{u}_D \) sums her monetary payoff \( u_D \), her expected joy of winning \( (1 - \theta)v \) and her relational emotions regarding Plaintiff’s outcome, \( \xi [u_P + \theta v] \).

The empirical literature on contests suggests that relational emotions are typically negative (\( \xi < 0 \)\(^{12} \)). Under this specification, each litigant has competitive preferences; her emotional payoff increases when the other litigant’s monetary payoff or expected joy of winning decreases. To our best knowledge, there is no empirical literature measuring emotional variables in litigated cases, but the specification of \( \xi < 0 \) is intuitively appealing given the adversarial nature of civil litigation, especially in common law jurisdictions. However, the Emotional Litigation Game also allows for the possibility of atypical cases characterized by \( 0 < \xi < 1 \)^{13}.

**Remark 1.** Unlike the empirical literature on contests, the empirical literature on bargaining and other more "cooperative" games reveal the possibilities that subjects have concerns for relative payoff, inequality or joint surplus (see section\(^{7} \)). The present Emotional Litigation Game captures these possibilities by assuming zero joy of winning (\( v = 0 \)) and adopting appropriate specifications and interpretations of the parameter \( \xi \).

First, specifying \( \xi > -1 \), a rearrangement of the litigants’ emotional payoffs (3) and (4) gives

\[
\frac{\tilde{u}_P}{1 + \xi} = u_P - \left( \frac{\xi}{1 + \xi} \right) (u_P - u_D), \quad \frac{\tilde{u}_D}{1 + \xi} = u_D - \left( \frac{\xi}{1 + \xi} \right) (u_D - u_P)
\]

\(^{11}\)See section\(^{1} \) for a discussion of the experimental literature supporting the existence of non-monetary utilities of winning, which we call joy of winning to avoid confusion with relational emotions.

\(^{12}\)See section\(^{1} \).

\(^{13}\)We exclude the possibility that each litigant values the other litigant’s monetary payoff (or expected joy of winning) more than her own; that is, \( \xi \geq 1 \). Some algebra using equations (3) and (4) will reveal that, in the limiting case of \( \xi = 1 \), each litigant acts to minimize total litigation costs. An examination of the first order conditions in system (5) in section\(^{3} \) will also reveal that allowing for \( \xi \geq 1 \) would render the model uninteresting, because each litigant would only have incentives to exert zero effort in any equilibrium.
where the function \( \hat{u}_P/(1 + \xi) \) is a strictly positive affine transformation of Plaintiff’s emotional payoff function \( \hat{u}_P \) in (3); these two functions thus represent the same underlying preferences. (An analogous logic applies to Defendant.) Hence the Emotional Litigation Game captures concerns for relative payoffs by interpreting the weight \( \xi/(1 + \xi) \) as the monetary value of relative payoffs. If \(-1 < \xi < 0\), then each litigant’s emotional payoff increases when her relative payoff increases; that is, she has competitive preferences. If \( \xi > 0 \), then each litigant’s emotional payoff decreases when her relative payoff increases; that is, she has inequality aversion (see, for example, Fehr and Schmidt 1999).

Secondly, specifying \( 0 < \xi < 1 \), a rearrangement of the litigants’ emotional payoffs (3) and (4) gives

\[
\hat{u}_P = (1 - \xi)u_P + \xi(u_P + u_D), \quad \hat{u}_D = (1 - \xi)u_D + \xi(u_P + u_D)
\]

where each litigant’s emotional payoff is a weighted average of her monetary payoff and joint surplus. Hence the Emotional Litigation Game captures concerns for joint surplus by interpreting the weight \( \xi \) as the extent to which each litigant values joint surplus.

Moreover, the mere presence of the joy of winning — without concerns for relative outcomes — may be sufficient to explain the over-exertion of efforts in contest experiments (see, for example, Sheremeta 2010, pp. 738-739 and the papers surveyed there). The present Emotional Litigation Game captures this possibility by specifying a positive joy of winning \( (v > 0) \) and zero relational emotions \( (\xi = 0) \).

We now state assumptions to guarantee equilibrium existence and uniqueness. On its subdomain \( \mathbb{R}^2_{++} \), the success function \( \theta(\cdot) \) is twice continuously differentiable and satisfies the following Assumptions 1-6, where Assumptions 5 and 6 also constrain the degree of homogeneity \( k \) of the cost function, the proportion \( \tilde{\lambda} \) of costs recoverable and the extent of relational emotions \( \xi \).

**Assumption 1.** Holding the efforts and the prior constant, whether a litigant is labeled "Plaintiff" or "Defendant" does not affect her posterior probability of success. Formally, \( \theta(e_1, e_2; \mu_0) = 1 - \theta(e_2, e_1; 1 - \mu_0) \), for any positive real numbers \( e_1, e_2 > 0 \) and \( 0 < \mu_0 < 1 \).

**Assumption 2.** Holding the prior constant, proportionate changes in effort levels do not affect Plaintiff’s posterior probability of success. Formally, \( \theta(e_P, e_D; \mu) = \theta(xe_P, xe_D; \mu) \), for all scalar \( x > 0 \).

**Assumption 3.** Holding the prior and Defendant’s effort constant, Plaintiff’s posterior probability of success is strictly increasing with and concave in her effort. Formally, \( \frac{\partial \theta}{\partial e_P} > 0 \) and \( \frac{\partial^2 \theta}{\partial e_P^2} \leq 0 \).

**Assumption 4.** Holding the efforts constant, Plaintiff’s posterior probability of success is strictly increasing with her prior probability of success. Formally, \( \frac{\partial \theta}{\partial \mu} > 0 \).
Assumptions 1-4 capture intuitions regarding the properties of reasonable success functions. Assumption 1 requires a litigant’s posterior probability of success to be unaffected by merely changing her label from "Plaintiff" — whose effort, prior and posterior probability of success are respectively denoted $e_P, \mu, \theta$ — to "Defendant" — whose effort, prior and posterior probability of success are respectively denoted $e_D, 1-\mu, 1-\theta$. In other words, the parameter $\mu$ captures any asymmetry that does not vary with litigation efforts. Under Assumption 2, proportionate changes in effort levels do not vary Plaintiff’s probability of success. Assumptions 3-4 further require that an increase in Plaintiff’s prior probability of success or effort strictly increases her posterior probability of success.

**Assumption 5.** Interdependence in payoffs is limited in the following precise sense:

1. The cost-shifting rule $\hat{\lambda}$ and relational emotions $\xi$ satisfy $\hat{\lambda}(1-\xi) \leq 1$.

2. Suppose $\hat{\lambda}(1-\xi) = 1$ and the cost function is linear, $k = 1$. Then Plaintiff does not win almost surely by exerting infinitely more effort than Defendant does. Formally,

$$\hat{\lambda}(1-\xi) = 1 \quad \text{implies} \quad \lim_{e_P/e_D \to +\infty} \theta < 1.$$  

Assumption 5 restricts the combined "strength" of the cost-shifting rule $\hat{\lambda}$ and relational emotions $\xi$. Part 1 ensures that each litigant’s marginal costs of exerting effort is always positive; allowing for $\hat{\lambda}(1-\xi) > 1$ might induce zero marginal costs for some extremely asymmetric effort pairs. Part 2 ensures that, in special cases involving "strong" negative relational emotions and cost shifting (precisely, $\hat{\lambda}(1-\xi) = 1$) and a linear cost function ($k = 1$), Plaintiff does not have incentives to make explosive efforts ($e_P/e_D \to +\infty$) with an expectation that she almost surely inflicts strong negative relational emotions and unbounded litigant costs on Defendant.

**Assumption 6.** For the interested triple of parameters $(\hat{\lambda}, k, \xi)$, the following condition holds

$$\frac{\partial^2}{\partial e_p^2} \left( \frac{\theta}{1-\hat{\lambda}(1-\xi)\theta} \right) < \frac{C''(e_P)}{C'(e_P)}.$$  

Assumption 6 is a technical assumption that ensures Plaintiff’s emotional payoff is strictly quasiconcave in her own effort. It ensures that Plaintiff’s distorted posterior probability of success — $\theta/(1-\hat{\lambda}(1-\xi)\theta)$ — be small compared to the curvature of the cost function.

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14 For instance, if being called "Plaintiff" requires a litigant to discharge a more onerous burden of proof than if she were called "Defendant", then the parameter $\mu$ should reflect such asymmetry.

15 System 5 to be stated below will formalize these intuitive observations.

16 See the Proof of Lemma 1 in Appendix A.
The exogenous parameters and the litigants’ payoff functions are common knowledge between them. To focus on the study of litigation efforts and probabilities of success, further assume there is no settlement or risk of default.

The solution concept adopted is a pure-strategy Nash equilibrium that is non-trivial in the sense of comprising positive efforts by both litigants. A pair of positive efforts is such an equilibrium if given the other litigant’s effort level, each litigant chooses an effort level to maximize her emotional payoff.

3 Equilibrium: Existence, Uniqueness, and the Role of Emotional Variables

This section proves the existence and uniqueness of a non-trivial Nash equilibrium in the Emotional Litigation Game. It then presents our main result revealing how the presence of relational emotions and joy of winning affects equilibrium efforts.

Lemma 1 allows any equilibrium to be characterized by a system of first order conditions (FOCs). Appendix A contains all proofs.

Lemma 1. Each litigant’s emotional payoff function is strictly quasiconcave in her own effort.

Lemma 1 implies given the other litigant’s effort, a litigant’s FOC characterizes her best reply. A substitution exercise using equations (1)-(4) reveals that a pair of positive efforts \((e_P, e_D) \in \mathbb{R}_{++}^2\) constitutes a Nash equilibrium if and only if it satisfies system (5):

\[
\begin{align*}
0 &= \frac{\partial \theta}{\partial e_P} (1 - \xi)[1 + \nu + \tilde{\lambda} C(e_P) + \tilde{\lambda} C(e_D)] - [1 - \tilde{\lambda}(1 - \xi)\theta]C'(e_P) \\
0 &= \frac{\partial (1 - \theta)}{\partial e_D} (1 - \xi)[1 + \nu + \tilde{\lambda} C(e_P) + \tilde{\lambda} C(e_D)] - [1 - \tilde{\lambda}(1 - \xi)(1 - \theta)]C'(e_D).
\end{align*}
\]

System (5) reveals how the cost-shifting rule \(\tilde{\lambda}\), relational emotions \(\xi\) and joy of winning \(\nu\) affect a litigant’s incentives to exert costly effort. For instance, holding Defendant’s effort \(e_D\) fixed, more cost shifting (\(\tilde{\lambda}\) increases) increases Plaintiff’s marginal benefits of exerting effort \(e_P\) by shifting a greater proportion of her costs — \(C(e_P)\) — into the "prize" of winning: \((1 - \xi)[1 + \nu + \tilde{\lambda} C(e_P) + \tilde{\lambda} C(e_D)]\). This shift also reduces her marginal costs of exerting effort: \([1 - \tilde{\lambda}(1 - \xi)(1 - \theta)]C'(e_P)\). Performing a similar role as the cost-shifting rule, more negative relational emotions (\(\xi\) decreases) increase Plaintiff’s marginal benefits of exerting effort by scaling up the "prize" of winning. A decreased \(\xi\) also reduces her marginal costs of exerting effort. By comparison, a greater joy of winning \(\nu\) increases Plaintiff’s marginal benefits of exerting effort, but does not affect her marginal

\[17\]Theorem 8 of [Diewert et al. 1981] holds any local maximizer of a strictly quasiconcave function is the unique global maximizer.
costs. The same observations apply to Defendant’s incentives to exert costly effort when Plaintiff’s effort is fixed. These intuitive observations have profound equilibrium implications, as Corollary 1 below will reveal.

Lemma 2 finds a unique, positive effort ratio which will be used to characterize the nontrivial Nash equilibrium. To simplify notation, define an auxiliary variable $s = e_D/e_P$ whenever Plaintiff’s effort $e_P > 0$; $s$ is the ratio of Defendant’s effort relative to Plaintiff’s. Assumption 2 implies that for any two pairs of positive efforts $(e_P, e_D), (e'_P, e'_D) \in \mathbb{R}^2_{++}$ satisfying $e_D/e_P = e'_D/e'_P$, the success function satisfies $\theta(e_P, e_D; \mu) = \theta(e'_P, e'_D; \mu)$. By a slight abuse of notation, denote $\theta(s; \mu) = \theta(e_P, e_D; \mu)$ and $\theta_s = \frac{\partial}{\partial s} \theta(s; \mu)$.

**Lemma 2.** There exists a unique positive effort ratio $s^* > 0$ that satisfies

$$s^* = \left[ \frac{1 - \tilde{\lambda}(1 - \xi)\theta(s^*; \mu)}{1 - \tilde{\lambda}(1 - \xi)[1 - \theta(s^*; \mu)]} \right]^{1/k}.$$  \(6\)

Proposition 1 establishes the existence and uniqueness of a non-trivial Nash equilibrium. It also characterizes the litigants’ relative efforts in equilibrium.

**Proposition 1.** There exists a unique Nash equilibrium with positive efforts $(e_P^*, e_D^*)$, which is characterized by

$$e_P^* = \left[ \frac{C(1)}{(1 + \upsilon)(1 - \xi)} \left[ ks^{k-1} \frac{[1 - \tilde{\lambda}(1 - \xi)(1 - \theta(s^*; \mu))]}{-\theta_s(s^*; \mu)} - \tilde{\lambda}(1 - \xi)(1 + s^{k}) \right] \right]^{-1/k}$$

$$e_D^* = s^* e_P^*$$

where Lemma 2 gives $s^*$.

Proposition 1 finds and characterizes the unique non-trivial Nash equilibrium of the Emotional Litigation Game; all subsequent references to the Game’s equilibrium refer to this non-trivial Nash equilibrium. Although the expressions for the equilibrium efforts are complicated, Lemma 2 immediately reveals that the application of the American rule ($\tilde{\lambda} = 0$) leads to equal equilibrium efforts ($s^* = e_D^*/e_P^* = 1$); under other cost-shifting rules, $s^* = 1$ also holds in the limit when relational emotions $\xi \to 1$. Remark 2 below will reveal the equilibrium relative efforts under other cost-shifting rules.

To simplify subsequent discussion and attract the comparative-static analysis of Chen and Rodrigues-Neto (2017) that covers all parameters except the joy of winning ($\upsilon$) and relational emotions ($\xi$), Corollary 1 below will reveal the exact roles that $\upsilon$ and $\xi$ play in equilibrium. To facilitate

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18Section 4 will reveal that more negative relational emotions or more cost shifting increases the equilibrium relative effort and probability of success of the relatively more advantageous litigant.
presentation, fix and suppress the relative-advantages parameter $\mu$ and the cost function $C(\cdot)$. Let $\mathbb{G}(\xi, \upsilon, \tilde{\lambda})$ denote the Emotional Litigation Game when a generic triple $\xi, \upsilon, \tilde{\lambda}$ of parameters respectively capture the relational emotions, the joy of winning, and the cost-shifting rule. Using Proposition [1] let $e^*_p(\xi, \upsilon, \tilde{\lambda})$ and $\theta^*(\xi, \upsilon, \tilde{\lambda})$ respectively denote Plaintiff’s equilibrium effort and probability of success in $\mathbb{G}(\xi, \upsilon, \tilde{\lambda})$. Similarly, let $e^*_D(\xi, \upsilon, \tilde{\lambda})$ and $s^*(\xi, \upsilon, \tilde{\lambda}) = e^*_D(\xi, \upsilon, \tilde{\lambda})/e^*_p(\xi, \upsilon, \tilde{\lambda})$ denote Defendant’s equilibrium (absolute) effort and relative effort. Finally, in the special case of $\upsilon = \xi = 0$, call the game a Monetary Litigation Game; intuitively, the litigants act only to maximize their monetary payoffs (see equations (1), (2)).

Presenting our main result, Corollary[1] relates the equilibrium of an Emotional Litigation Game to that of a Monetary Litigation Game with a different cost-shifting rule characterized by $\tilde{\lambda}(1 - \xi)$.

**Corollary 1.** Consider the equilibrium $(e^*_p(\xi, \upsilon, \tilde{\lambda}), e^*_D(\xi, \upsilon, \tilde{\lambda}))$ of the Emotional Litigation Game $\mathbb{G}(\xi, \upsilon, \tilde{\lambda})$ and the equilibrium $(e^*_p(0, 0, \tilde{\lambda}(1 - \xi)), e^*_D(0, 0, \tilde{\lambda}(1 - \xi)))$ of the Monetary Litigation Game $\mathbb{G}(0, 0, \tilde{\lambda}(1 - \xi))$. Each litigant’s equilibrium effort in $\mathbb{G}(\xi, \upsilon, \tilde{\lambda})$ is $(1 + \upsilon)^{1/k}(1 - \xi)^{1/k}$ times her equilibrium effort in $\mathbb{G}(0, 0, \tilde{\lambda}(1 - \xi))$; the litigant has the same relative effort and posterior probability of success in these equilibria. Formally,

\[ e^*_p(\xi, \upsilon, \tilde{\lambda}) = (1 + \upsilon)^{1/k}(1 - \xi)^{1/k} e^*_p(0, 0, \tilde{\lambda}(1 - \xi)), \]

\[ e^*_D(\xi, \upsilon, \tilde{\lambda}) = (1 + \upsilon)^{1/k}(1 - \xi)^{1/k} e^*_D(0, 0, \tilde{\lambda}(1 - \xi)), \]

and

\[ s^*(\xi, \upsilon, \tilde{\lambda}) = s^*(0, 0, \tilde{\lambda}(1 - \xi)), \quad \theta^*(\xi, \upsilon, \tilde{\lambda}) = \theta^*(0, 0, \tilde{\lambda}(1 - \xi)). \]

Corollary[1] reveals the different implications that relational emotions and joy of winning have on equilibrium efforts. Suppose relational emotions are negative in the Emotional Litigation Game $\mathbb{G}(\xi, \upsilon, \tilde{\lambda})$; that is, $\xi < 0$. Such negative relational emotions affect each litigant’s equilibrium effort directly and indirectly. Indirectly, negative relational emotions render each litigant’s equilibrium effort in $\mathbb{G}(\xi, \upsilon, \tilde{\lambda})$ to be proportionate to her equilibrium effort in the Monetary Litigation Game $\mathbb{G}(0, 0, \tilde{\lambda}(1 - \xi))$, in which the cost-shifting rule is scaled up by $(1 - \xi)$. Directly, each litigant’s equilibrium effort in $\mathbb{G}(\xi, \upsilon, \tilde{\lambda})$ is also scaled up by $(1 - \xi)^{1/k}$ compared to her equilibrium effort in $\mathbb{G}(0, 0, \tilde{\lambda}(1 - \xi))$. The opposite direct and indirect effects arise in the presence of positive relational emotions, $0 < \xi < 1$. However, only relational emotions have both such direct and indirect effects on equilibrium efforts. A positive joy of winning $\upsilon > 0$ directly scales up equilibrium efforts by $(1 + \upsilon)^{1/k}$, but does not vary the effects of cost shifting.

Hence, unlike the joy of winning, the presence of relational emotions modifies the effects of cost shifting. For instance, equilibrium litigation efforts given negative relational emotions ($\xi < 0$)
and the cost-shifting rule $\lambda$ are enlargements of equilibrium litigation efforts given pure self interest ($\xi = 0$) and a greater cost-shifting rule $\lambda(1 - \xi)$; in other words, the presence of negative relational emotions strengthens the cost-shifting rule. The opposite is true in the presence of positive emotion, $0 < \xi < 1$. Because it is typical to have negative relational emotions $\xi < 0$ in a litigated case (see section $[5]$), the "true" effects of cost shifting are greater than what they would be if the litigants were purely self-interested. Section $[5]$ will further develop the normative implications of this result.

Moreover, a litigant has the same relative effort and probability of success in the equilibrium of the Emotional Litigation Game $G(\xi, \nu, \lambda)$ and in the equilibrium of the Monetary Litigation Game $G(0, 0, \lambda(1 - \xi))$. This is because her (absolute) efforts in these equilibria are proportional, so are the other litigant’s. Thus each litigant’s relative effort is the same in these equilibria. This immediately implies she has the same probability of success in these equilibria because the success function $\theta$ only (directly) depends on relative advantages and relative efforts.

Corollary $[1]$ reveals a bijective relationship between each litigant’s equilibrium efforts in the Emotional Litigation Game $G(\xi, \nu, \lambda)$ and the Monetary Litigation Game $G(0, 0, \lambda(1 - \xi))$. For that reason, call $G(0, 0, \lambda(1 - \xi))$ the transformed Monetary Litigation Game of $G(\xi, \nu, \lambda)$. Except in the special case of $\lambda = 0$, $\xi = 0$ or $\nu = 0$, transforming $G(\xi, \nu, \lambda)$ to $G(0, 0, \lambda(1 - \xi))$ requires changing three parameters: the joy of winning from $\nu$ to 0, the relational emotions from $\xi$ to 0, and the cost-shifting rule from $\lambda$ to $\lambda(1 - \xi)$. Section $[4]$ below will use this property to simplify the comparative-statics analysis.

Remark 2. The Litigation Game that Chen and Rodrigues-Neto (2017) presented is a Monetary Litigation Game in the present sense. The description of equilibrium outcomes by Chen and Rodrigues-Neto (2017) in their Proposition 1 applies to the present Emotional Litigation Game:

1. If the American rule applies or relative advantages are equal, then the litigants exert the same levels of effort in equilibrium. Formally, $\lambda = 0$ or $\mu = 0.5$ implies $e^*_{p}(\xi, \nu, \lambda) = e^*_{p}(\xi, \nu, \lambda)$. Moreover, equal relative advantages imply equal equilibrium probabilities of success. Formally, $\mu = 0.5$ implies $\theta^*(\xi, \nu, \lambda) = 0.5$.

2. If the cost-shifting rule allows the winner to recover at least some of her costs from the loser, then in equilibrium, the relatively more advantageous litigant exerts relatively more effort and has a relatively greater probability of success. Formally, $\lambda > 0$ and $\mu > 0.5$ (respectively, $\mu < 0.5$) implies $e^*_{p}(\xi, \nu, \lambda) > e^*_{p}(\xi, \nu, \lambda)$ and $\theta^*(\xi, \nu, \lambda) > 0.5$ ($e^*_{p}(\xi, \nu, \lambda) < e^*_{p}(\xi, \nu, \lambda)$ and $\theta^*(\xi, \nu, \lambda) < 0.5$).
4 Comparative Statics

This section considers the equilibrium implications of variations in the parameters of the Emotional Litigation Game.

4.1 Relative Efforts and Probabilities of Success

This subsection reveals the different implications that relational emotions and joy of winning have on equilibrium efforts and probabilities of success. Corollary 2 first considers the implications arising from changes in relational emotions.

Corollary 2. Consider the equilibrium of the Emotional Litigation Game $G(\xi, \upsilon, \bar{\lambda})$.

1. Suppose the American rule applies or the litigants’ relative advantages are equal. Then each litigant’s equilibrium relative effort and probability of success do not change as relational emotions $\xi$ change. Formally, $\bar{\lambda} = 0$ or $\mu = 0.5$ implies $\frac{d}{d\xi}s^*(\xi, \upsilon, \bar{\lambda}) = 0$ and $\frac{d}{d\xi}\theta^*(\xi, \upsilon, \bar{\lambda}) = 0$.

2. Suppose the cost-shifting rule allows for at least some recovery and Plaintiff is relatively more advantageous. More negative relational emotions ($\xi$ decreases) increase her equilibrium relative effort and probability of success. Formally, $\bar{\lambda} > 0$ and $\mu > 0.5$ imply $\frac{d}{d\xi}s^*(\xi, \upsilon, \bar{\lambda}) > 0$ and $\frac{d}{d\xi}\theta^*(\xi, \upsilon, \bar{\lambda}) < 0$.

3. Suppose the cost-shifting rule allows for at least some recovery and Defendant is relatively more advantageous. More negative relational emotions ($\xi$ decreases) increase her equilibrium relative effort and probability of success. Formally, $\bar{\lambda} > 0$ and $\mu < 0.5$ imply $\frac{d}{d\xi}s^*(\xi, \upsilon, \bar{\lambda}) < 0$ and $\frac{d}{d\xi}\theta^*(\xi, \upsilon, \bar{\lambda}) > 0$.

Corollary 2 reveals how variations in relational emotions $\xi$ affect equilibrium effort ratios and probabilities of success. Consider cases in which the cost-shifting rule allows the winner to recover at least some of her litigation costs from the loser ($\bar{\lambda} > 0$) and one litigant is relatively more advantageous ($\mu \neq 0.5$). Then as relational emotions become more negative ($\xi$ decreases), that litigant exerts relatively more effort and enjoys a greater probability of success in equilibrium.

Corollary 3. Consider the equilibrium of the Emotional Litigation Game $G(\xi, \upsilon, \bar{\lambda})$. Variations in the joy of winning $\upsilon$ do not affect each litigant’s relative effort and probability of success. Formally, $\frac{d}{d\upsilon}s^*(\xi, \upsilon, \bar{\lambda}) = 0$ and $\frac{d}{d\upsilon}\theta^*(\xi, \upsilon, \bar{\lambda}) = 0$.

Corollaries 2 and 3 reveal that relational emotions and joy of winning have very different effects on equilibrium relative efforts and probabilities of success. A greater joy of winning $\upsilon$ affects the litigants in a symmetric manner; to the same extent for both litigants, a greater $\upsilon$ increases
the marginal benefits of exerting effort to increase the probability of success. Corollary 3 thus reveals that equilibrium efforts and probabilities of success remain constant when $\nu$ increases. By comparison, relational emotions $\xi$ interact with the cost-shifting rule (see Corollary 1). Amplifying the effects of cost shifting, more negative relational emotions ($\xi$ decreases) affect the litigants’ incentives asymmetrically. Corollary 2 confirms that such asymmetric effects distort equilibrium efforts and probabilities of success in favor of the relatively more advantageous litigant.

### 4.2 Monetary Consequences

This subsection considers how changes in relational emotions or joy of winning affect the litigants’ monetary payoffs and costs, assuming that these changes do not stop the case from proceeding to litigation. Only private costs and benefits are considered here.

Define (equilibrium) litigation expenditure $C^*(\xi, \nu, \bar{\lambda})$ of the Emotional Litigation Game $\mathcal{G}(\xi, \nu, \bar{\lambda})$ as the sum of Plaintiff and Defendant’s respective litigation costs:

$$C^*(\xi, \nu, \bar{\lambda}) = C(e_p^*(\xi, \nu, \bar{\lambda})) + C(e_D^*(\xi, \nu, \bar{\lambda})).$$  (7)

Changes in $\xi$ or $\nu$ affect each litigant’s incentives to exert efforts in equilibrium. Changes in equilibrium efforts then affect litigation expenditure.

Corollary 4 below will reveal that more negative relational emotions ($\xi$ decreases) typically increase litigation expenditure. To facilitate presentation, define a function $\sigma: (-\infty, 1) \times [0, 1] \times [1, +\infty) \rightarrow (0, 0.5]$ by

$$\sigma(\xi, \bar{\lambda}, k) = \max \{\mu \in [0, 1] | \theta(s^*; \mu) \leq (3 - \bar{\lambda}(1 - \xi))/(4 - \bar{\lambda}(1 - \xi))\} - 0.5,$$

where Lemma 2 gives the equilibrium effort ratio $s^*$, which is a function of all parameters — including the degree of homogeneity $k$ of the cost function — except the joy of winning parameter $\nu$ (see Corollary 3). The function $\sigma(\cdot)$ chooses the maximum prior parameter $\mu$ that induces an equilibrium probability $\theta(s^*; \mu)$ no greater than a value between $2/3$ and $3/4$ — where that value depends on the cost-shifting rule $\bar{\lambda}$ and relational emotions $\xi$ — and deducts $0.5$.  

**Corollary 4.** Consider the equilibrium of the Emotional Litigation Games $\mathcal{G}(\xi, \nu, \bar{\lambda})$. If any one

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19 A study of social costs and benefits of litigation is beyond the scope of this paper. See Shavell (1997) for a discussion of the social suboptimality of private incentives to litigate.

20 To see that the function $\sigma$ exists and satisfies $0 < \sigma \leq 0.5$, suppose $\mu = 0.5$ and fix all other parameters. Remark 2 confirms that the symmetry between the litigants implies equal (equilibrium efforts and) probabilities of success, $\theta(1; 0.5) = 0.5$. Then the properties $\frac{d}{d\mu}\theta(s^*; \mu) > 0$ (from Remark 1) and $0.5 < 2/3 \leq (3 - \bar{\lambda}(1 - \xi))/(4 - \bar{\lambda}(1 - \xi))$ imply there exists some $\mu' \in (0.5, 1]$ satisfying $0.5 < \theta(s^*; \mu') \leq (3 - \bar{\lambda}(1 - \xi))/(4 - \bar{\lambda}(1 - \xi))$. The value of $\sigma$ is uniquely determined by the maximum of all such $\mu'$.

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of the following sufficient conditions holds, then as relational emotion become marginally more negative, litigation expenditure increases; that is, \( \frac{d}{d\xi} C^*(\xi, \nu, \lambda) < 0 \).

1. The American rule applies, \( \lambda = 0 \).

2. The relative advantages of the litigants are sufficiently balanced, in the precise sense of
\[ 0.5 - \sigma(\xi, \lambda, k) \leq \mu \leq 0.5 + \sigma(\xi, \lambda, k). \]

3. The cost function \( C(\cdot) \) is sufficiently convex, in the precise sense of \( k \geq 2 \).

Corollary 4 offers sufficient conditions for concluding that in equilibrium, more negative relational emotions (\( \xi \) decreases) lead to a greater litigation expenditure. Intuitively and as confirmed by the expressions for the litigants’ emotional payoffs (in equations (3), (4)), more negative relational emotions directly increase the emotional reward of winning, heightening incentives to exert costly efforts. Corollary 4 also reveals that more negative relational emotions strengthen the effects of cost shifting; this indirectly affects each litigant’s incentives to exert costly efforts. In equilibrium, the indirect effect on a litigant’s effort may or may not be in the same direction as the direct effect. However, litigation expenditure — which sums the litigants’ costs of exerting efforts in equilibrium — increases if both litigants exert more efforts, or if one litigant’s exertion of additional effort is not offset by a more rapid reduction in effort by the other litigant. Corollary 4 identifies sufficient conditions for concluding that overall, the direct and indirect effects of more negative relational emotions increase litigation expenditure.

Part 1 of Corollary 4 reveals that one such sufficient condition is the application of the American rule to allow for no recovery of the winner’s costs from the loser (\( \lambda = 0 \)). Intuitively, the absence of cost shifting removes the indirect effect that more negative relational emotions (decreased \( \xi \)) have on equilibrium efforts. Independently of the cost-shifting rule, more negative relational emotions directly increase each litigant’s equilibrium effort (see Corollary 1). In the absence of any countervailing indirect effect arising from the “scaling up” of the cost-shifting rule, a greater litigation expenditure follows.

Parts 2 and 3 of Corollary 4 utilize Corollaries 6 and 7 in Chen and Rodrigues-Neto (2017), which contain sufficient conditions for concluding that more cost shifting increases litigation expenditure in a Monetary Litigation Game. Intuitively, more cost shifting generally increases litigation expenditure. This is because more cost shifting reduces each litigant’s expected marginal cost of exerting effort by shifting away a greater proportion of her costs if she wins. More cost shifting also widens the difference in monetary outcome between winning and losing by increasing the recoverable-costs component of the "prize" of winning. These observations and their results apply to the present analysis because the present Corollary 4 reveals that relational emotions and the cost-shifting rule reinforce each other. Part 2 of the present Corollary 4 proves that if relative advantages of the litigants are sufficiently balanced, then more negative relational emotions (\( \xi \)
decreases) increase litigation expenditure. A litigant must have very poor prospects of success to reduce equilibrium effort — which further harms her prospects of success — in order to save costs. In cases characterized by sufficiently balanced relative advantages, no litigant has very poor prospects of success. Hence, in these cases, a decreased $\xi$ incentivizes the litigants collectively to exert more equilibrium efforts, leading to a greater litigation expenditure. The function $\sigma$ defines what is required for relative advantages to be "sufficiently balanced" in this sense; it marks the upper and lower bounds within which the relative-advantages parameter $\mu$ is sufficiently balanced. Moreover, part 3 of Corollary 4 proves that if the cost function is sufficiently convex ($k \geq 2$), then a decreased $\xi$ increases litigation expenditure. This holds even in extreme cases falling outside the scope of part 1 or 2.

Corollary 4 thus provides general sufficient conditions for concluding that more negative relational emotions lead to a greater litigation expenditure (in equilibrium). To fall outside the scope of any one of these sufficient conditions, a case needs to meet all of the following requirements: the American rule does not apply, $\lambda > 0$; the relative advantages of the litigants are sufficiently extreme in the sense of $\mu < 0.5 - \sigma$ or $\mu > 0.5 + \sigma$; and the cost function is insufficiently convex in the sense of $k < 2^2$.[21]

Corollary 5 reveals how changes in the joy of winning affect (individual) monetary payoffs and (collective) litigation expenditure. To facilitate presentation, let $u_p^*(\xi, \upsilon, \bar{\lambda})$ and $u_D^*(\xi, \upsilon, \bar{\lambda})$ respectively denote Plaintiff and Defendant’s equilibrium monetary payoffs (see equations (1), (2)).

**Corollary 5.** Consider the equilibrium of the Emotional Litigation Games $\mathcal{G}(\xi, \upsilon, \bar{\lambda})$.

1. A greater joy of winning $\upsilon$ decreases each litigant’s monetary payoff. Formally, $\frac{d}{d\upsilon} u_p^*(\xi, \upsilon, \bar{\lambda}) < 0$ and $\frac{d}{d\upsilon} u_D^*(\xi, \upsilon, \bar{\lambda}) < 0$.

2. A greater $\upsilon$ leads to a greater litigation expenditure. Formally, $\frac{d}{d\upsilon} C^*(\xi, \upsilon, \bar{\lambda}) > 0$.

Compared to Corollary 4, Corollary 5 establishes a more general result regarding the monetary implications of changes in the joy of winning $\upsilon$: a greater $\upsilon$ increases (collective) litigation expenditure in equilibrium, even in exceptional cases falling outside the scope of Corollary 4. A greater $\upsilon$ increases the marginal benefits of exerting costly efforts to win without affecting the equilibrium probabilities of success (see Corollary 3). However, unlike relational emotions ($\xi$), $\upsilon$ does not interact with the cost-shifting rule (see Corollary 1); no variable in this model offsets the heightened incentives to exert costly efforts arising from a greater $\upsilon$. Hence a lower individual monetary payoff and a greater litigation expenditure follow.

**Remark 3.** Corollary 4 implies that changes in the proportion of costs recoverable $\bar{\lambda}$, Plaintiff’s relative advantages $\mu$ or the cost function $C$ have the same equilibrium implications in the Emotional

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Litigation Game and in its transformed Monetary Litigation Game. Chen and Rodrigues-Neto (2017) offered a detailed analysis of these equilibrium implications. A brief summary of their Corollaries 2-8 is as follows:

1. In cases where one litigant is relatively more advantageous ($\mu \neq 0.5$), an increase in $\lambda$ increases that litigant’s equilibrium relative effort and probability of success. However, there is no such equilibrium effect if the relative advantages are equal ($\mu = 0.5$).

2. In cases where the cost-shifting rule allows for recovery ($\lambda > 0$), an increase in $\mu$ increases Plaintiff’s equilibrium relative effort and probability of success. However, if the American rule applies ($\lambda = 0$), then an increase in $\mu$ does not affect Plaintiff’s equilibrium relative effort, but it increases her equilibrium probability of success.

3. In cases where one litigant is relatively more advantageous ($\mu \neq 0.5$) and the cost-shifting rule allows recovery ($\lambda > 0$), an increase in $k$ decreases that litigant’s equilibrium relative effort and probability of success. In the other cases ($\mu = 0.5$ or $\lambda = 0$), an increase in $k$ has no such equilibrium effect.

Moreover, more cost shifting ($\lambda$ increases) generally leads to a more predictable outcome, in the sense of driving the equilibrium probability of success $\theta(s^*; \mu)$ closer to 1 (respectively, 0) if Plaintiff (Defendant) is relatively more advantageous, $\mu > 0.5$ ($< 0.5$). However, a greater $\lambda$ typically increases litigation expenditure. Depending on the properties of the success function and the cost function, a greater $\lambda$ also may increase or decrease accuracy in outcome, which is measured by the difference between the prior and equilibrium probabilities of success, $|\theta(s^*; \mu) - \mu|$.

5 Discussion

This section discusses some normative implications of the Emotional Litigation Game and future research directions.

5.1 Cost Shifting

There is a debate about the positive implications and normative merits of cost-shifting rules. For instance, using a generalized contest model with purely self-interested litigants, Chen and Rodrigues-Neto (2017) extended the result — first observed by Braeutigam et al. (1984), Katz (1987) and Plott (1987) — that more cost shifting tends to increase costs in litigated cases and distort litigation outcomes in favor of the litigant with stronger prior advantages. Moreover, models based on purely self-interested litigants with non-common priors (for example, Shavell 1982) or information asymmetry (see footnote 28) reveal that decisions to file suit or settle also depend on
cost shifting rules. However, the divergence of private and social incentives to litigate implies that cost shifting itself is unlikely to be sufficient to induce the socially optimally number of suits (see Shavell [1997]).

Most relevant to the debate about cost-shifting rules is the present result that the presence of negative relational emotions — that a litigant derives value from harming her adversary — amplifies the effects of cost shifting (see Corollary 1 in section 3). This result reveals that models based on purely self-interested litigants typically underestimate the full effects of cost shifting. The presence of preferences to harm an adversary is intuitively plausible in many litigated cases and frequently observed in contest experiments (see section 1). Thus a nominally low-powered cost-shifting rule tends to have the practical implications of a higher-powered cost-shifting rule. If the lawmaker (or the judge, when she has discretion over cost shifting) aims to effectuate a particular extent of cost shifting, then in the presence of significant negative relational emotions she should stipulate a nominally weak cost-shifting rule. As a real-world example, while many common law jurisdictions apply high-powered cost-shifting rules by default, judges often exercise their discretion to effectuate low-powered cost-shifting rules in cases involving emotionally-charged litigants with intertwined and conflicted interests, such as succession disputes. Our analysis reveals the behavioral-economic foundations of that judicial practice, and supports it. On the other hand, our analysis does not support adjusting the cost-shifting rule in response to a strong non-monetary joy of winning, which a litigant obtains upon winning the lawsuit rather than upon harming her adversary. A real-life example concerns disputes over properties that have non-monetary value to the litigants. Unlike relational emotions, such outcome-dependent value does not modify the effects of cost shifting (see Corollary 1 in section 3). Moreover, if the judge can observe whether emotions are present in individual cases while the lawmaker cannot, then the present analysis also reveals an advantage of conferral of judicial discretion over cost shifting: it enables the judge to adjust the cost-shifting rule to account for the presence or absence of different types of emotions.

5.2 Mediation

Mediation has arisen as a popular alternative to formal adjudication by a court of law. Mediation typically involves an independent specialist who facilitates discussions between the disputants and helps them reach an agreement regarding their dispute. Although participation in mediation is

\[ \xi < 0 \]

More precisely, in the presence of negative relational emotions \( \xi < 0 \), to effectuate the effects of a cost-shifting rule characterized by the loser bearing \( 0 < \lambda \leq 1 \) proportion of the winner’s costs requires stipulating a weaker cost-shifting rule characterized by \( \lambda = \lambda/(1 - \xi) \).

\[ \xi \]

For example, see the opinion of Justice Gaudron in the Australian case of Singer v Berghouse (1993) 114 ALR 521, [6].

\[ \xi \]

For example, according to the Restatement (Second) of Contracts, section 360, comment e (American Law Institute, 1981), in American common law jurisdictions "land has long been regarded as unique and impossible of duplication by the use of any amount of money". Restatements are authoritative statements of American law.
typically voluntary, some jurisdictions make it a mandatory prerequisite to obtaining a formal court hearing. For example, in Australia and California, mediation is usually compulsory for disputes concerning parenting arrangements after a divorce. In New South Wales and Ontario, mediation is compulsory for common succession disputes. Aside from adopting less formal procedures to reduce legal costs, mediation is typically designed to reduce acrimony between the disputants.

The present analysis provides behavioral-economic justifications for using mediation to resolve disputes involving emotionally-charged litigants, such as divorce and succession disputes. Mediation can be modeled as a contest in which the disputants exert costly efforts to increase their respective shares of the surplus arising from not proceeding to formal adjudication. To the extent that mediation achieves its stated goal of reducing acrimony, the present analysis reveals a corresponding reduction in the disputants’ costs; Corollary reveals that more positive relational emotions typically reduce costs, so does a reduced (non-monetary) joy of winning according to Corollary. These results together suggest that, ceteris paribus, mediation by reducing emotional motivations typically leads to positive monetary consequences for the disputants.

5.3 Unresolved Questions: Filing, Settlement, and Causes of Emotional Considerations

An important qualification of the present analysis is that decisions to file suit or settle are beyond the scope of the Emotional Litigation Game. Future research may explore how emotional considerations affect incentives to sue or settle. Katz and Sanchirico (2012) surveyed the vast economic literature on filing and settlement decisions. Shavell (1982) and others employed non-common-prior models to analyze these decisions under extreme cost-shifting rules that shift either all or none of the winner’s costs to the loser. An alternative approach uses contract-theoretic models to explain settlement outcomes when the litigants have asymmetric information. These models typically assume the litigants are purely self-interested. To take into account emotional considerations, future research may modify the present Emotional Litigation Game to an extensive-form game that models pre-litigation filing or settlement decisions. The litigants’ emotional payoffs in the equilibrium of the Emotional Litigation Game are their outside payoffs for failing to settle in the pre-litigation stages of such an extensive-form game. Through changing these outside payoffs, variations in emotional variables in litigation (see section 4) may have filing or settlement

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26 See Supreme Court of New South Wales Practice Note No. SC EQ 7, 7 and Ontario Rules of Civil Procedure rule 75.1.
27 See, for example, subsection 3161(a) of the California Family Code (2005).
implications.

The present analysis also does not consider the causes of emotional motivations. The present Emotional Litigation Game directly includes emotional variables in individual payoffs, and offers experimental findings as evidence (see section 1). This exogenous approach reveals the profound implications of these variables, but is silent on their causes. Postlewaite (1998) discussed the advantages and disadvantages of different approaches to incorporating social variables. In particular, while there is a strong evolutionary argument for the exogenous approach, the alternative approach of endogenously deriving individual concerns for social variables is better at explaining how the standard economic variables give rise to these concerns. For instance, Cole et al. (1992) constructed a matching model in which non-market decisions endogenously generate concerns for relative outcomes, and applied that model to explain cross-country differences in economic growth rates. Charness and Rabin (2002) (in appendix 1) and Rabin (1993) offered models that endogenously generate intentions-based reciprocity. Sano (2014) applied Rabin’s (1993) approach to study a symmetric Tullock contest with a linear cost function. Future research may generate intentions-based reciprocity in the present contest model in order to capture whole classes of success functions, cost functions and asymmetric relative advantages.
A Appendix: Proofs

Proof of Lemma[1]

This proof establishes the result for Plaintiff. Defendant’s result follows symmetric steps. This proof takes the following steps: (i) establish that if Plaintiff’s FOC holds at a pair of efforts, then her SOC is negative at that pair; (ii) using the results established in step (i), a theorem by [Diewert et al. (1981)] proves that Plaintiff’s payoff function is strictly quasiconcave in her own effort.

Step (i)

Take the partial derivatives of Plaintiff’s emotional payoff function in (3) with respect to her effort $e_p$ to obtain

$$
\frac{\partial \bar{u}_p}{\partial e_p} = \frac{\partial \theta}{\partial e_p} (1 - \xi)[1 + \nu + \bar{\lambda}C(e_p) + \bar{\lambda}C(e_D)] - [1 - \bar{\lambda}(1 - \xi)\theta]C'(e_p) \tag{8}
$$

$$
\frac{\partial^2 \bar{u}_p}{\partial e_p^2} = \frac{\partial^2 \theta}{\partial e_p^2} (1 - \xi)[1 + \nu + \bar{\lambda}C(e_p) + \bar{\lambda}C(e_D)] + 2\bar{\lambda}(1 - \xi) \frac{\partial \theta}{\partial e_p} C'(e_p) \tag{9}
$$

$$
- [1 - \bar{\lambda}(1 - \xi)\theta]C''(e_p).
$$

Suppose Plaintiff’s FOC holds, then some algebra using equation (8) reveals

$$
(1 - \xi)[1 + \nu + \bar{\lambda}C(e_p) + \bar{\lambda}C(e_D)] = \frac{[1 - \bar{\lambda}(1 - \xi)\theta]C'(e_p)}{\partial \theta/\partial e_p}.
$$

A substitution exercise using equation (9) gives

$$
\frac{\partial^2 \bar{u}_p}{\partial e_p^2} = \frac{\partial^2 \theta}{\partial e_p^2} \left[ [1 - \bar{\lambda}(1 - \xi)\theta]C'(e_p) \right] + 2\bar{\lambda}(1 - \xi) \frac{\partial \theta}{\partial e_p} C'(e_p) - [1 - \bar{\lambda}(1 - \xi)\theta]C''(e_p)
$$

$$
= C'(e_p) [1 - \bar{\lambda}(1 - \xi)\theta] \left[ \frac{[1 - \bar{\lambda}(1 - \xi)\theta] \frac{\partial \theta}{\partial e_p} + 2\bar{\lambda}(1 - \xi) \left( \frac{\partial \theta}{\partial e_p} \right)^2}{[1 - \bar{\lambda}(1 - \xi)\theta] \frac{\partial \theta}{\partial e_p}} - \frac{C''(e_p)}{C'(e_p)} \right] < 0
$$

where the last inequality uses Assumptions (5), (6).

Step (ii)

Corollary 9.3 of [Diewert et al. (1981)] holds that a twice continuously differentiable function $f$ defined on an open $S$ is strictly quasiconcave if and only if $y^0 \in S$, $w^T w = 1$ and $w^T \nabla f(y^0)w = 0$ implies $w^T \nabla^2 f(y^0)w < 0$; or $w^T \nabla^2 f(y^0)w = 0$ and $g(z) \equiv f(y^0 + zw)$ does not attain a local minimum at $z = 0$. We apply their result.

Fix Defendant’s effort $e_D = e_1$ for some arbitrary $e_1 > 0$, and consider Plaintiff’s emotional
payoff function $\bar{u}_P(\cdot)$. Suppose $e_P > 0$, $w^T w = 1$ and

$$0 = w^T \nabla \bar{u}_P(e_P, e_1)w = w^T \frac{\partial}{\partial e_P} \bar{u}_P(e_P, e_1)w.$$ 

That $w^T w = 1$ implies $w \neq 0$. Hence $\frac{\partial}{\partial e_P} \bar{u}_P(e_P, e_1) = 0$. Then step (i) proves

$$0 > \frac{\partial^2}{\partial e_P^2} \bar{u}_P(e_P, e_1) = \nabla^2 \bar{u}_P(e_P, e_1).$$

That $w \neq 0$ implies $w^T \nabla^2 \bar{u}_P(e_P, e_1)w < 0$. Hence an application of Corollary 9.3 of Diewert et al. (1981) proves $\bar{u}_P$ is strictly quasiconcave in $e_P$. □

**Proof of Lemma 2**

That the success function $\theta$ and the cost-shifting rule $\lambda$ satisfy Assumptions 1-6 in the present Emotional Litigation Game implies $\theta$ and a different cost-shifting rule defined by $\lambda = \bar{\lambda}(1 - \xi)$ satisfy Assumptions 1-6 on the Litigation Game constructed by Chen and Rodrigues-Neto (2017). Lemma 2 in their paper proves that there exists a unique real number $s^*$ satisfying

$$s^{*k} = \frac{1 - \lambda \theta(s^*; \mu)}{1 - \bar{\lambda}(1 - \theta(s^*; \mu))}.$$ 

Choosing $s = s^*$ gives the result for the present Emotional Litigation Game. □

**Proof of Proposition 1**

This proof will first establish that the pair $(e_P^*, e_D^*)$ satisfies both Plaintiff and Defendant’s FOCs in system (5), thereby characterizing a Nash equilibrium. It will then prove the other direction and uniqueness.

*Step (i)*

Let $s = s^*$ and use the expression for $e_P^*$ to obtain

$$e_P^{*k} = \frac{-(1 + \nu)(1 - \xi)s\theta_s}{C(1)[ks^k[1 - \bar{\lambda}(1 - \xi)(1 - \theta)] + \bar{\lambda}(1 - \xi)(1 + s^k)s\theta_s]}$$

$$= \frac{-(1 + \nu)(1 - \xi)s\theta_s}{C(1)[k[1 - \bar{\lambda}(1 - \xi)\theta] + \bar{\lambda}(1 - \xi)(1 + s^k)s\theta_s]}$$

where the last equality uses the property $1 - \bar{\lambda}(1 - \xi)\theta = s^k[1 - \bar{\lambda}(1 - \xi)(1 - \theta)]$ from Lemma 2.

Then

$$\frac{-(1 + \nu)(1 - \xi)s\theta_s}{e_P^*} = C(1)[k[1 - \bar{\lambda}(1 - \xi)\theta] + \bar{\lambda}(1 - \xi)(1 + s^k)s\theta_s]e_p^{*k-1}$$

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\[-(1 + \nu)(1 - \xi)s\theta_s e^*_p = C(1)[1 - \bar{\lambda}(1 - \xi)\theta]ke^{*k-1}_p + C(1)\bar{\lambda}(1 - \xi)(1 + s^k)s\theta_se^{*k-1}_p\]

\[-(1 + \nu)(1 - \xi)s\theta_s e^*_p - C(1)\bar{\lambda}(1 - \xi)(1 + s^k)s\theta_se^{*k-1}_p = C(1)[1 - \bar{\lambda}(1 - \xi)\theta]ke^{*k-1}_p\]

\[-(1 - \xi)s\theta_s e^*_p \left[1 + \nu + \bar{\lambda}C(1)(1 + s^k)e^{*k}_p\right] = C(1)[1 - \bar{\lambda}(1 - \xi)\theta]ke^{*k-1}_p\]

\[\frac{\partial \theta}{\partial e_p}(1 - \xi)\left[1 + \nu + \bar{\lambda}C(e^*_p) + \bar{\lambda}C(e^*_D)\right] = [1 - \bar{\lambda}(1 - \xi)\theta]C'(e^*_p)\]

where the last equality uses the properties that $C(\cdot)$ is homogeneous of degree $k$, $s^k e^{*k}_p = e^{*k}_D$. Hence the pair $(e^*_p, e^*_D)$ satisfies Plaintiff’s FOC.

Now consider the expression for $e^*_D$

\[e^{*k}_D = s^k e^{*k}_p = \frac{-(1 + \nu)(1 - \xi)s^{k+1}\theta_s}{C(1)\left[k s^k[1 - \bar{\lambda}(1 - \xi)(1 - \theta)] + \bar{\lambda}(1 - \xi)(1 + s^k)s\theta_s\right]}\]

a rearrangement of which gives:

\[-(1 + \nu)(1 - \xi)s\theta_s e^*_D = C(1)\left[k s^k[1 - \bar{\lambda}(1 - \xi)(1 - \theta)] + \bar{\lambda}(1 - \xi)(1 + s^k)s\theta_s\right] e^{*k-1}_D e^{*k}_D\]

\[-(1 + \nu)(1 - \xi)s\theta_s e^*_D = [1 - \bar{\lambda}(1 - \xi)(1 - \theta)]C(1)ke^{*k-1}_D + \bar{\lambda}(1 - \xi)C(1)(1 + s^k)s\theta_se^{*k-1}_D \frac{e^{*k-1}_D}{s^k}\]

\[-(1 + \nu)(1 - \xi)s\theta_s e^*_D - \bar{\lambda}(1 - \xi)C(1)(1 + s^k)s\theta_se^{*k-1}_D \frac{e^{*k-1}_D}{s^k} = [1 - \bar{\lambda}(1 - \xi)(1 - \theta)]C(1)ke^{*k-1}_D\]

\[-\bar{\lambda}(1 - \xi)s\theta_s e^*_D e^{*k}_D \left[1 + \nu + \bar{\lambda}C(1)\left(e^{*k}_D e^{*k}_D + e^{*k}_D\right)\right] = [1 - \bar{\lambda}(1 - \xi)(1 - \theta)]C(1)ke^{*k-1}_D\]

\[\frac{\partial (1 - \theta)}{\partial e_D}(1 - \xi)\left[1 + \nu + \bar{\lambda}C(e^*_p) + \bar{\lambda}C(e^*_D)\right] = [1 - \bar{\lambda}(1 - \xi)(1 - \theta)]C'(e^*_D)\]

which implies the pair $(e^*_p, e^*_D)$ satisfies Defendant’s FOC.

**Step (ii)**

Suppose $(e'_{p}, e'_{D}) \in \mathbb{R}^{2}_{++}$ is a Nash equilibrium with positive efforts. Denote $s' = e'_{D}/e'_{p}$. Some
algebra reveals
\[ e_p^k = \frac{(e_p' + e_D')^k}{(1 + s')^k} \quad e_D^k = \frac{s^k(e_p' + e_D')^k}{(1 + s')^k}. \]

Substituting these into Plaintiff and Defendant’s FOCs in system (5), some algebra reveals:
\[
\frac{-(1 + \nu)(1 - \xi)s(1 + s)^k \theta_s}{C(1)\left[k[1 - \bar{\lambda}(1 - \xi)\theta] + \bar{\lambda}(1 - \xi)(1 + s^k)s\theta_s\right]} \bigg|_{s=s'} = (e_p' + e_D')^k
\]
\[
= \frac{-(1 + \nu)(1 - \xi)s(1 + s)^k \theta_s}{C(1)\left[k s^k[1 - \bar{\lambda}(1 - \xi)(1 - \theta)] + \bar{\lambda}(1 - \xi)(1 + s^k)s\theta_s\right]} \bigg|_{s=s'}
\]
where the first equality (respectively, second equality) is derived from Plaintiff’s (Defendant’s) FOC. Then some algebra using the equality of both sides will reveal that \( s = s' \) induces \( 1 - \bar{\lambda}(1 - \xi)\theta = s^k[1 - \bar{\lambda}(1 - \xi)(1 - \theta)] \). Hence the uniqueness limb of Lemma 2 implies \( s' = s^* \).

Then obtain from the definition of \( e_p' \) in Proposition 1
\[
e_p' + e_D' = \left[ \frac{-(1 + \nu)(1 - \xi)s(1 + s)^k \theta_s}{C(1)\left[k s^k[1 - \bar{\lambda}(1 - \xi)(1 - \theta)] + \bar{\lambda}(1 - \xi)(1 + s^k)s\theta_s\right]} \right]^{1/k} = (1 + s^*)e_p'
\]
where the properties \( e_p' + e_D' = (1 + s')e_p' \) and \( s' = s^* \) imply \( e_p' = e_p^* \). Similarly, use the properties \( e_p' + e_D' = e_D'(1 + s')/s' \) and \( s' = s^* \) to obtain \( e_D' = e_D^* \). □

**Proof of Corollary 1**

An application of Lemma 2 and Proposition 1 gives the result. □

**Proof of Corollary 2**

To facilitate presentation, define a function \( \lambda = \bar{\lambda}(1 - \xi) \). An application of Corollary 1 and the chain rule reveals
\[
\frac{d}{d\xi} s^*(\xi, \nu, \bar{\lambda}) = \frac{d}{d\xi} s^*(0, 0, \lambda) = \frac{d\lambda}{d\xi} \frac{d}{d\lambda} s^*(0, 0, \lambda) = -\bar{\lambda} \frac{d}{d\lambda} s^*(0, 0, \lambda).
\]

Then part 1 follows from letting \( \bar{\lambda} = 0 \), and parts 2-3 an application of Corollary 2 in Chen and Rodrigues-Neto (2017). □

**Proof of Corollary 3**

Lemma 2 reveals that the equilibrium effort ratio \( s^* \) does not depend on the value of joy of winning \( \nu \). The success function \( \theta \) also does not depend on \( \nu \). □

**Proof of Corollary 4**
Using equation (7), Corollary 5 and the homogeneity of the cost function $C(\cdot)$, some algebra obtains

$$C^*(\xi, \nu, \tilde{\lambda}) = C(e^*_p(\xi, \nu, \tilde{\lambda})) + C(e^*_d(\xi, \nu, \tilde{\lambda}))$$

$$= C((1 + \nu)^{1/k}(1 - \xi)^{1/k}e^*_p(0, 0, \tilde{\lambda}(1 - \xi))) + C((1 + \nu)^{1/k}(1 - \xi)^{1/k}e^*_d(0, 0, \tilde{\lambda}(1 - \xi)))$$

$$= (1 + \nu)(1 - \xi)C(1)[e^*_p(0, 0, \tilde{\lambda}(1 - \xi))] + C(1)(1 - \xi)[e^*_d(0, 0, \tilde{\lambda}(1 - \xi))]$$

$$= (1 + \nu)(1 - \xi)C^*(0, 0, \tilde{\lambda}(1 - \xi))$$

where $C^*(0, 0, \tilde{\lambda}(1 - \xi))$ is the (equilibrium) litigation expenditure in the transformed Monetary Litigation Game with a different cost-shifting rule defined by $\lambda = \tilde{\lambda}(1 - \xi)$. Then an application of the product rule and the chain rule reveals

$$\frac{d}{d\xi} C^*(\xi, \nu, \tilde{\lambda}) = \frac{d}{d\xi} \left[(1 + \nu)(1 - \xi)C^*(0, 0, \lambda)\right] = (1 + \nu) \left[(1 - \xi)\frac{d}{d\xi} C^*(0, 0, \lambda) - C^*(0, 0, \lambda)\right]$$

$$= (1 + \nu) \left[(1 - \xi)\frac{d\lambda}{d\xi} \frac{d}{d\lambda} C^*(0, 0, \lambda) - C^*(0, 0, \lambda)\right]$$

$$= -(1 + \nu) \left[C^*(0, 0, \lambda) + (1 - \xi)\tilde{\lambda} \frac{d}{d\lambda} C^*(0, 0, \lambda)\right].$$

Then part 1 follows from letting $\tilde{\lambda} = 0$ and noting $C^*(0, 0, \lambda) > 0$. Parts 2 and 3 respectively follow from Corollaries 6 and 7 in [Chen and Rodrigues-Neto, 2017].

**Proof of Corollary 5**

**Part 1**

The proof for this part will establish the result for Plaintiff’s monetary payoff; similar steps gives the result for Defendant’s.

Using equations (1), (10) and Corollary 5, some algebra reveals

$$\frac{d}{dv} u^*_p(\xi, \nu, \tilde{\lambda}) = \frac{d}{dv} \left[\theta^*(\xi, \nu, \tilde{\lambda})[1 + \tilde{\lambda}C^*(\xi, \nu, \tilde{\lambda})] - C(e^*_p(\xi, \nu, \tilde{\lambda})) - \tilde{\lambda}C(e^*_d(\xi, \nu, \tilde{\lambda}))\right]$$

$$= \frac{d}{dv} \left[\theta^*(\xi, \nu, \tilde{\lambda})[1 + \tilde{\lambda}(1 + \nu)(1 - \xi)C^*(0, 0, \tilde{\lambda}(1 - \xi))]\right]$$

$$- \frac{d}{dv} \left[(1 + \nu)(1 - \xi)[C(e^*_p(0, 0, \tilde{\lambda}(1 - \xi))) + \tilde{\lambda}C(e^*_d(0, 0, \tilde{\lambda}(1 - \xi))]\right]$$

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\[= [1 + \tilde{\lambda}(1 + \nu)(1 - \xi)C^*(0, 0, \tilde{\lambda}(1 - \xi))] \frac{d}{d\nu} \theta^*(\xi, \nu, \tilde{\lambda})
\]
\[- \theta^*(\xi, \nu, \tilde{\lambda})\tilde{\lambda}(1 - \xi)C^*(0, 0, \tilde{\lambda}(1 - \xi))
\]
\[- (1 - \xi)[C(e^*_p(0, 0, \tilde{\lambda}(1 - \xi))) + \tilde{\lambda}C(e^*_D(0, 0, \tilde{\lambda}(1 - \xi)))]\]

where Corollary 3 reveals \( \frac{d}{d\nu} \theta^*(\xi, \nu, \tilde{\lambda}) = 0 \). Hence \( \frac{d}{d\nu} u^*_p(\xi, \nu, \tilde{\lambda}) < 0 \).

**Part 2**

The result follows from differentiating both sides of equation (11) with respect to \( \nu \). \( \Box \)

**References**


