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By

Ben Chen
and
José A.
Rodrigues-Neto

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Cost Shifting in Civil Litigation: A General Theory*

Ben Chen[†] José A. Rodrigues-Neto[‡]

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Abstract

We model civil litigation as a contest between a plaintiff and a defendant. A success function describes the litigants' respective posterior probabilities of success based on their simultaneously-chosen efforts and an exogenous prior reflecting their relative advantages. The present success function satisfies general assumptions which capture frequently-used functional forms. These assumptions represent natural intuitions regarding the properties of reasonable success functions, and enable the results arising from the present model to reach a great degree of generality. Another generalization is the use of an exogenous proportion to characterize a cost-shifting rule that allows the winner to recover that proportion of her litigation costs from the loser. There exists a unique Nash equilibrium with positive efforts. In equilibrium, more cost shifting makes the outcome of the case more predictable, but may increase the litigants' collective expenditure and decrease their collective welfare.

Keywords: cost shifting, legal predictability, litigation costs, legal accuracy, contest theory.

JEL Classification: C72, K41.

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[†]Corresponding author. Columbia Law School, New York, NY - 10027, United States; Research School of Economics, the Australian National University, Canberra, ACT - 2601, Australia. URL: `https://sites.google.com/view/nehcneb`. E-mail: nehc.neb@gmail.com. This author is partly supported by a Herman N. Finkelstein Memorial Fellowship and an Australian Government Research Training Program Scholarship.

[‡]Research School of Economics, the Australian National University, Canberra, ACT - 2601, Australia. URL: `http://rodriguesneto.weebly.com`. E-mail: jarnwi@yahoo.com.

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1 Introduction

A typical civil lawsuit involves two opposing litigants, a plaintiff who seeks judicial remedies at the expense of a defendant. Participation in litigation is costly. A litigant's costs include paying for lawyers, conducting discovery of evidence, researching the law, preparing and making legal arguments, and other costly activities taken to maximize her own payoff. The amount of costs involved in running a lawsuit is a major concern to litigants and the society at large. A recent survey of corporate counsel from various companies across the globe reports the average annual litigation spend to be US$1 million.[1] The protection of rights and enforcement of duties under substantive laws — such as property, contracts and torts — are dependent on the ability of the judicial system to operate fairly and efficiently. Understanding the economics and strategic interaction of civil litigation is thus fundamental to the functioning of a society governed by the rule of law.

Modern judicial systems apply a **cost-shifting rule** to allocate litigation costs between the winner and loser of a civil suit.[2] Influencing the strategic interaction of the litigants, the cost-shifting rule affects the predictability and accuracy of the outcome of the case, as well as the costs spent on litigation. On one end of the costs-shifting spectrum is the traditional **American rule** which requires that each litigant bears her own costs. On the other end is the traditional **English rule** which requires that the loser pays the winner's costs. Most modern legal systems across the globe apply intermediate cost-shifting rules that operate somewhere in between the extremes (Katz and Sanchirico 2012, pp. 273-75). Innovating contest-theoretic techniques, we construct a general model of litigation to capture a great diversity of judicial systems, and apply that model to analyze intermediate and extreme cost-shifting rules.

A litigation model typically includes a **success function** that maps litigation efforts to a litigant's probability of success. The specific functional form of the success function describes the practical operation of a *specific* judicial system. Imposing general assumptions on the success function *without* specifying its function form expands the descriptive scope of the litigation model to cover a whole class of judicial systems. Economic analysis premised on a success function that satisfies general assumptions have broader implications than those premised on a success function that takes a specific functional form. Hence we construct a litigation model that employs general and reasonable assumptions to characterize its success function. Departing from the standard practice of specifying a functional form for the success function, the present model captures a large class of success functions, including the functional forms frequently used in the existing civil-litigation

[1]See the 2016 *Annual Litigation Trends Survey* by Norton Rose Fulbright, an international law firm, at `http://www.nortonrosefulbright.com/files/20160915-2016-litigation-trends-annual-survey-142485.pdf`.

[2]Unless stated otherwise, litigation costs in this paper include attorneys' fees.

literature as well as their convex combinations.[3]

We model litigation as a simultaneous-move game of complete information with two risk-neutral players — a plaintiff and a defendant — each of whom exerts costly effort to maximize her own payoff. A generally-formulated success function describes the litigants' respective posterior probabilities of success based on their efforts and an exogenous prior. The prior reflects the relative advantages of the litigants. The defendant pays a monetary judgment sum to the plaintiff if and only if the plaintiff wins. A cost-shifting rule allows the winner to recover an exogenous proportion of her litigation costs from the loser. Characterizing the cost-shifting rule as a fixed proportion of costs recoverable captures the extreme American and English rules as well as the intermediate rules that shift a part of the winner's costs.

The present formulation of the success function is novel and general; it does not take a functional form, but satisfies general assumptions (Assumptions 1-6 in section 2). Roughly, under these assumptions: a litigant's posterior probability of success is unaffected by a mere change in her label as "plaintiff" or "defendant", or by proportionate changes in efforts; her posterior probability of success is strictly increasing with her prior probability of success, and is strictly increasing with her effort at a diminishing rate; the curvature of the success function is sufficiently small compared to that of the cost function — ensuring the quasiconcavity of the litigant's payoff function; in the special case where the cost function is linear and the English rule applies to allow full recovery of the winner's costs from the loser, a litigant cannot win almost surely by exerting infinitely more effort than the other litigant does. These assumptions capture a large class of success functions; in particular, they capture the Tullock success function — which is the standard in rent-seeking and contest-theory literatures — that expresses a contestant's probability of success as the ratio of her effort relative to total efforts.

This **Litigation Game** has a unique Nash equilibrium with positive effort levels. In equilibrium, the relatively more advantageous litigant is more likely to win, and her equilibrium probability of success increases if the proportion of costs recoverable increases. Hence more cost shifting reduces uncertainty, making the outcome of the case more predictable ex ante. Moreover, litigation efforts may cause **distortion** in the sense of driving posterior probabilities of success away from the prior. If this obfuscatory effect disappears when the litigants exert equal efforts, then more cost shifting necessarily increases distortion in equilibrium.

Influencing litigation efforts in equilibrium, the cost-shifting rule also affects the costs of exerting such efforts. Call the sum of both litigants' costs **litigation expenditure**. In the present Litigation Game, litigation expenditure equals the negative of the sum of the litigants' payoffs because any judgment in favor of the plaintiff or shifting of costs amounts to a transfer of money between the

[3]Subsection 8.1 will reveal that the ability to capture the convex combinations of different success functions enables the model to cover cases where the identity of the judge is uncertain.

litigants. If the relative advantages of the litigants are sufficiently balanced or the cost function is sufficiently convex,[4] then more cost shifting increases litigation expenditure in equilibrium. In other cases, how cost shifting affects litigation expenditure depends on the curvatures of the success function and cost function. Intuitively, an increase in the proportion of costs recoverable raises the stakes by widening the difference in monetary outcome between winning and losing, and reduces the expected marginal cost of exerting effort (Katz and Sanchirico 2012, p. 275). Unless she has very poor prospects, a litigant has incentives to exert more effort in equilibrium to take advantage of a more generous cost-shifting rule.

Even though in many jurisdictions most civil lawsuits settle before the court gives judgment (Hodges et al. 2009, p. 165), they settle in the shadow of the law; that is, the litigants to a dispute reach a settlement with an expectation of what the outcome would be if the court were to adjudicate their dispute. Studying litigation outcomes in the absence of a settlement is thus absolutely essential to understanding settlement negotiations. The (equilibrium) litigation expenditure that arises in the present Litigation Game is the surplus that the litigants would share in a pre-game that models their settlement negotiations, and the litigants' (equilibrium) payoffs in the Litigation Game are their outside options in that pre-game. This paper thus provides the parameters for future research projects that more comprehensively study settlement negotiations.[5]

The choice of cost-shifting rules affects the policy goals of reducing litigation expenditure and improving legal predictability and accuracy. This paper establishes that a more generous cost-shifting rule improves legal predictability in equilibrium, but increases litigation expenditure in cases with balanced advantages or sufficiently convex cost functions. This paper also identifies sufficient conditions for concluding that distortion to the litigants' relative advantages is monotonic with the proportion of costs recoverable. How distortion affects legal accuracy depends on the extent to which relative advantages reflect the inherent merits of the case.

The present generalization of the success function overcomes many limitations that arise from specifying the function form of the success function in a litigation model. First, empirical work on litigation needs to wrestle with the problem of selection bias that arises from decisions to file or settle suits. That problem hiders efforts to ascertain empirically which of the existing functional forms (for example, compare the success functions in Plott 1987 and Carbonara et al. 2015) prevails. By comparison, positive predictions arising from the present axiomatized success function extend as far as the generality and defensibility of its assumptions. Secondly, the present generalization of the success function expands the range of real-world scenarios beyond those captured by the existing functional forms. Consider a scenario in which the litigants exert efforts before they observe the identity of the judge who is assigned to their case. Suppose each of the potential judges rules

[4]Section 5 precisely defines the notions of sufficiently balanced advantages and sufficiently convex cost functions.
[5]Subsection 5.3 and section 9 contain discussions of future research directions.

according to a different success function. Assuming the litigants have a common prior probability for each judge being assigned to their case, we prove that a special case of the Litigation Game captures this scenario. Alternatively, consider a scenario in which a litigant has a positive posterior probability of losing even if the other litigant exerts zero effort. The present generalization captures this alternative scenario, but the oft-used Tullock success function does not.[6]

This paper builds on the vast body of literature on the economics of cost shifting, the seminal papers in which are surveyed by Katz and Sanchirico (2012). To our best knowledge, only a few authors have considered intermediate cost shifting when litigation efforts are endogenous. The majority of these authors formulate that an exogenous *quantity* marks the limit below which the winner's costs are fully recoverable and above which her costs are fully unrecoverable (for example, Hyde and Williams 2002, Carbonara et al. 2015, Farmer and Pecorino 2016, pp. 214-15). A minority of authors formulate that an exogenous *proportion* of the winner' costs are recoverable (for example, Plott 1987, Hause 1989, Gong and McAfee 2000, Luppi and Parisi 2012). We further develop the proportion formulation, and generalizes the success functions and cost functions used by these authors.[7] However, we do not consider optimism or divergent beliefs regarding probabilities of success, while some authors do (for example, Hause 1989, Hyde and Williams 2002).

Intermediate cost-shifting rules reflect the reality in modern judicial systems, including the modern American and English systems. After the American Revolution, American jurisdictions eventually departed from the English position of shifting the "necessary" or "reasonable" costs of conducting litigation, for reasons including a failure to increase statutory caps on costs recoverable and distrust of lawyers. The general application of the American rule was subject to exceptions including bad faith proceedings which are considered unwarranted, baseless or vexatious, and contempt proceedings enforcing prior judgments. The U.S. Federal Rules of Civil Procedure now provide for shifting of costs other than lawyers' fees by default, and allow for shifting of lawyers' fees in narrow circumstances.[8] Most non-American jurisdictions now apply intermediate cost-shifting rules (Katz and Sanchirico 2012, pp. 273-75). There is a recoverability gap in many jurisdictions that allow for cost shifting by judicial discretion; respondents to a 2009 survey reveal this gap to be 25 percent of the winner's actual costs in England, 30-45 percent in Australia, and 33-50 percent

[6]Subsection 8.3 will explore the relationship between the present assumptions and the Tullock success function.

[7]That the proportion formulation of cost-shifting rules generalizes the American and English rules (by setting the proportion to zero or one) implies it generalizes the quantity formulation in the following sense. Suppose, under the quantity formulation, litigation costs in an equilibrium fall below the limiting quantity. Then infinitesimal changes in efforts affect each litigant's payoff in the same way as if the English rule applied, and the English rule would induce the same equilibrium. Alternatively, suppose equilibrium litigation costs under the quantity formulation exceed the limiting quantity, then the following steps would obtain the same equilibrium from the American rule: include in the (fixed) judgment sum in dispute the part of costs that do not exceed the limiting quantity; and apply the American rule to render fully unrecoverable the part of costs that exceed the quantity.

[8]See the U.S. Supreme Court's interpretation of rule 54(d)(1) of the Federal Rules of Civil Procedure in the case of *Marx v. General Revenue Corporation*, 133 S. Ct. 1166, 1175 (2013).

in Singapore (Hodges et al. 2009, p. 21). Hence the heavily-analyzed "American rule" (no shifting of the winner's costs) and "English rule" (full shifting of the winner's costs) should be understood as ideal extremes that bound the cost-shifting spectrum.

This paper builds on the literature on contest theory. Tullock (2001) formulates the standard success function, which gives a contestant's probability of success as the ratio of her own effort relative to the aggregate effort of all contestants. Cornes and Hartley (2005), (2012) refine the Tullock contest model to incorporate risk aversion and general technologies. Einy et al. (2015) prove the existence of pure-strategy Bayesian-Nash equilibria in the Tullock contest model with incomplete information. Serena and Corchón (2017) offer a recent survey of the contest-theory literature, and Vojnović (2016) provides a textbook treatment.

The rest of this paper is structured as follows. Section 2 constructs a model of litigation that simultaneously generalizes the success function, cost function and cost-shifting rule. A large class of success functions and cost functions, including those commonly used to study litigation, are special cases of the present assumptions. Moreover, the model's formulation of cost-shifting rules covers the extreme ones as well as the intermediate ones which operate in the vast majority of jurisdictions across the globe. Section 3 finds and characterizes the unique nontrivial Nash equilibrium, making explicit and quantifying the strategic aspects of civil litigation. Section 4 studies how the equilibrium relative efforts and probabilities of success change in response to infinitesimal changes in, respectively, the litigants' relative advantages, the cost-shifting rule, and the cost function. Sections 6-5 respectively analyze how cost shifting affects legal predictability, distortion to relative advantages, and litigation expenditure and welfare. Section 8 illustrates how our litigation model captures uncertainty regarding the identity of the judge, and the frequently-used Tullock contest model. Section 9 concludes with a discussion of the normative implications and limitations of our positive predictions. Appendix A contains all proofs. Appendix B contains calculations that facilitate the presentation of examples.

2 The Litigation Game

The **Litigation Game** is a simultaneous-move game of complete information characterized by two players, Plaintiff and Defendant, their common set of actions $\mathbb{R}_+$ and respective payoff functions $u_P, u_D : \mathbb{R}_+^2 \to \mathbb{R}$. Their payoff functions and all exogenous parameters are common knowledge. Each player's payoff is her expected monetary outcome in litigation; she is implicitly assumed to be risk neutral.[9]

[9]The simplifying assumption of risk neutrality is particularly apt to describe litigants which are large corporations, each holding a portfolio of lawsuits, and the dispute between them does not concern a sum that is large relative to their wealth.

Plaintiff and Defendant respectively exert $e_P, e_D \geq 0$ levels of effort. Let each litigant's cost of exerting effort be given by a homogeneous cost function $C : \mathbb{R}_+ \to \mathbb{R}_+$ with an exogenous degree of homogeneity $k \geq 1$. Given a pair of efforts (e_P, e_D), the judicial process determines whether, under the law, Defendant is to transfer a judgment sum 1 to Plaintiff. This transfer takes place with (posterior) probability given by a success function $\theta : \mathbb{R}^2_+ \to [0, 1]$ satisfying Assumptions 1-6 to be set out below.

The function $\theta(\cdot)$ has an exogenous parameter $0 < \mu < 1$ which represents Plaintiff's prior probability of success. Defendant's prior probability of success is $1 - \mu$. Plaintiff (respectively, Defendant) is **relatively more advantageous** if $\mu > 0.5$ ($\mu < 0.5$). Relative advantages reflect institutional factors that do not vary with litigation efforts but influence the outcome of the case. These factors may be case-specific, such as inherent merits or prejudices against some legally irrelevant characteristic of a litigant. The judge also may rely on her own personal and professional experiences, and may hold a systemically biased view of the law. The litigants must take these institutional factors as given.

An exogenous parameter $0 \leq \lambda \leq 1$ characterizes the applicable **cost-shifting rule**. This parameter is the proportion of the winner's costs recoverable from the loser. Important instances of all cost-shifting rules are the extremes. The **English rule** characterized by $\lambda = 1$ allows for full recovery of the winner's costs from the loser, whereas the **American rule** characterized by $\lambda = 0$ allows for no recovery.

Remark 1. *We interpret litigation efforts and probabilities of success as follows. Suppose, given the facts that characterize the relevant dispute and the litigation efforts, a random variable will realize one of two outcomes — "Plaintiff wins" or "Defendant wins" — at the end of the litigation process. Before such realization, the litigants first exert some minimum sunk efforts to initiate the litigation process, and then exert additional efforts to influence the realization of the outcome. Sunk efforts are referable to activities to acquire knowledge of the "rules of the game", commence legal proceedings and present the bare minimum "amounts" of evidence and legal arguments to obtain a judicial ruling; these activities may involve initial consultations with lawyers, filing the required documents and giving notice to the interested persons. Additional efforts refer to activities beyond the bare minimum, such as conducting extensive discovery, adducing voluminous evidence and making lengthy legal arguments.*[10] *Plaintiff's (respectively, Defendant's) prior probability of success μ (respectively, $1 - \mu$) is the probability that the outcome realizes in her favor conditional on exertion of sunk efforts and* no *additional efforts. The variables e_P and e_D are the additional efforts that the litigants exert to influence the realization of the outcome. The facts, sunk efforts*

[10]For simplicity, we assume there is no agency cost in the relationship between each litigant and her lawyer(s). An extension of the Litigation Game, which is beyond the scope of this paper, may model the principal-agent relationship between a litigant and her lawyer.

and the practical operation of the judicial system affect the prior probabilities of success, but additional efforts do not affect these probabilities. Plaintiff's (respectively, Defendant's) posterior probability of success θ (respectively, $1-\theta$) is the probability that the outcome realizes in her favor after exertion of sunk efforts and additional efforts. Because the Litigation Game is a model of the litigants' strategic interaction after *their exertion of sunk efforts, we call variables e_P and e_D "efforts" and drop the "additional" label for simplicity.*

On its subdomain $\mathbb{R}^2_{++}$, the success function $\theta(\cdot)$ is twice continuously differentiable and satisfies Assumptions 1-6.

Assumption 1. *Holding the efforts and the prior constant, whether a litigant is labeled "Plaintiff" or "Defendant" does not affect her posterior probability of success. Formally, $\theta(e_1, e_2; \mu_0) = 1 - \theta(e_2, e_1; 1 - \mu_0)$, for any real numbers $e_1, e_2 > 0$ and $0 < \mu_0 < 1$.*

Assumption 2. *Holding the prior constant, proportionate changes in effort levels do not affect Plaintiff's posterior probability of success. Formally, $\theta(e_P, e_D; \mu) = \theta(xe_P, xe_D; \mu)$, for all scalar $x > 0$.*

Assumption 1 requires a litigant's posterior probability of success to be unaffected by merely changing her label from "Plaintiff" — whose effort, prior and posterior probabilities of success are respectively denoted e_P, μ, θ — to "Defendant" — whose effort, prior and posterior probabilities of success are respectively denoted $e_D, 1 - \mu, 1 - \theta$. Assumption 2 further requires Plaintiff's probability of success to be unaffected by proportionate changes in effort levels.

Assumption 3. *Holding the efforts constant, Plaintiff's posterior probability of success is strictly increasing with her prior probability of success. Formally, $\frac{\partial \theta}{\partial \mu} > 0$.*

Assumption 4. *Holding the prior and Defendant's effort constant, Plaintiff's posterior probability of success is strictly increasing with and concave in her effort. Formally, $\frac{\partial \theta}{\partial e_P} > 0$ and $\frac{\partial^2 \theta}{\partial e_P^2} < 0$.*

Assumption 3 requires that holding all else constant, an increase in Plaintiff's prior probability of success strictly increases her posterior probability of success. Assumption 4 further requires that holding all else constant, more effort by Plaintiff strictly increases her posterior probability of success but at a diminishing rate.

Assumption 5. *For the interested pair of cost-shifting rule characterized by λ and cost function characterized by k, the following condition holds*

$$\frac{\frac{\partial^2}{\partial e_P^2}\left(\frac{\theta}{1-\lambda\theta}\right)}{\frac{\partial}{\partial e_P}\left(\frac{\theta}{1-\lambda\theta}\right)} < \frac{C''(e_P)}{C'(e_P)}. \tag{1}$$

Assumption 5 requires that the curvature of the ratio $\theta/(1-\lambda\theta)$ — being Plaintiff's distorted[11] posterior probability of success — be small when compared to the curvature of the applicable cost function, in the precise sense described by condition (1). This technical assumption ensures that Plaintiff's payoff function is strictly quasiconcave in her own effort.[12]

Assumption 6. *If the cost function is linear and the English rule applies to allow full recovery of the winner's costs from the loser, then as she exerts infinitely more effort than Defendant does, Plaintiff's probability of success does not approach* 1. *Formally, that* $k = \lambda = 1$ *implies* $\lim_{e_D/e_P \to 0} \theta < 1$.

In the special case where the marginal cost of exerting effort is constant and the English rule applies, Assumption 6 prevents Plaintiff from winning almost surely (and recovering all her costs almost surely) by exerting infinitely more effort than Defendant does. This technical assumption prevents Plaintiff from incurring explosive litigation costs under the expectation that all her costs are borne by Defendant.

Remark 2. *Assumption 3 reflects the observation that as the litigants' relative advantages (which the exogenous prior captures) play a greater role in determining the outcome of the case, the relatively more advantageous litigant is more likely to succeed. However, the judge must attribute some weight to the litigants' efforts; she sees their evidence and hears their arguments. The judge cannot ignore litigation efforts because in an adversarial system of civil litigation, which the Litigation Game aims to capture, greater constitutional and moral principles mandate that litigants be given an opportunity to present their case and have their arguments heard. The judge also may have to give adequate reasons.*[13] *Assumption 4 thus reflects the observation that the litigants' participation in the litigation process is not in vein.*

Plaintiff and Defendant respectively have payoff functions $u_P, u_D : \mathbb{R}^2_+ \to \mathbb{R}$ given by

$$u_P = \theta[1-(1-\lambda)C(e_P)] - (1-\theta)[C(e_P) + \lambda C(e_D)] \tag{2}$$

$$u_D = -\theta[1 + C(e_D) + \lambda C(e_P)] - (1-\theta)(1-\lambda)C(e_D). \tag{3}$$

Plaintiff's payoff u_P is the weighted average of her monetary outcome in the event that she wins, $1-(1-\lambda)C(e_P)$, and her monetary outcome in the event that she loses, $-C(e_P) - \lambda C(e_D)$. Weights θ and $1-\theta$ are her probabilities of winning and losing respectively.

[11]The ratio $\theta/(1-\lambda\theta)$ is distorted by the applicable cost-shifting rule λ in sense that it lies between Plaintiff's posterior probability of success θ and her relative posterior probability of success $\theta/(1-\theta)$; that is, $\theta \le \theta/(1-\lambda\theta) \le \theta/(1-\theta)$.

[12]Subsection 8.3 will discuss the extent to which Assumption 5 guarantees equilibrium existence in existing models of litigation using the Tullock success function.

[13]An early constitutional protection of procedural fairness was clause 39 of the Magna Carta 1215. Modern constitutional protections include the Due Process Clause of the Fifth and Fourteenth Amendments to the United States Constitution, and articles 6 and 45 of the European Convention on Human Rights and Fundamental Freedoms.

Defendant's payoff u_D is the weighted average of her monetary outcome in the event that she loses, $-1-C(e_D)-\lambda C(e_P)$, and her monetary outcome in the event that she wins, $-(1-\lambda)C(e_D)$. Weights θ and $1-\theta$ are respectively her probabilities of losing and winning.

The exogenous parameters and the litigants' payoff functions are common knowledge between them. The cost-shifting rule (λ) is common knowledge because it represents matters of law and community values. The prior probability of success μ is also common knowledge between the litigants because they have had the opportunity to observe the true facts and circumstances of the case as well as the institutional factors of the judicial system. Similarly, the degree of homogeneity of the cost function (k) is common knowledge because it reflects legal services commonly available in the market. To focus on the study of litigation efforts and probabilities of success, further assume there is no settlement or risk of default.

The solution concept adopted is a Nash equilibrium that is nontrivial in the sense of comprising positive efforts by both litigants. A pair of positive efforts denoted (e_P^*, e_D^*) is a nontrivial Nash equilibrium if given the other player's effort level, each player chooses an effort level to maximize her payoff.

To facilitate presentation, let $\Lambda \subset [0,1]$ represent a collection of cost-shifting rules that shift some $\lambda \in \Lambda$ proportion of the winner's costs to the loser. Let $K \subset [1,+\infty)$ represent a collection of homogeneous cost functions of some degree $k \in K$. Let $\Theta(\Lambda, K)$ denote the set of twice continuously differentiable functions $\theta : \mathbb{R}_+^2 \to [0,1]$ that satisfy Assumptions 1-6 when the applicable pair of cost-shifting rule and cost function is characterized by some $(\lambda, k) \in \Lambda \times K$. Then $\Theta([0,1],[1,+\infty))$ denotes the set of twice continuously differentiable functions that satisfy Assumptions 1-6 for all $0 \leq \lambda \leq 1$ and all $k \geq 1$. Unless stated otherwise, all lemmas, propositions and corollaries assume the success function $\theta \in \Theta(\{\lambda\},\{k\})$ where (λ, k) characterizes the pair of cost-shifting rule and cost function that applies to the case between the litigants.

Remark 3. *The Litigation Game captures uncertainty regarding the identity of the judge who will be assigned to the litigants' case. Consider a modified model in which a judge chosen from a finite collection of $n \geq 1$ judges will hear and decide the case. A success function $\theta_i(\cdot) \in \Theta(\{\lambda\},\{k\})$ characterizes judge $i \in \{1,2,...,n\}$. Judge i rules in favor of Plaintiff with posterior probability $\theta_i(e_P, e_D; \mu)$ and in favor of Defendant with posterior probability $1-\theta_i(e_P, e_D; \mu)$. Plaintiff and Defendant exert efforts* before *they observe the identity of the chosen judge. They assign a common prior probability $p_i \geq 0$ to judge i being chosen, where $\sum_{i=1}^n p_i = 1$. The prior belief $(p_1, p_2, ..., p_n)$ is common knowledge between the litigants.*

Subsection 8.1 will reveal a special case of the Litigation Game that adopts the success function $\theta = \sum_{i=1}^n p_i\theta_i$ captures this modified model.

3 Equilibrium Existence and Uniqueness

This section proves the existence and uniqueness of a nontrivial Nash equilibrium. Lemma 1 allows any nontrivial Nash equilibrium to be characterized by a system of first order conditions (hereinafter, FOCs). Appendix A contains all proofs.

Lemma 1. *Each litigant's payoff function is strictly quasiconcave in her own effort.*

Lemma 1 implies given the other litigant's effort, a litigant's FOC characterizes her best reply.[14] A pair of positive efforts $(e_P^*, e_D^*) \in \mathbb{R}^2_{++}$ constitutes a Nash equilibrium if and only if it satisfies system (4):

$$\begin{cases} \frac{\partial u_P}{\partial e_P} = \frac{\partial \theta}{\partial e_P}[1 + \lambda C(e_P) + \lambda C(e_D)] - (1 - \lambda\theta)C'(e_P) = 0 \\ \frac{\partial u_D}{\partial e_D} = \frac{\partial (1-\theta)}{\partial e_D}[1 + \lambda C(e_P) + \lambda C(e_D)] - (1 - \lambda(1 - \theta))C'(e_D) = 0. \end{cases} \tag{4}$$

To simplify notation, define an auxiliary variable $s = e_D/e_P$ whenever Plaintiff's effort $e_P > 0$; s is the ratio of Defendant's effort relative to Plaintiff's. Assumption 2 implies that for any two pairs of positive efforts $(e_P, e_D), (e'_P, e'_D) \in \mathbb{R}^2_{++}$ such that $e_D/e_P = e'_D/e'_P$, the success function satisfies $\theta(e_P, e_D; \mu) = \theta(e'_P, e'_D; \mu)$. By a slight abuse of notation, denote $\theta(s; \mu) = \theta(e_P, e_D; \mu)$, $\theta_s = \frac{\partial \theta}{\partial s}$ and $\theta_{ss} = \frac{\partial^2 \theta}{\partial s^2}$.

Lemma 2 finds a unique, positive effort ratio $s^* > 0$ which will be used to characterize the nontrival Nash equilibrium.

Lemma 2. *There exists a unique positive effort ratio $s^* > 0$ such that $s = s^*$ satisfies*

$$s^k = \frac{1 - \lambda\theta}{1 - \lambda(1 - \theta)}. \tag{5}$$

The value of s^* satisfies the following properties:

1. *If the American rule applies (that is, $\lambda = 0$), then $s^* = 1$.*
2. *If the cost-shifting rule allows the winner to recover at least some costs from the loser (that is, $\lambda > 0$) and Plaintiff's prior probability of success $\mu > 0.5$ (respectively, $= 0.5$, < 0.5), then $s^* < 1$ (respectively, $= 1$, > 1).*

To facilitate presentation, define a real number $A > 0$ by

$$A = \left[\frac{-s(1 + s)^k \theta_s}{C(1)\left[ks^k[1 - \lambda(1 - \theta)] + \lambda s(1 + s^k)\theta_s\right]} \right]^{1/k} \tag{6}$$

[14] Theorem 8 of Diewert et al. (1981) holds any local maximizer of a strictly quasiconcave function is the unique global maximizer.

where $s = s^*$ given by Lemma 2.[15]

Proposition 1 establishes the existence and uniqueness of a nontrivial Nash equilibrium. It also reveals the litigants' relative efforts in equilibrium.

Proposition 1. *There exists a unique Nash equilibrium with positive efforts* (e_P^*, e_D^*)*, which is characterized by*

$$e_P^* = \frac{A}{1+s^*} \qquad e_D^* = \frac{s^* A}{1+s^*},$$

where s^* *and* A *are positive real numbers given by Lemma 2 and equation (6) respectively.*

This Nash equilibrium satisfies the following properties:

1. *If the American rule applies or relative advantages are equal, then the litigants exert the same levels of effort in equilibrium. Formally,* $\lambda = 0$ *or* $\mu = 0.5$ *implies* $e_P^* = e_D^*$*. Moreover, that relative advantages are equal implies posterior probabilities of success are equal in equilibrium. Formally,* $\mu = 0.5$ *implies* $\theta(e_P^*, e_D^*; \mu) = 0.5$.
2. *If the cost-shifting rule allows the winner to recover at least some costs from the loser, then the relatively more advantageous litigant exerts relatively more effort and has a relatively greater posterior probability of success in equilibrium. Formally,* $\lambda > 0$ *and* $\mu > 0.5$ *(respectively,* $\mu < 0.5$*) implies* $e_P^* > e_D^*$ *and* $\theta(e_P^*, e_D^*; \mu) > 0.5$ *(*$e_P^* < e_D^*$ *and* $\theta(e_P^*, e_D^*; \mu) < 0.5$*).*

Proposition 1 reveals that the applicable cost-shifting rule and the prior determine the litigants' relative efforts in the nontrivial Nash equilibrium. That the American rule applies to deny the winner of any recovery of her costs is sufficient to induce equal equilibrium efforts. If the cost-shifting rule allows at least some recovery and she is relatively more advantageous (respectively, relatively less advantageous), then Plaintiff's equilibrium effort is greater than (smaller than) Defendant's. The litigants exert equal efforts in equilibrium if their relative advantages are equal.

All subsequent analyses of equilibrium properties are referable to the unique nontrivial Nash equilibrium characterized by Proposition 1. We are not interested in any equilibrium that is trivial in the sense that at least one litigant exerts zero effort.

As a preliminary to subsequent discussions on the equilibrium implications of variations in exogenous paremeters, Corollary 1 ensures that a nontrivial Nash equilibrium actually exists within the interested range of exogenous parameters.[16]

[15]Part 9 of Lemma 4, a technical lemma contained in Appendix A, implies $A > 0$.

[16]Subsection 8.3 will reveal that even the standard Tullock success function does not induce a nontrivial Nash equilibrium under some special pairs of cost-shifting rule and cost function. Hence we are careful to confine the subsequent results to pairs of cost-shifting rule and cost function that actually induce a nontrivial Nash equilibrium under the relevant success function.

Corollary 1. *Consider a success function $\theta \in \Theta(\{\bar{\lambda}\}, \{\underline{k}\})$ and a pair of cost-shifting rule and success function characterized by some $0 \leq \bar{\lambda} \leq 1$ and $\underline{k} \geq 1$. There exists a unique nontrivial Nash equilibrium under the same success function θ and any pair of cost-shifting rule and cost function characterized by $\lambda \leq \bar{\lambda}$ and $k \geq \underline{k}$.*

Given a success function $\theta \in \Theta(\{\bar{\lambda}\}, \{\underline{k}\})$ where $\bar{\lambda}$ characterizes the most generous cost-shifting rule and $\underline{k}$ the least convex cost function which arouse our interest, Corollary 1 enables us to analyse equilibrium properties for all combinations of cost-shifting rule $\lambda \leq \bar{\lambda}$ and cost function $k \geq \underline{k}$. Corollary 1 also enables us to analyse how equilibrium properties respond to variations in the applicable pair of cost-shifting rule and cost function, to the extent that such variations do not extend beyond the scope of $[0, \bar{\lambda}] \times [\underline{k}, +\infty)$.[17] It follows that if $\theta \in \Theta(\{1\}, \{1\})$, then we can analyse equilibrium properties for all cost-shifting rules and cost functions.

4 Comparative Statics

This section calculates the equilibrium effects of variations in the prior, cost-shifting rule and degree of homogeneity of the cost function. To facilitate presentation, let $\theta^* = \theta(e_P^*, e_D^*; \mu)$ denote Plaintiff's posterior probability of success in the nontrivial Nash equilibrium, and call it her **equilibrium probability of success**. Defendant's equilibrium probability of success is $1 - \theta^*$. Call $s^* = e_D^*/e_P^*$ Defendant's **equilibrium relative effort**. Plaintiff's equilibrium relative effort is $1/s^*$.

Corollary 2 reveals how equilibrium relative efforts and probabilities of success respond to infinitesimal variations in the applicable cost-shifting rule.

Corollary 2. *Consider the nontrivial Nash equilibrium (e_P^*, e_D^*).*

1. *Suppose Plaintiff's is relatively more advantageous. Then her equilibrium relative effort and probability of success are increasing with the proportion of costs recoverable. Formally, $\mu > 0.5$ implies $\frac{d(1/s^*)}{d\lambda} > 0$ and $\frac{d\theta^*}{d\lambda} > 0$.*
2. *Suppose the relative advantages are equal. Then each litigant's equilibrium relative effort and probability of success do not change with the proportion of costs recoverable. Formally, $\mu = 0.5$ implies $\frac{ds^*}{d\lambda} = 0$ and $\frac{d\theta^*}{d\lambda} = 0$.*
3. *Suppose Defendant's is relatively more advantageous. Then her equilibrium relative effort and probability of success are increasing with the proportion of costs recoverable. Formally, $\mu < 0.5$ implies $\frac{ds^*}{d\lambda} > 0$ and $\frac{d(1-\theta^*)}{d\lambda} > 0$.*

Corollary 2 proves that the relative advantages of the litigants determine how equilibrium relative efforts and probabilities of success respond to infinitesimal variations in the applicable

[17] By a slight abuse of notation, let $[0, \bar{\lambda}] = \{0\}$ if $\bar{\lambda} = 0$.

cost-shifting rule. If one litigant is relatively more advantageous (that is, $\mu \neq 0.5$), then parts 1 and 3 prove that she exerts relatively more effort in equilibrium. Parts 1 and 3 also reveal that more cost shifting increases the equilibrium probability of success for the relatively more advantageous litigant. Intuitively, more cost shifting incentivizes the relatively more advantageous litigant — who has better prior prospects of winning — to exert relatively more effort. Then that both relative advantages and relative effort are in favor of the relatively more advantageous litigant; a greater equilibrium probability of success for her follows.

Corollary 3. *Consider the nontrivial Nash equilibrium* (e_P^*, e_D^*).

1. *If the American rule applies, then an increase in a litigant's prior probability of success does not affect her equilibrium relative effort, but increases her equilibrium probability of success. Formally,* $\lambda = 0$ *implies* $\frac{d(1/s^*)}{d\mu} = 0$, $\frac{d\theta^*}{d\mu} > 0$ *for Plaintiff and* $\frac{ds^*}{d(1-\mu)} = 0$, $\frac{d(1-\theta^*)}{d(1-\mu)} > 0$ *for Defendant.*
2. *If the cost-shifting rule allows the winner to cover at least some costs from the loser, then a litigant's relative effort and equilibrium probability of success are increasing with her prior probability of success. Formally,* $\lambda > 0$ *implies* $\frac{d(1/s^*)}{d\mu} > 0$, $\frac{d\theta^*}{d\mu} > 0$ *for Plaintiff and* $\frac{ds^*}{d(1-\mu)} > 0$, $\frac{d(1-\theta^*)}{d(1-\mu)} > 0$ *for Defendant.*

Corollary 3 proves that the equilibrium effects of changes in a litigant's relative advantages depends on the applicable cost-shifting rule. Part 1 proves that under the American rule (that is, $\lambda = 0$), becoming more advantageous does not incentivize a litigant to exert relatively more effort in equilibrium. Nonetheless, the increase in her prior probability of success (which represents her relative advantages) has a direct effect that improves her equilibrium probability of success. Part 2 proves that if cost shifting takes place (that is, $\lambda > 0$), becoming more advantageous incentivizes a litigant to exert relatively more effort in equilibrium. Then the increase in her prior probability of success directly improves her equilibrium probability of success, and indirectly does so through increasing her relative effort.

Remark 4. *Corollary 3 implies that in equilibrium, each of the effort ratio* s^* *and the posterior probability* θ^* *is a bijective function of the prior probability* μ*. This result does* not *suggest that the judge, who does not know* μ*, has enough information to infer it and decide the case without giving weight to litigation efforts. As discussed in Remark 1, we interpret the prior and litigation efforts to influence the posterior probabilities that a random variable — representing the practical operation of the judicial system — realizes one of two values: "Plaintiff wins" or "Defendant wins". Consistently with reality, the judge observes the* realized *value of this random variable, but does not observe the effort ratio or posterior probability. Hence the judge has insufficient information to infer* μ*.*

Corollary 4. *Consider the nontrivial Nash equilibrium* (e_P^*, e_D^*).

1. *If the American rule applies or the relative advantages are equal, then changes in the degree of homogeneity of the cost function does not affect each litigant's relative effort and probability of success in equilibrium. Formally, if* $\lambda = 0$ *or* $\mu = 0.5$*, then* $\frac{ds^*}{dk} = 0$ *and* $\frac{d\theta^*}{dk} = 0$.
2. *If the cost-shifting rule allows the winner to recover at least some costs from the loser and one litigant's case is relative more advantageous, then that litigant's equilibrium relative effort and probability of success are increasing with the degree of homogeneity of the cost function. Formally, that* $\lambda > 0$ *and* $\mu > 0.5$ *(respectively,* $\mu < 0.5$*) implies* $\frac{d(1/s^*)}{dk} > 0$ *and* $\frac{d\theta^*}{dk} > 0$ *(respectively,* $\frac{ds^*}{dk} > 0$ *and* $\frac{d(1-\theta^*)}{dk} > 0$*).*

Corollary 4 proves the effects of changes in the degree of homogeneity (that is, k) of the cost function depends on the applicable cost-shifting rule. The value of k corresponds to the convexity of the cost function; as k increases, the cost function becomes more convex. Part 1 proves a more convex cost function does not affect a litigant's equilibrium relative effort or probability of success in cases where the American rule applies ($\lambda = 0$) or neither litigant is relatively more advantageous ($\mu = 0.5$). In other cases, part 2 proves a more convex cost function incentivizes the relatively more advantageous litigant to exert relatively more effort in equilibrium, which indirectly increases her equilibrium probability of success.

5 Cost Shifting Affects Expenditure and Welfare

This section considers the effects of cost shifting on litigation costs and welfare in the nontrivial Nash equilibrium (e_P^*, e_D^*). **Litigation expenditure** (in equilibrium), denoted C^*, is defined as the sum of Plaintiff and Defendant's respective litigation costs in equilibrium

$$C^* = C(e_P^*) + C(e_D^*). \tag{7}$$

Litigation welfare (in equilibrium), denoted U^*, is defined as the sum of Plaintiff and Defendant's respective equilibrium payoffs

$$U^* = u_P(e_P^*, e_D^*; \mu, \lambda, k) + u_D(e_P^*, e_D^*; \mu, \lambda, k) \tag{8}$$

where Plaintiff and Defendant's respective equilibrium efforts e_P^*, e_D^* are functions of exogenous parameters μ, λ, k.

Some algebra reveals that litigation welfare is the negative of litigation expenditure; that is, $U^* = -C^*$. This follows from the assumption of risk neutrality in money; that each litigant

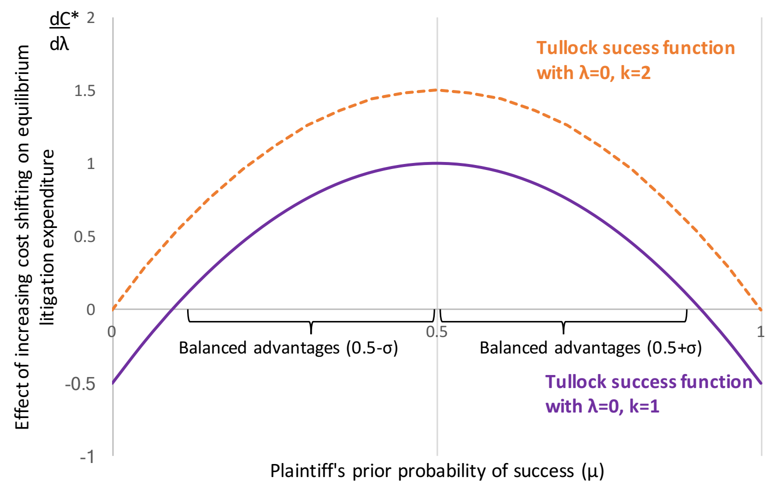

Figure 1: How equilibrium litigation expenditure responds to more cost shifting in the $\mathbb{T}_1$ Game and $\mathbb{T}_2$ Game, where the American rule ($\lambda = 0$) is the baseline rule.

has constant marginal valuation of the judgment sum, and of costs. The judgment sum may be transferred from Defendant to Plaintiff, but such transfer, if it takes place, would not "lose value" and would be cancelled out in the summation of payoffs to derive litigation welfare, U^*. Similarly, the applicable cost-shifting rule may require the loser to pay some or all of the winner's costs — also without "losing value" — and such payment would be cancelled out in the summation of the litigants' payoffs. Only the litigants' costs remain after the summation of their payoffs. Hence, as the litigants incur more costs to increase their individual payoffs, they decrease their collective welfare. In the nontrivial Nash equilibrium, that litigation expenditure is always positive (that is, $C^* > 0$) also implies that litigation welfare is always negative (that is, $U^* < 0$).

Assumptions 1-6 are not sufficient for answering the question whether more cost shifting increases litigation expenditure in every case. To see this, consider Figure 1. For each value of Plaintiff's prior probability of success μ and under the American rule (that is, $\lambda = 0$), Figure 1 depicts how litigation expenditure responds to infinitesimally more cost shifting (that is, $\frac{dC^*}{d\lambda}$). The purple solid curve represents the $\mathbb{T}_1$ Game, which has a linear cost function characterized by $k = 1$. The orange dashed curve represents the $\mathbb{T}_2$ Game, which has a strictly convex cost function characterized by $k = 2$. Each of these Games adopts the Tullock success function θ_T given by (13). Consider the $\mathbb{T}_1$ Game. In cases characterized by sufficiently balanced relative advantages (here, cases with μ satisfying $\mu' < \mu < \mu''$), more cost shifting increases litigation expenditure (that is, $\frac{dC^*}{d\lambda} > 0$). In cases characterized by extreme relative advantages (here, cases with $\mu < \mu'$ or $\mu > \mu''$), more cost shifting *decreases* litigation expenditure (that is, $\frac{dC^*}{d\lambda} < 0$). In borderline cases characterized by $\mu = \mu'$ or $\mu = \mu''$, more cost shifting does not affect litigation expenditure (that is, $\frac{dC^*}{d\lambda} = 0$). However, in the $\mathbb{T}_2$ Game, more cost shifting increases litigation expenditure in *all* cases.

5.1 Sufficiently Balanced Relative Advantages or Sufficiently Convex Cost Functions

Motivated by the special cases depicted in Figure 1, this subsection considers the effect of cost shifting on (equilibrium) litigation expenditure in cases characterized by sufficiently balanced relative advantages or sufficiently convex cost functions. As a preliminary, Corollary 5 characterizes the sufficient and necessary condition for litigation expenditure to be increasing with the proportion of costs recoverable.

Corollary 5. *Consider the nontrivial Nash equilibrium. Litigation expenditure C^* is increasing with the cost-shifting rule λ if and only if the following condition holds:*

$$-\frac{(2\theta-1)s\theta_{ss}}{\theta_s} > (2\theta-1)\left[1-\frac{k\lambda(2\theta-1)}{2-\lambda}\right]+\frac{\alpha(2-\lambda)s\theta_s}{k(1-\lambda\theta)[1-\lambda(1-\theta)]}-\frac{\alpha}{2-\lambda} \tag{9}$$

where $s = s^*$ given by Lemma 2.

Corollary 5 identifies condition (9) as the the sufficient and necessary condition for more cost shifting to increase litigation expenditure. Condition (9) requires that the relative curvature of the success function θ with respect to effort ratio (that is, $-\frac{\theta_{ss}}{\theta_s}$) to be sufficiently large in equilibrium. Corollaries 6 and 7 will use condition (9) and Lemma 2 to ascertain how cost shifting affects litigation expenditure given sufficiently balanced advantages or sufficiently convex cost functions.

To facilitate presentation, define a function $\sigma : [0,1]\times[1,+\infty) \to (0,0.5]$ by[18]

$$\sigma(\lambda,k) = \max\{\mu\in[0,1]\,|\,\theta^* \le (3-\lambda)/(4-\lambda)\} - 0.5.$$

Corollary 6. *Consider two cost-shifting rules $0 < \lambda_1 < \lambda_2 \le 1$, where the success function $\theta \in \Theta(\{\lambda_2\},\{k\})$ and (equilibrium) litigation expenditure is denoted C_1^* under λ_1 and C_2^* under λ_2. Suppose relative advantages are sufficiently balanced in the precise sense of $0.5-\sigma(\lambda_2,k) \le \mu \le 0.5+\sigma(\lambda_2,k)$. Then increasing the proportion of costs recoverable from λ_1 to λ_2 increases litigation expenditure. Formally, $0.5-\sigma(\lambda_2,k) \le \mu \le 0.5+\sigma(\lambda_2,k)$ implies $C_2^* > C_1^*$.*

Corollary 6 proves that if relative advantages of the litigants are sufficiently balanced, more cost shifting increases litigation expenditure. Intuitively, more cost shifting increases litigation

[18]To see that the function $\sigma(\cdot)$ exists and $0 < \sigma(\lambda,k) \le 0.5$, first fix a pair of cost-shifting rule λ and cost function k and use part 1 of Proposition 1 to obtain that $\mu = 0.5$ implies $\theta^* = 0.5$ in equilibrium, which in term implies $\theta^*(4-\lambda) > 3-\lambda$. Then the property $\frac{d\theta^*}{d\mu} > 0$ from Corollary 3 implies a one-to-one relationship between μ and θ^*. Hence there exists at most one $0.5 < \mu'' \le 1$ that induces $\theta^*(4-\lambda) = 3-\lambda$, and that all $0.5 < \mu < \min\{1,\mu''\}$ induces $\theta^*(4-\lambda) < 3-\lambda$.

expenditure if both litigants exert more efforts in equilibrium, or if one litigant's exertion of additional effort is not offset by a more rapid reduction in effort by the other litigant. More cost shifting allows a litigant to recover a greater proportion of her costs if she wins, and widens the difference in monetary outcome between wining and losing. A litigant must have very poor prospects of success to reduce equilibrium effort — which further harms her prospects of success — in order to save costs. In cases characterized by sufficiently balanced relative advantages, no litigant has very poor prospects of success. Hence, in these cases, more cost shifting incentivizes the litigant collectively to exert more equilibrium efforts. The function $\sigma(\cdot)$ defines what is required for relative advantages to be "sufficiently balanced" in this sense. As a function of the applicable cost-shifting rule λ and cost function k, $\sigma(\cdot)$ marks the upper and lower bounds within which the prior — being the parameter that represents relative advantages — is considered sufficiently balanced.

Corollary 7. *Consider two cost-shifting rules $0 \leq \lambda_1 < \lambda_2 \leq 1$, where the success function $\theta \in \Theta(\{\lambda_2\}, \{k\})$ and (equilibrium) litigation expenditure is denoted C_1^* under λ_1 and C_2^* under λ_2. If the cost function is sufficiently convex in the sense that its degree of homogeneity $k \geq 2$, then increasing the proportion of costs recoverable from λ_1 to λ_2 increases litigation expenditure. Formally, $k \geq 2$ implies $C_2^* > C_1^*$.*

Corollary 7 proves that if the cost function is sufficiently convex, then more cost shifting increases litigation expenditure (in equilibrium). This holds even in extreme cases which fall outside the scope of Corollary 6 due one litigant having very favorable prior probability of success. Hence Corollaries 6 and 7 together provide general conditions under which more cost shifting increases litigation expenditure. This result expands a finding in the existing literature, that the English rule (full recovery of the winner's costs) encourages greater legal expenditure in litigated cases than the American rule (no recovery of the winner's costs) does.[19]

5.2 Extreme Relative Advantages and Insufficiently Convex Cost Functions

As a result of Corollaries 6 and 7, only in exceptional cases characterized by very one-sided prior and insufficiently convex cost functions may it be possible for litigation expenditure to be nonincreasing with the proportion of costs recoverable. We now propose an additional condition, captured by Assumption 7, that is sufficient for concluding that even in these exceptional cases, more cost shifting increases litigation expenditure.

Assumption 7. *Suppose the prior is very favorable to Plaintiff in the sense that $\mu > 0.5 + \sigma(\lambda, k)$, and the cost function is insufficiently convex in the sense that $k < 2$. If Plaintiff's effort is no less*

[19]For example, Plott (1987), Braeutigam et al. (1984) and Katz (1987).

than some positive effort by Defendant (that is, $0 < s \leq 1$), then one of the following condition holds:

$$-\frac{(2\theta-1)s\theta_{ss}}{\theta_s} > (2\theta-1)\left[1-\frac{k\lambda(2\theta-1)}{2-\lambda}\right]+\frac{\alpha(2-\lambda)s\theta_s}{k(1-\lambda\theta)[1-\lambda(1-\theta)]}-\frac{\alpha}{2-\lambda} \tag{10}$$

or

$$-\frac{(2\theta-1)s\theta_{ss}}{\theta_s} > (2\theta-1)\left[1-\frac{k(1-s^k)}{1+s^k}\right]+\frac{\beta(1+s^k)^2 s\theta_s}{k(2-\lambda)s^k}-\frac{\beta}{2-\lambda}. \tag{11}$$

Assumption 7 requires the relative curvature of the success function with respect to effort ratio (that is, $-\frac{\theta_{ss}}{\theta_s}$) to be sufficiently large. For example, the $\mathbb{L}_k$ Game, which adopts the success function θ_L defined in (17), satisfies Assumption 7.[20]

To facilitate presentation, let $\Theta_7(\Lambda, K)$ denote the set of twice continuously differentiable functions $\theta : \mathbb{R}_+^2 \to [0,1]$ that satisfy Assumption 7 when the applicable cost-shifting rule and cost function are characterized some $\lambda \in \Lambda \subset [0,1]$ and some $k \in K \subset [0,+\infty)$ respectively.

Proposition 2. *Consider two cost-shifting rules $0 \leq \lambda_1 < \lambda_2 \leq 1$, where the success function $\theta \in \Theta(\{\lambda_2\},\{k\}) \cap \Theta_7([\lambda_1,\lambda_2],\{k\})$ and (equilibrium) litigation expenditure is C_1^* under λ_1 and C_2^* under λ_2. Then increasing the proportion of costs recoverable from λ_1 to λ_2 increases litigation expenditure. Formally, $\theta \in \Theta(\{\lambda_2\},\{k\}) \cap \Theta_7([\lambda_1,\lambda_2],\{k\})$ implies $C_2^* > C_1^*$.*

Proposition 2 proves that adding Assumption 7 is sufficient for concluding that in all cases, more cost shifting increases litigation expenditure. This holds even if one litigant has very favorable relative advantages and the cost function is insufficiently convex.

5.3 Settlement

When applied to study the litigants' settlement decisions, the Litigation Game gives the range of acceptable settlement amounts. Assuming that Plaintiff settles if she is indifferent between settling or litigating, the lower bound of the range of acceptable settlement offers occurs when Defendant makes a take-it-or-leave it offer which is equal to Plaintiff's equilibrium payoff in proceeding to litigation, that is, $u_P(e_P^*, e_D^*; \mu, \lambda, k)$. The upper bound occurs when Plaintiff makes a take-it-or-leave-it offer which is equal to the magnitude of Defendant's equilibrium payoff in litigation, that

[20] Using Appendix B, some algebra will reveal that $\mu \geq 0.5$ and $0 < s \leq 1$ implies

$$-s\,\frac{\partial^2\theta_L}{\partial s^2}\bigg/\frac{\partial\theta_L}{\partial s} \geq 1-\frac{k(1-s^k)}{1+s^k}.$$

The property $\theta_s < 0$ from Lemma 4, a technical Lemma in Appendix A, implies $\theta_L \in \Theta_7([0,1],[1,+\infty))$. Hence the $\mathbb{L}_k$ Game satisfies Assumption 7 under any cost-shifting rule $0 \leq \lambda \leq 1$ and any cost function $k \geq 1$.

is, $-u_D(e_P^*, e_D^*; \mu, \lambda, k)$. The range of mutually acceptable settlement amounts is the closed interval $[u_P(e_P^*, e_D^*; \mu, \lambda, k), -u_D(e_P^*, e_D^*; \mu, \lambda, k)]$. The length of this interval is

$$-u_D(e_P^*, e_D^*; \mu, \lambda, k) - u_P(e_P^*, e_D^*; \mu, \lambda, k) = -U^* = C^* > 0.$$

Hence settlement and avoidance of litigation will generate surplus to the litigants. Corollaries 6-7 reveal that more cost shifting increases litigation expenditure (C^*) at least in cases where the litigants' relative advantages are sufficiently balanced or the cost function is sufficiently convex. In these cases, more cost shifting by increasing the size of the surplus arising from settlement disincentivizes the litigants from bringing their case to litigation. However, the extent of any reduction in overall litigation expenditure of all litigants in a society depends on exogenous factors such as the distribution of relative advantages.

To provide a comprehensive analysis of litigation efforts under different cost-shifting rules, the present paper largely abstracts away from pre-litigation behaviors and the society's perspective on litigation. For instance, outside the present scope is a comprehensive treatment of settlement negotiations to divide the saved litigation expenditure. The present definition of litigation expenditure (see (7)) also only represents the litigation costs borne by those litigants who proceed to litigation. This definition does not include the public costs borne by the judicial system or the society at large, such as the costs of providing judges to adjudicate cases, running and maintaining courts and enforcing judgments. Nor does this definition attempt to capture how private and public litigation costs change in response to decisions to bring suit, settle or proceed to litigation. A more comprehensive (and complex) model that includes the society's perspective on the costs and benefits of litigation is required to resolve issues regarding the optimal balance between the litigants' private interests and the interests of the society. These issues, and those that section 9 below will identify, cannot be resolved without a comprehensive analysis of how cost-shifting rules affect private litigation efforts, and the present paper offers that analysis.

6 Cost Shifting Affects Legal Predictability

This section reveals how changes in the applicable cost-shifting rule affects legal predictability in equilibrium. Consider two arbitrary cost-shifting rules $0 \leq \lambda_1, \lambda_2 \leq 1$ and a success function $\theta \in \Theta(\{\lambda_1, \lambda_2\}, \{k\})$. Corollary 1 proves the existence and uniqueness of a nontrivial Nash equilibrium under each of these cost-shifting rules. Let θ_1^*, θ_2^* denote Plaintiff's equilibrium probabilities of success under λ_1, λ_2 respectively. We say the cost-shifting rule λ_2 makes the outcome of the case

more predictable than the cost-shifting rule λ_1 does if and only if

$$|\theta_2^* - 0.5| > |\theta_1^* - 0.5|. \tag{12}$$

Intuitively, the worst scenario for legal predictability occurs when the litigants win with equal probabilities in equilibrium. Then θ_2^* is better for legal predictability than θ_1^* is in the sense that θ_2^* is further away from 0.5 than θ_1^* is. Changing the applicable cost-shifting rule from λ_1 to λ_2 improves legal predictability from Plaintiff's perspective by changing her equilibrium probability of success from θ_1^* to θ_2^*. The same reasoning applies to legal predictability from Defendant's perspective; condition (12) is equivalent to $|(1-\theta_2^*) - 0.5| > |(1-\theta_1^*) - 0.5|$, where $1-\theta_2^*$, $1-\theta_1^*$ are Defendant's equilibrium probabilities of success under λ_2, λ_1 respectively.

In an equal-advantage case ($\mu = 0.5$), part 2 of Corollary 2 renders trivial the question whether more cost shifting improves or impairs legal predictability. This is because variations in the cost-shifting rule does not affect equilibrium relative efforts or probabilities of success. Intuitively, no litigant has an advantage over the other. Any variation in the cost-shifting rule affects the incentives of both litigants equally, and in equilibrium they exert equal efforts and win with equal probabilities.

Applying the results in Corollary 2 to cases in which one litigant is relatively more advantageous, Corollary 8 reveals how variations in cost shifting affects legal predictability.

Corollary 8. *Consider two cost-shifting rules $0 \leq \lambda_1 < \lambda_2 \leq 1$, where the success function $\theta \in \Theta(\{\lambda_2\}, \{k\})$ and Plaintiff's equilibrium probability of success is θ_1^* under λ_1 and θ_2^* under λ_2. If one litigant is relatively more advantageous, then increasing the applicable cost-shifting rule from λ_1 to λ_2 makes the outcome of the case more predictable. Formally, $\mu \neq 0.5$ implies $|\theta_2^* - 0.5| > |\theta_1^* - 0.5|$.*

Corollary 8 proves that if one litigant is relatively more advantageous ($\mu \neq 0.5$), then more cost shifting improves legal predictability in equilibrium. Intuitively, the relatively more advantageous litigant is more likely to win in equilibrium (according to Proposition 1), and more cost shifting ($\lambda_1 \to \lambda_2$) incentivizes her further to increase her equilibrium effort relative to the other litigant's (according to Corollary 2). This increase in her relative effort further increases her equilibrium probability of success, thereby improving legal predictability in her favor ($|\theta_2^* - 0.5| > |\theta_1^* - 0.5|$).

Figure 2 illustrates Corollary 8 with a special case of the Litigation Game, called the $\mathbb{T}_1$ Game, that adopts a linear cost function ($k = 1$) and an oft-used Tullock success function $\theta_T : \mathbb{R}_+^2 \to [0, 1]$ where

$$\theta_T(e_P, e_D; \mu) = \begin{cases} \frac{\mu e_P}{\mu e_P + (1-\mu) e_D} & \text{if } e_P + e_D \neq 0 \\ \mu & \text{otherwise} \end{cases} \tag{13}$$

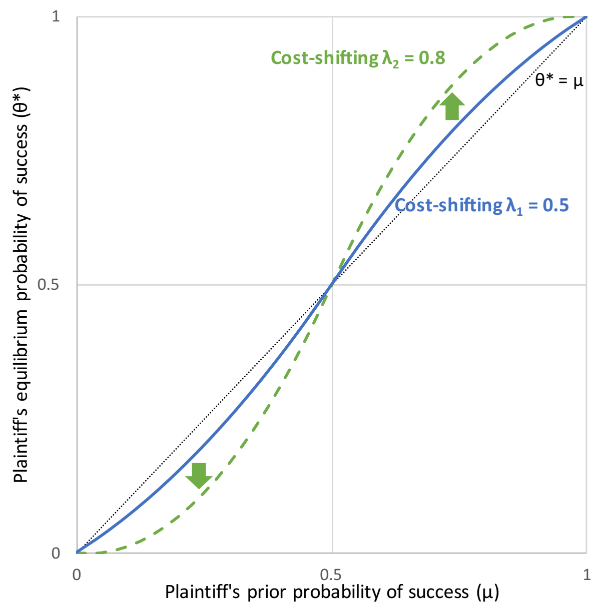

Figure 2: Plaintiff's equilibrium probabilities of success as functions of her prior probability of success under two cost-shifting rules, $\lambda_1 = 0.5$ and $\lambda_2 = 0.8$, in the $\mathbb{T}_1$ Game.

where some algebra using Appendix B reveals $\theta_T \in \Theta([0,1),[1,+\infty))$.[21] Figure 2 plots the relationship between Plaintiff's equilibrium probability of success θ^* and prior probability of success μ under two cost-shifting rules $\lambda_1 = 0.5$ and $\lambda_2 = 0.8$. The blue solid curve (respectively, green dashed curve) depicts θ^* as a function of μ when $\lambda_1 = 0.5$ (respectively, $\lambda_2 = 0.8$). For all $\mu \neq 0.5$, the value of θ^* on the green dashed curve is further away from 0.5 compared to that on the blue solid curve.

7 Cost Shifting Distorts Relative Advantages

This section explores how changes in the applicable cost-shifting rule influence the extent to which the litigants' relative advantages affects their equilibrium probabilities of success. Define **distortion** $\Delta : (0,1) \times [0,1] \times [1,+\infty) \to \mathbb{R}$ by the magnitude of the difference between Plaintiff's equilibrium probability of success θ^* and prior probability of success μ

$$\Delta(\mu, \lambda, k) = |\theta^* - \mu|, \tag{14}$$

where θ^* is a function of μ, the applicable cost-shifting rule λ and the degree of homogeneity k of the cost function. Distortion from Defendant's perspective is $\Delta(\mu, \lambda, k) = |1 - \theta^* - (1 - \mu)|$, which is the magnitude of the difference between her equilibrium probability of success $1 - \theta^*$ and prior

[21]Subsection 8.3 will study more general formulations of the Tullock success function.

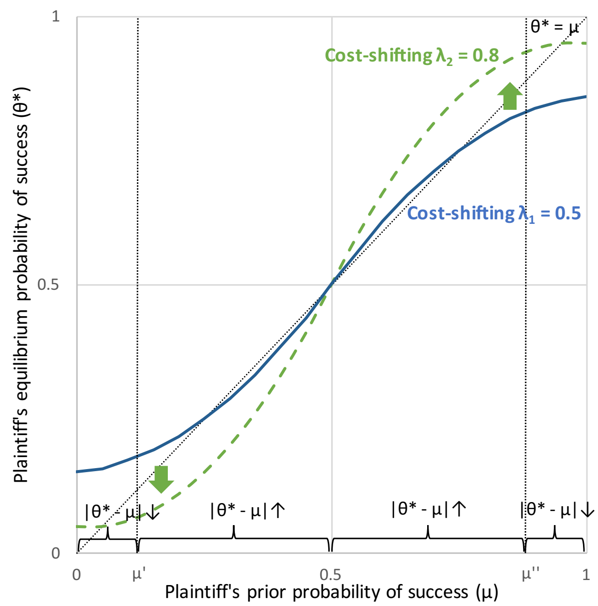

Figure 3: Plaintiff's equilibrium probabilities of success as functions of her prior probability of success under two cost-shifting rules, $\lambda_1 = 0.5$ and $\lambda_2 = 0.8$.

probability of success $1 - \mu$.

Intuitively, distortion measures the extent to which litigation efforts drive equilibrium probabilities of success away from the litigants' relative advantages, as captured by the prior. A large (respectively, small) distortion means that, compared to relative advantages, litigation efforts have a significant (insignificant) influence on equilibrium probabilities of success. If changing a cost-shifting rule increases (respectively, decreases) distortion, then this change increases (decreases) the influence that litigation efforts have on equilibrium probabilities of success.

Our previous assumptions are not sufficient for answering the question how cost shifting affects distortion in every case. That question is trivial in equal-advantages cases ($\mu = 0.5$) because, as part 2 of Corollary 2 proves, each litigant's equilibrium probability of success is not affected by any variation in the cost-shifting rule. However, if the relative advantages are unequal ($\mu \neq 0.5$), how cost shifting affects distortion is not immediately clear. To illustrate potential complexities, consider Figure 3, which depicts for a special case of the Litigation Game the relationship between Plaintiff's equilibrium probability of success θ^* and her prior probability of success μ under two cost-shifting rules, $\lambda_1 = 0.5$ and $\lambda_2 = 0.8$. The blue solid curve (respectively, green dashed curve) depicts θ^* as a function of μ when $\lambda_1 = 0.5$ (respectively, $\lambda_2 = 0.8$). For any case characterized by a μ satisfying $\mu' < \mu < 0.5$ or $0.5 < \mu < \mu''$, the value of θ^* on the green dashed curve is further away from μ compared to that on the blue solid curve. In these cases, increasing the cost-shifting rule from λ_1 to λ_2 increases distortion. For any case characterized by a μ satisfying $\mu < \mu'$ or $\mu > \mu''$, the value of θ^* on the green dashed curve is closer to μ compared to that on the blue solid curve. In these cases, increasing the cost-shifting rule from λ_1 to λ_2 decreases distortion. For a case characterized by $\mu = \mu'$ or $\mu = \mu''$, increasing the cost-shifting rule from λ_1 to λ_2 does not affect distortion.

7.1 When More Cost Shifting Increases Distortion

We now propose additional conditions that are sufficient to answer the question whether more cost shifting increases or decreases distortion in cases where one litigant is relatively more advantageous.

Assumption 8. *If litigation efforts are equal, then Plaintiff's posterior probability of success equals her prior probability of success. Formally, $e_P = e_D$ implies $\theta(e_P, e_D; \mu) = \mu$.*

Assumption 8 imposes a condition in respect of generic pairs of efforts (e_P, e_D), not just the equilibrium pair of efforts (e_P^*, e_D^*). It requires that a litigant's posterior probability of success accurately reflects her relative advantages if litigation efforts are equal. Intuitively, under Assumption 8, equal efforts do not distort the litigants' relative advantages. Satisfaction of Assumption 8 does not depend on the applicable cost-shifting rule or cost function.

Adding Assumption 8, Proposition 3 proves the relatively more advantageous litigant has an equilibrium probability of success that is no smaller than her prior probability of success. To facilitate presentation, let Θ_8 denote twice continuously differentiable functions $\theta : \mathbb{R}_+^2 \to [0, 1]$ that satisfy Assumption 8.

Proposition 3. *Suppose the success function $\theta \in \Theta(\{\lambda\}, \{k\}) \cap \Theta_8$ and one litigant is relatively more advantageous (that is, $\mu \neq 0.5$). In the nontrivial Nash equilibrium, the equilibrium probability of success of the relatively more advantageous litigant is no smaller than her prior probability of success. Her equilibrium probability of success is greater than her prior probability of success if the cost-shifting rule makes at least some costs recoverable. Formally:*

1. *That $\mu > 0.5$ implies $\theta^* \geq \mu$, holding strictly if $\lambda > 0$.*
2. *That $\mu < 0.5$ implies $1 - \theta^* \geq 1 - \mu$, holding holds strictly if $\lambda > 0$.*

Corollary 9. *Consider two cost-shifting rules $0 \leq \lambda_1 < \lambda_2 \leq 1$, where the success function $\theta \in \Theta(\{\lambda_2\}, \{k\}) \cap \Theta_8$. If one litigant is relatively more advantageous, then increasing the applicable cost-shifting rule from λ_1 to λ_2 increases distortion in equilibrium. Formally, $\mu \neq 0.5$ implies $\Delta(\mu, \lambda_2, k) > \Delta(\mu, \lambda_1, k)$.*

Using the results in Proposition 3, Corollary 9 proves that adding Assumption 8 is sufficient for concluding that in any unequal-advantages case, more cost shifting increases distortion in equilibrium. Intuitively, Assumption 8 ensures that in any unequal-advantages case and under any cost-shifting rule, the equilibrium probability of success of the more advantageous litigant is no smaller than her prior probability of success. Then allowing for more cost shifting increases her relative effort (according to Corollary 2), which further pushes her equilibrium probability of success above her prior probability of success.

Figure 2 illustrates Corollary 9 using the $\mathbb{T}_1$ Game, which satisfies Assumption 8. Figure 2 plots the relationship between Plaintiff's equilibrium probability of success θ and her prior probability of success μ under two cost-shifting rules $\lambda_1 = 0.5$ and $\lambda_2 = 0.8$. The blue solid curve (respectively, green dashed curve) depicts θ^* as a function of μ when $\lambda_1 = 0.5$ ($\lambda_2 = 0.8$). For all $\mu \neq 0.5$, the value of θ^* on the green dashed curve is further away from μ compared to that on the blue solid curve.

7.2 When More Cost Shifting Decreases Distortion

We now propose an alternative assumption that imposes a sufficient condition for ensuring that more cost shifting decreases distortion in unequal-advantages cases. To facilitate presentation, denote $\theta_\mu = \frac{\partial\theta}{\partial\mu}$, $\theta_{\mu\mu} = \frac{\partial^2\theta}{\partial\mu^2}$, $\theta_{ss} = \frac{\partial^2\theta}{\partial s^2}$ and $\theta_{s\mu} = \frac{\partial^2\theta}{\partial s\partial\mu}$, and define functions $\alpha, \beta : \mathbb{R}_{++} \to \mathbb{R}_{++}$ by

$$\alpha(s; \mu, \lambda, k) = k(1 - \lambda\theta)[1 - \lambda(1 - \theta)] + \lambda(2 - \lambda)s\theta_s$$

$$\beta(s; \mu, \lambda, k) = \frac{k(2 - \lambda)^2 s^k}{(1 + s^k)^2} + \lambda(2 - \lambda)s\theta_s$$

where $\theta \in \Theta(\{\lambda\}, \{k\})$, and μ, λ, k are exogenous parameters in functions α, β.[22]

Assumption 9. *If Plaintiff is relatively more advantageous (that is,* $\mu > 0.5$*) and her effort is no less than some positive effort by Defendant (that is,* $0 < s \leq 1$*), then at least one of the following conditions holds:*

$$\alpha^2\theta_{\mu\mu} - 2\lambda(2 - \lambda)\alpha s\theta_\mu\theta_{s\mu} \geq -\lambda^2(2 - \lambda)^2 s\theta_\mu^2\left\{s\theta_{ss} + \left[1 - \frac{k\lambda(2\theta - 1)}{2 - \lambda}\right]\theta_s\right\} \tag{15}$$

or

$$\beta^2\theta_{\mu\mu} - 2\lambda(2 - \lambda)\beta s\theta_\mu\theta_{s\mu} \geq -\lambda^2(2 - \lambda)^2 s\theta_\mu^2\left\{s\theta_{ss} + \left[1 - \frac{k(1 - s^k)}{1 + s^k}\right]\theta_s\right\}. \tag{16}$$

Given a pair of cost-shifting rule λ and a cost function k, Assumption 9 imposes restrictions on the first, second and cross derivatives of the success function $\theta(s; \mu)$.[23] As Proposition 4

[22]Part 9 of Lemma 4, a technical lemma in Appendix A, implies $\alpha, \beta > 0$.

[23]Part 6 of Lemma 4, a technical lemma in Appendix A, reveals that Assumptions 1, 3 and 4 imply $\theta_\mu > 0, \theta_s < 0$ and $\theta_{ss} > 0$. Assumption 9 imposes additional restrictions in respect of their magnitude.

will prove, adding Assumption 9 ensures that Plaintiff's equilibrium probability of success is convex (respectively, concave) in her prior probability of success when her case is relatively more (respectively, less) advantageous. To facilitate presentation, let $\Theta_9(\Lambda, K)$ denote the set of twice continuously differentiable functions $\theta : \mathbb{R}_+^2 \to [0, 1]$ that satisfy Assumption 9 when the applicable cost-shifting rule and cost function are characterized by $\lambda \in \Lambda \subset [0, 1]$ and $k \in K \subset [0, +\infty)$ respectively.

Proposition 4. *Suppose the success function $\theta \in \Theta(\{\lambda\}, \{k\}) \cap \Theta_9(\{\lambda\}, \{k\})$ and one litigant is relatively more advantageous (that is, $\mu \neq 0.5$). If her case is relatively more (respectively, less) advantageous, then Plaintiff's equilibrium probability of success is a convex (respectively, concave) function of her prior probability of success. The convexity (respectively, concavity) is strict if condition (15) or (16) holds strictly. Formally:*

1. *That $\mu > 0.5$ implies $\frac{d^2\theta^*}{d\mu^2} \geq 0$, holding strictly if condition (15) or (16) holds strictly.*
2. *That $\mu < 0.5$ implies $\frac{d^2\theta^*}{d\mu^2} \leq 0$, holding strictly if condition (15) or (16) holds strictly.*

Corollary 10. *Consider two cost-shifting rules $0 \leq \lambda_1 < \lambda_2 \leq 1$, where the success function $\theta \in \Theta(\{\lambda_2\}, \{k\}) \cap \Theta_9(\{\lambda_1, \lambda_2\}, \{k\})$. If one litigant is relatively more advantageous, then reducing the cost-shifting rule from λ_2 to λ_1 increases distortion in equilibrium. Formally, $\mu \neq 0.5$ implies $\Delta(\mu, \lambda_2, k) < \Delta(\mu, \lambda_1, k)$.*

Using the results in Proposition 4, Corollary 10 proves that adding Assumption 9 is sufficient for concluding that in any unequal-advantages case, reducing cost shifting increases distortion in equilibrium. Intuitively and as confirmed by Proposition 4, Assumption 9 imposes conditions on the success function to ensure that in any unequal-advantages case and under any cost-shifting rule, the equilibrium probability of success of the more advantageous litigant is no greater than her prior probability of success. Then reducing cost shifting, which decreases her relative effort (according to Corollary 2), further pushes her equilibrium probability of success below her prior probability of success. This in term increases distortion.

Figure 4 illustrates Corollary 10 using a special case of the Litigation Game, called the $\mathbb{L}_k$ Game, that adopts a strictly convex cost function (that is, $k > 1$) and the following linear success function $\theta_L : \mathbb{R}_+^2 \to [0, 1]$

$$\theta_L(e_P, e_D; \mu) = \begin{cases} \mu\eta + (1-\eta)\frac{e_P}{e_P+e_D} & \text{if } e_P + e_D \neq 0 \\ \mu & \text{otherwise} \end{cases} \tag{17}$$

where an exogenous weight $0 < \eta < 1$ determines the relative influences of the prior and of the

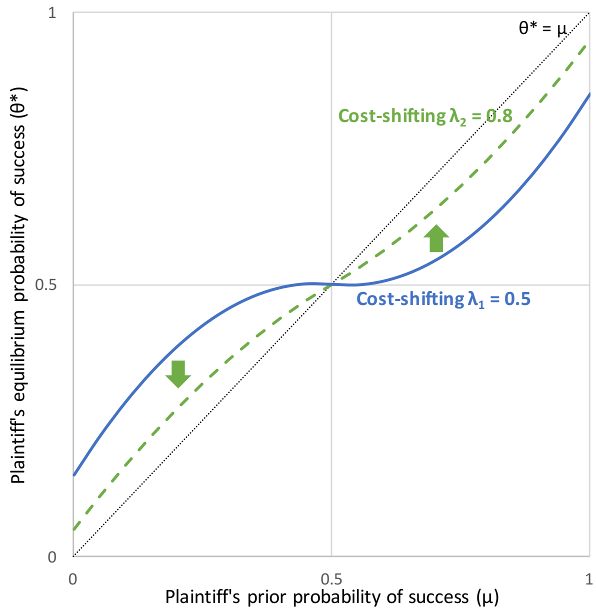

Figure 4: Plaintiff's equilibrium probabilities of success as functions of her prior probability of success under two cost-shifting rules, $\lambda_1 = 0.5$ and $\lambda_2 = 0.8$, in the $\mathbb{L}_k$ Game.

litigation efforts on the posterior probabilities of success.[24] The $\mathbb{L}_k$ Game satisfies Assumption 9 under any pair of cost-shifting rule and cost function.[25] Figure 4 depicts the relationship between Plaintiff's equilibrium probability of success θ^* and her prior probability of success μ under two cost-shifting rules $\lambda_1 = 0.5$ and $\lambda_2 = 0.8$. The blue solid curve (respectively, green dashed curve) depicts θ^* as a function of μ when $\lambda_1 = 0.5$ (respectively, $\lambda_2 = 0.8$). For all $\mu \neq 0.5$, the value of θ^* on the green dashed curve is closer to μ compared to that on the blue solid curve.

8 Remarks on Generality

8.1 Multiple Judges

This subsection demonstrates that the Litigation Game as formulated in section 2 captures uncertainty in respect of the judge who hears and decides the case.

[24]An increase in η represents an increase in the relative weight that the judicial process gives to the relative advantages, and a corresponding decrease in the relative weight that it gives to the litigants' relative effort level.

[25]Using Appendix B, some algebra reveals that $\frac{\partial^2\theta_L}{\partial^2\mu} = \frac{\partial^2\theta_L}{\partial s \partial \mu} = 0$ for all μ, and that $\mu > 0.5$ and $0 < s \leq 1$ imply

$$s\frac{\partial^2\theta_L}{\partial s^2} + \left[1 - \frac{k(1-s^k)}{1+s^k}\right]\frac{\partial\theta_L}{\partial s} \geq 0,$$

holding strictly if $k > 1$. Hence condition (16) holds for all $\lambda \in [0,1]$ and all $k \geq 1$, implying that $\theta_L \in \Theta_8([0,1],[1,+\infty))$.

Consider the following modification of the Litigation Game, called the **Litigation Game with Multiple Judges**. Suppose that a judge chosen from a finite collection of $n \geq 1$ judges will hear the dispute. A judge denoted $i \in \{1, 2, ..., n\}$ rules in favor of Plaintiff with posterior probability $\theta_i(e_P, e_D; \mu)$ where $\theta_i \in \Theta(\{\lambda\}, \{k\})$, and rules in favor of Defendant with probability $1 - \theta_i(e_P, e_D; \mu)$. Before the identity of the judge is revealed, the litigants observe Plaintiff's relative advantages μ, the cost function k and cost-shifting rule λ, and exert effort levels e_P, e_D. It is common knowledge that the litigants assign the prior probability $p_i \geq 0$ to judge i being chosen, where $\sum_{i=1}^n p_i = 1$. Plaintiff and Defendant's payoff functions are respectively $\widetilde{u}_P, \widetilde{u}_D : \mathbb{R}^2_+ \to \mathbb{R}$ where

$$\widetilde{u}_P = \mathbb{E}\{\theta_i[1 - (1 - \lambda)C(e_P)] - (1 - \theta_i)[C(e_P) + \lambda C(e_D)]\},$$

$$\widetilde{u}_D = \mathbb{E}\{-\theta_i[1 + C(e_D) + \lambda C(e_P)] - (1 - \theta_i)(1 - \lambda)C(e_D)\}$$

where $\mathbb{E}$ is the expectation operator with respect to $(p_i)_{i=1}^n$.

Remark 5. *In reality, some courts disclose the identity of the judicial officer randomly assigned to the case only late in the litigation process, sometimes on the day of the hearing. This practice is justified on grounds including promotion of judicial independence and impartiality as well as discouragement of "judge shopping".*[26] *Under this practice, the litigants only find out about the identity of the judge after they have exerted significant litigation efforts. The Litigation Game with Multiple Judges captures this practice by specifying that the litigants exert efforts before they observe the identity of the judge.*

The Litigation Game with Multiple Judges is captured by a special case of the original Litigation Game formulated in section 2. To see this, construct a success function $\theta = \sum_{i=1}^n p_i\theta_i$. Lemma 3 establishes that $\theta \in \Theta(\{\lambda\}, \{k\})$.

Lemma 3. *Consider a finite collection of success functions $\theta_1, \theta_2, ..., \theta_n \in \Theta(\{\lambda\}, \{k\})$. If a success function θ is their convex combination, then $\theta \in \Theta(\{\lambda\}, \{k\})$. Formally, for weights $p_1, p_2, ..., p_n \geq 0$ satisfying $\sum_{i=1}^n p_i = 1$, that $\theta_1, \theta_2, ..., \theta_n \in \Theta(\{\lambda\}, \{k\})$ and $\theta = \sum_{i=1}^n p_i\theta_i$ implies $\theta \in \Theta(\{\lambda\}, \{k\})$.*

An application of Lemma 3 proves that a special case of the original Litigation Game that adopts the success function $\theta = \sum_{i=1}^n p_i\theta_i$ falls within the scope of Assumptions 1-6.

Now, some algebra using Plaintiff's payoff in the Litigation Game with Multiple Judges and the linearity of the expectation operation reveals

$$\widetilde{u}_P = \mathbb{E}\{\theta_i[1 + \lambda C(e_P) + \lambda C(e_D)] - C(e_P) - \lambda C(e_D)\}$$

[26]For example, some courts in Australia and Europe follow this practice. See, generally, Wallace et al. (2014), especially pp. 687-89.

$$= \sum_{i=1}^{n} (p_i\theta_i)[1 + \lambda C(e_P) + \lambda C(e_D)] - C(e_P) - \lambda C(e_D)$$

$$= \theta[1 + \lambda C(e_P) + \lambda C(e_D)] - C(e_P) - \lambda C(e_D) = u_P$$

where u_P is Plaintiff's payoff (given by (2)) in the (original) Litigation Game formulated in section 2. Similarly, obtain $\widetilde{u}_D = u_D$ (given by (3)) for Defendant, where u_D is Defendant's payoff in the Litigation Game.

Hence the Litigation Game with Multiple Judges is a special case of the (original) Litigation Game that adopts the success function $\theta = \sum_{i=1}^{n} p_i\theta_i$. Then the Litigation Game with Multiple Judges has a unique nontrivial Nash equilibrium as established by Proposition 1, and attracts the previous analyses of equilibrium properties of the Litigation Game.

8.2 Arbitrary Judgment Sum

This subsection demonstrates that the Litigation Game as formulated in section 2 captures any positive judgment sum.

Consider the following modification of the Litigation Game, called the **Litigation Game with Arbitrary Judgment Sum**. Suppose the judgment sum which may be awarded to Plaintiff is characterized by an exogenous parameter $J > 0$. The cost function $\bar{C} : \mathbb{R}_+ \to \mathbb{R}$ is homogenous of degree $\bar{k} \geq 1$. Plaintiff and Defendant respectively have payoff functions $\bar{u}_P, \bar{u}_D : \mathbb{R}_+^2 \to \mathbb{R}$ where

$$\bar{u}_P = \theta[J - (1 - \lambda)\bar{C}(e_P)] - (1 - \theta)[\bar{C}(e_P) + \lambda\bar{C}(e_D)],$$

$$\bar{u}_D = -\theta[J + \bar{C}(e_D) + \lambda\bar{C}(e_P)] - (1 - \theta)(1 - \lambda)\bar{C}(e_D).$$

The Litigation Game with Arbitrary Judgment Sum is captured by the baseline Litigation Game as formulated in subsection 2. To see this, consider a baseline Litigation Game with the judgment sum 1 and a cost function $C(\cdot)$ defined by the degree of homogeneity $k = \bar{k}$ and $C(1) = \bar{C}(1)/J$. From equation (2), Plaintiff's payoff function in this game is:

$$u_P = \theta[1 - (1 - \lambda)C(e_P)] - (1 - \theta)[C(e_P) + \lambda C(e_D)]$$

$$u_P = \theta\left[1 - \frac{(1 - \lambda)\bar{C}(1)e_P^k}{J}\right] - (1 - \theta)\left[\frac{\bar{C}(1)e_P^k}{J} + \frac{\lambda\bar{C}(1)e_D^k}{J}\right]$$

$$\Leftrightarrow \qquad Ju_P = \theta\left[J - (1 - \lambda)\bar{C}(1)e_P^k\right] - (1 - \theta)\left[\bar{C}(1)e_P^k + \lambda\bar{C}(1)e_D^k\right] = \bar{u}_P$$

where $\bar{u}_P$ is Plaintiff's payoff function in the Litigation Game with Arbitrary Judgment Sum. Similar steps establish $Ju_D = \bar{u}_D$, where u_D is Defendant's payoff function in the baseline Litigation Game.

Hence each litigant's payoff function in the Litigation Game with Arbitrary Judgment Sum is a positive affine transformation of her payoff function in the baseline Litigation Game that adopts the judgment sum 1 and the cost function $C(\cdot)$ characterized by $k = \bar{k}$ and $C(1) = \bar{C}(1)/J$. Then the Litigation Game with Arbitrary Judgment Sum has a unique nontrivial Nash equilibrium as established by Proposition 1, and attracts the same analyses of equilibrium properties as those that apply to the baseline Litigation Game.

8.3 Relationship with the Tullock Success Function

This subsection considers the extent to which Assumptions 1-6 generalize the Tullock success function, which is commonly used to model litigation.[27]

Consider the following set of properties on success functions. There are $n > 1$ players. The probability of success of player $i \in \{1, ..., n\}$ is denoted $\phi_i(e)$, where $e = (e_1, ..., e_i, ..., e_n)$ is a vector of nonnegative strategies satisfying $e \neq 0$. Let $\phi : e \mapsto (\phi_1(e), ..., \phi_i(e), ..., \phi_n(e))$ be a success function satisfying Assumption 10.

Assumption 10. *The contest success function ϕ satisfies the following properties:*

1. $1 > \phi_i(e) \geq 0$ *and* $\sum_i \phi_i(e) = 1$*; if* $e_i > 0$ *then* $\phi_i(e) > 0$.
2. $\phi_i(e)$ *is strictly increasing in* e_i *and nonincreasing in* e_j, $j \neq i$.
3. $\phi_i(e_1, ..., e_{k-1}, 0, e_{k+1}, ..., e_n) = \frac{\phi_i(e)}{1-\phi_k(e)}$ *for every* $i \neq k$.
4. $\phi_i(e) = \phi_i(xe)$ *for every* i*, every* $x > 0$ *where* $xe = (xe_1, ..., xe_i, ..., xe_n)$.

Building on Skaperdas (1996), Clark and Riis (1998) prove that Assumption 10 holds if and only if the success function ϕ takes the following form

$$\phi_i(e) = \frac{z_i e_i^r}{\sum_{j=1}^n z_j e_j^r}$$

where $r, z_i, z_j > 0$ are constants.

Assumption 10 does not generalize Assumptions 1-6. To see this, consider the success function θ_L given by (17), which is adopted in the $\mathbb{L}_k$ Game. Suppose Defendant does not exert any effort and Plaintiff exerts some positive effort, that is, $e_P > e_D = 0$. Then Plaintiff's probability of success is $\theta_L = \mu\eta + (1-\eta)$ and Defendant's is $1 - \theta_L = (1-\mu)\eta$. That $\theta_L \neq \theta_L/[1-(1-\theta_L)] = 1$ implies property 3 of Assumption 10 is not satisfied. Hence $\theta_L(\cdot)$ falls within the scope of Assumptions 1-6, but outside that of Assumption 10.

[27]See the discussion of the existing literature in section 1.

Remark 6. *In a two-player litigation model, property 3 of Assumption 10 essentially requires that a litigant wins almost surely if the other litigant exerts zero effort. In the real world, that property does not hold because litigants may lose uncontested suits.*[28] *By comparison, Assumptions 1-6 permit a litigant to lose with positive probability even if the other litigant exerts zero effort.*

Within the confines of a two-player model (that is, $n = 2$), the success function ϕ falls within the scope of Assumptions 1-6. To see this, construct a **Tullock Game** by making the following modifications to the Litigation Game:

1. The success function is ϕ, where the parameter z_1 (respectively, z_2) is interpreted to capture Plaintiff's (Defendant's) relative advantages.
2. The cost function $\widehat{C}(\cdot)$ is homogeneous of degree $\widehat{k} \geq \gamma$.
3. Plaintiff's respective payoff functions are $\widehat{u}_P, \widehat{u}_D : \mathbb{R}^2_+ \to \mathbb{R}$ where

$$\widehat{u}_P = \phi[1 - (1-\lambda)\widehat{C}(e_P)] - (1-\phi)[\widehat{C}(e_P) + \lambda\widehat{C}(e_D)]$$

$$\widehat{u}_D = -\phi[1 + \widehat{C}(e_D) + \lambda\widehat{C}(e_P)] - (1-\phi)(1-\lambda)\widehat{C}(e_D).$$

The Tullock Game is equivalent to a special case of the Litigation Game, denoted the $\mathbb{T}_k$ Game. The following properties characterize the $\mathbb{T}_k$ Game:

1. Plaintiff's prior probability is $\mu = \frac{z_1}{z_1+z_2}$.
2. The success function is $\theta(e_P, e_D; \mu) = \phi\left(e_P^{1/r}, e_D^{1/r}\right)$, which is expressed by (13).
3. The cost function $C(\cdot)$ is characterized by $C(1) = \widehat{C}(1)$ and the degree of homogeneity $k = \widehat{k}/\gamma$.
4. Plaintiff and Defendant's respective payoff functions are u_P given by (2) and u_D given by (3).

Some algebra using Appendix B reveals the success function θ_T satisfies Assumptions 1-4, 6 and

$$\frac{\frac{\partial^2}{\partial {e_P}^2}\left(\frac{\theta_T}{1-\lambda\theta_T}\right)}{\frac{\partial}{\partial e_P}\left(\frac{\theta_T}{1-\lambda\theta_T}\right)} \leq 0 \leq \frac{C''(e_P)}{C'(e_P)}$$

where the first weak inequality holds strictly if $\lambda < 1$ and with equality if $\lambda = 1$, and the second weak inequality holds strictly if $k > 1$ and with equality if $k = 1$. Hence condition (1) in Assumption

[28]For example, rule 65 of the U.S. Federal Rules of Civil Procedure imposes stringent restrictions on the availability of ex parte temporary restraining orders, which are granted by judicial discretion. See the opinion of the U.S. Supreme Court in the case of Granny Goose Foods, Inc. v. Brotherhood of Teamsters and Auto Truck Drivers Local No. 70 of Alameda County, 415 U.S. 423 (1974).

5 is satisfied if and only if $k > 1$ or $\lambda < 1$; that is, $\theta_T \in \Theta([0,1),[1,+\infty)) \cup \Theta([0,1],(1,+\infty))$, but $\theta_T \notin \Theta(\{1\},\{1\})$. If the cost-shifting rule permits less than full recovery ($\lambda < 1$) or the cost function is strictly convex ($k > 1$), then Proposition 1 proves the existence and uniqueness of a nontrivial Nash equilibrium in the $\mathbb{T}_k$ Game (and, equivalently, in the Tullock Game).

Given the interested cost-shifting rule, Assumption 5 provides a sufficient condition to ensure each litigant's payoff function is strictly quasiconcave in her effort. Assumption 5 is crucial to our equilibrium-existence result (Proposition 1), and explains the existence results regarding the English rule ($\lambda = 1$) in the existing literature. Using a special case of the $\mathbb{L}_k$ Game, Plott (1987) (at 189) proves the English rule induces a nontrivial Nash equilibrium if the litigants' efforts do *not* completely determine posterior probabilities of success. Plott (1987) (at 189) also proves the English rule does *not* induce a nontrivial Nash equilibrium if the litigants' efforts completely determine posterior probabilities of success. Using special cases of the $\mathbb{T}_k$ Game, Farmer and Pecorino (1999) (at 281-82) and Carbonara et al. (2015) (at 8-9) establish the English rule induces a nontrivial Nash equilibrium if and only if litigation efforts are insufficiently influential on posterior probabilities of success. The conditions guaranteeing equilibrium existence in their models are special cases of the present Assumption 5. However, Assumption 5 does not cover the litigation models of Hause (1989), Hyde and Williams (2002). Their models allow for generally-formulated success functions and divergent beliefs regarding posterior probabilities of success, but assume the English rule induces a Nash equilibrium.[29] With a minor modification to introduce divergent beliefs, the $\mathbb{T}_k$ Game can provide a counter-example that satisfies the conditions imposed by Hause (1989) or Hyde and Williams (2002) but does not induce a nontrivial Nash equilibrium under the English rule.

8.4 How Changes in Relative Advantages Affect Welfare and Expenditure

A conventional wisdom in the existing literatures on Tullock contest models is a more asymmetric contest reduces rent dissipation, thereby increasing the collective welfare of the contestants.[30] This subsection shows that this conventional wisdom does not necessary hold in the Litigation Game. It further proposes a sufficient condition that ensures this conventional wisdom holds.

In the Litigation Game, the contest between the litigants becomes more asymmetric as the prior becomes more one-sided. The collective welfare of the litigants is given by the notion of litigation welfare U^*, which is the negative of litigation expenditure (see section 5). Our previous assumptions are not sufficient for answering the question whether litigation expenditure increases or decreases when the prior becomes more one-sided. Figure 5 illustrates potential complexities.

[29] Hause (1989), pp. 165-66, and Hyde and Williams (2002), pp.137-39, 147-48.

[30] For such a result in a Tullock contest with asymmetric technologies, see Cornes and Hartley (2005) 940-41. For such a result in a model of litigation, see Carbonara et al. (2015) 6-7.

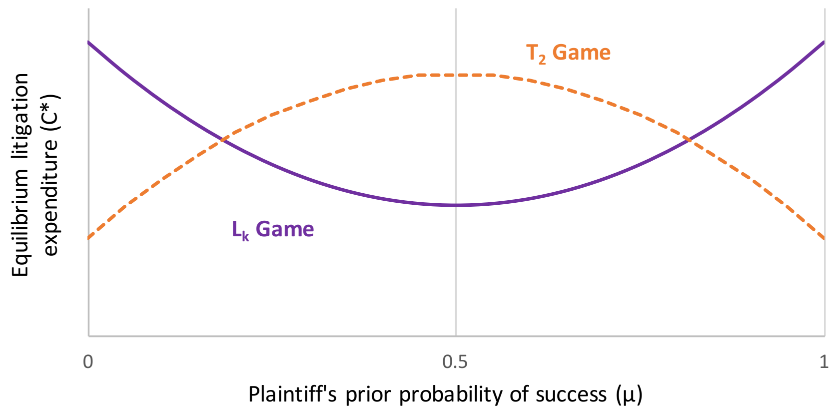

Figure 5: Litigation expenditure as a function of Plaintiff's prior probability of success in the $\mathbb{L}_k$ Game and $\mathbb{T}_2$ Game.

Figure 5 depicts litigation expenditure (C^*) as a function of Plaintiff's prior probability of success μ in the $\mathbb{L}_k$ Game and in another special case of the Litigation Game, called the $\mathbb{T}_2$ Game, that adopts the Tullock success function θ_T given by (13) and a homogenous cost function of degree $k = 2$. The purple solid curve represents the $\mathbb{L}_k$ Game and the orange dashed curve the $\mathbb{T}_2$ Game. In the $\mathbb{L}_k$ Game, litigation expenditure increases when the prior becomes more one-sided. By contrast, in the $\mathbb{T}_2$ Game, litigation expenditure decreases when the prior becomes more one-sided.

Motivated by Figure 5, we propose Assumption 11 as a sufficient condition for ensuring that litigation expenditure increases when the prior becomes more one-sided

Assumption 11. *Suppose Plaintiff's case is relatively more advantageous (that is, $\mu > 0.5$) and her effort is no less than Defendant's positive (that is, $0 < s \leq 1$). Then at least one of the following conditions holds:*

$$\lambda(2-\lambda)\theta_\mu\left\{s\theta_{ss} + \left[1 - \frac{k(1-s^k)}{(1+s^k)}\right]\theta_s\right\} \geq \beta\theta_{s\mu} \tag{18}$$

or

$$\lambda(2-\lambda)\theta_\mu\left\{s\theta_{ss} + \left[1 - \frac{k\lambda(2\theta-1)}{2-\lambda}\right]\theta_s\right\} \geq \alpha\theta_{s\mu}. \tag{19}$$

Assumption 11 imposes restrictions on the curvature of the success function θ with respect to Defendant's relative effort s and Plaintiff's prior probability of success parameter μ. For example, the success function θ_L given by (17), which the $\mathbb{L}_k$ Game adopts, satisfies condition (18) strictly (respectively, with equality) if the applicable pair of cost-shifting rule and cost function satisfies $\lambda > 0$ and $k > 1$ (respectively, $\lambda = 0$ or $k = 1$).

Proposition 5 proves adding Assumption 11 is sufficient for concluding that as the prior becomes more favorable to one litigant, litigation expenditure does not decrease. To facilitate presentation, let $\Theta_{11}(\{\lambda\},\{k\})$ denote the set of twice continuously differentiable functions $\theta : \mathbb{R}_+^2 \to [0,1]$ that satisfy Assumption 11 when the applicable pair of cost-shifting rule and cost function is characterized by (λ, k).

Proposition 5. *Suppose the success function $\theta \in \Theta(\{\lambda\},\{k\}) \cap \Theta_{11}(\{\lambda\},\{k\})$. Then as the prior becomes more favorable to one litigant, (equilibrium) litigation expenditure does not decrease; it increases if condition (18) or (19) holds strictly. Formally, that $\mu > 0.5$ (respectively, $\mu < 0.5$) implies $\frac{dC^*}{d\mu} \geq 0$ (respectively, $\frac{dC^*}{d\mu} \leq 0$), holding strictly if condition (18) or (19) holds strictly.*

Proposition 5 identifies a class of success functions — those that satisfy Assumption 11 in addition to Assumptions 1-6 — under which litigation welfare weakly increases (and litigation expenditure weakly decreases) when the prior becomes more balanced. Any one of these success functions violates the conventional wisdom that a more asymmetric contest increases the collective welfare of the contestants.

We now propose alternative conditions the satisfaction of which is sufficient to uphold this conventional wisdom. Consider Assumption 12, which imposes restrictions on the curvature of the success function θ with respect to Defendant's relative effort s and Plaintiff's prior probability of success μ.

Assumption 12. *Suppose Plaintiff's case is relatively more advantageous (that is, $\mu > 0.5$) and her effort is no less than some positive effort by Defendant (that is, $0 < s \leq 1$). Then at least one of the following conditions holds:*

$$\lambda(2-\lambda)\theta_\mu\left\{s\theta_{ss} + \left[1 - \frac{k(1-s^k)}{(1+s^k)}\right]\theta_s\right\} \leq \beta\theta_{s\mu} \tag{20}$$

or

$$\lambda(2-\lambda)\theta_\mu\left\{s\theta_{ss} + \left[1 - \frac{k\lambda(2\theta-1)}{2-\lambda}\right]\theta_s\right\} \leq \alpha\theta_{s\mu}. \tag{21}$$

Assumption 12 captures the Tullock success function θ_T given by (13). Some algebra reveals that θ_T satisfies condition (21) strictly (respectively, with equality) if the applicable cost function is characterized by $\lambda < 1$ (respectively, $\lambda = 1$)

Proposition 6 proves that adding Assumption 11 is sufficient for concluding that as the prior becomes more favorable to one litigant, litigation expenditure does not increase. To facilitate presentation, let $\Theta_{12}(\{\lambda\},\{k\})$ denote the set of twice continuously differentiable functions $\theta : \mathbb{R}_+^2 \to [0,1]$ that satisfy Assumption 11 when the applicable pair of cost-shifting rule and cost function is characterized by (λ, k).

Proposition 6. *Suppose the success function* $\theta \in \Theta(\{\lambda\}, \{k\}) \cap \Theta_{12}(\{\lambda\}, \{k\})$. *Then as the prior becomes more favorable to one litigant, (equilibrium) litigation expenditure does not increase; it decreases if condition (20) or (21) holds strictly. Formally, if* $\mu > 0.5$ *(respectively,* $\mu < 0.5$*), then* $\frac{dC^*}{d\mu} \leq 0$ *(respectively,* $\frac{dC^*}{d\mu} \geq 0$*), holding strictly if condition (20) or (21) holds strictly.*

Proposition 6 proves that if the success function satisfies Assumption 11 in addition to Assumptions 1-6, then litigation welfare weakly decreases (and litigation expenditure weakly increases) when the prior becomes more balanced. Such a success function respects the conventional wisdom that a more asymmetric contest increases the collective welfare of the contestants.

9 Normative Discussion

This section discusses the normative implications and limitations of our positive predictions regarding how cost shifting affects legal predictability, accuracy and expenditure. Fixing the pre-litigation behaviors (for example, the injurious activity in a tort case), section 2 develops a contest model of litigation to analyze the litigants' strategic interaction under different cost-shifting rules governing the allocation of litigation costs. This Litigation Game generalizes large classes of success functions, cost functions and cost-shifting rules. In particular, the present characterization of cost-shifting rules covers the extreme ones that shift either all or none of the winner's costs to the loser, as well as the intermediate ones that shift a proportion of such costs. Premising on the unique nontrivial Nash equilibrium that Section 3 finds and characterizes, the positive predictions of the Litigation Game are general and thus facilitate a normative analysis of whole classes of judicial systems. Yet the Litigation Game is about efforts to litigate; it does not model every behavior that is or should be subject to legal regulation. The following thus elicits the normative implications of cost shifting to the extent that litigation efforts are the only variables, and discusses how these results facilitate future research into other normatively relevant variables.

First, cost shifting affects the policy objective of improving predictability in the judicial determination of the outcomes of cases. Vague standards (such as "reasonableness"), the open texture of language and judicial discretion are among the factors that render imperfectly predictable the application of substantive law in a case and therefore the outcome of the case. From a utilitarian perspective, improved legal predictability better enables individuals and businesses to make plans for the future. Legal predictability is also attractive to the liberal ideal of the rule of law because it gives fair notice to individuals of the legal consequences of their choices and holds public officials accountable for their exercises of public powers.[31] Fixing the pre-litigation behaviors, Corollary 8

[31]For a survey of the philosophical literatures on the rule of law and its relationship with legal predictability and accessibility, see *The Rule of Law* (June 22, 2016), Stanford Encyclopaedia of Philosophy, `https://plato.stanford.edu/entries/rule-of-law/`. For a survey of the seminal papers on the complex relationship between

in section 6 establishes that more cost shifting unambigously drives equilibrium litigation efforts to improve legal predictability. This result reveals the desirability of cost shifting to the extent that litigation efforts are the only variables and the society aims to improve legal predictability. Future research may consider a pre-game in which the litigants choose their pre-litigation behaviors, and study how more cost shifting by improving legal predictability in the Litigation Game affects those choices.

Secondly, there is a complex relationship between cost-shifting rules and the policy objective of deciding cases accurately to reflect their inherent merits. Using a standard model of tort liability for harmful activities, Kaplow and Shavell (1996) illustrate that greater accuracy ameliorates the misalignment between the levels of precautions that informed injurers take and the magnitude of the harm that they are likely to generate. However, they also reveal that greater accuracy has limited incentive-alignment effects when the injurers are not informed. They fix the amount of litigation costs and vary the pre-litigation behaviors (levels of precaution and decisions to learn about harm), while we fix the pre-litigation behaviors and vary litigation costs. We nonetheless reach a similarly complex conclusion regarding how cost-shifting rule by determining equilibrium litigation efforts (and costs) affects accuracy. Assuming that a litigant's prior probability of success (that is, her relative advantages before exerting efforts, see section 2) accurately reflects the inherent merits of her case,[32] section 7 reveals that the properties of the success function determine whether more cost shifting increases or decreases the difference between her prior and equilibrium probabilities of success. That difference, which we call distortion (see (14) in section 7), measures accuracy in the expected outcome of the case. Section 7 also provides exact sufficient conditions (Assumptions 8, 9) for ensuring that distortion is monotonic with the proportion of costs recoverable. Future research may seek empirical evidence on whether a particular judicial system satisfies one of these conditions to ascertain the relationship between cost shifting and accuracy in that system.

Thirdly, the present results regarding litigation efforts under different cost-shifting rules do not resolve the question of how to optimize the amount of civil suits. Shavell (1997) observes that a private litigant's decisions to bring suit, settle or incur litigation costs are socially suboptimal due to two externalities: the negative externality arising from her lack of incentives to consider the costs that she exerts on others, such as the other litigant or the state; and the positive externality arising from her lack of incentives to consider the social benefits of litigation, such as deterrence of future injuries. In particular, he uses examples with fixed litigation costs to illustrate that a suboptimal amount of suits arise when the cost-shifting rule shifts all of the winner's costs to the loser. Section 5 reveals a similarly complex conclusion when litigation costs vary and cost-shifting rules shift

high probability suits induced by cost shifting and the substantive behaviors giving rise to the dispute, see Katz and Sanchirico (2012), pp. 278-86.

[32] The existing models of cost shifting generally adopt this assumption (for example, Plott 1987, p. 188, Katz 1988, p. 129-30, Gong and McAfee 2000, p. 223, Carbonara et al. 2015, p. 5).

some proportion of the winner's costs. Assuming that litigation takes place, Corollaries 6-7 reveal that more cost shifting increases private litigation costs in equilibrium at least in cases where the litigants' relative advantages are sufficiently balanced or the cost function is sufficiently convex. If we account for the possibly of settlement in these cases, then more cost shifting by increasing the size of the surplus arising from settlement incentivizes the litigants to settle rather than litigate. Building on the present comprehensive analysis of private costs in litigation, future research may consider how cost shifting affects the pre-litigation decisions to file suit or settle and how these decisions relate to the society's interests.

Finally, future research may introduce additional features to the present model of litigation. For example, introducing divergence in the litigants' valuation of the judgment sum would enable an analysis of the judge's decisions regarding both the winner of the case and the magnitude of the judgment sum. Introducing budget constraints also would enable an analysis of the implications of divergence in the litigants' wealth levels or provisions of legal aid. Moreover, it may be fruitful to study cost-shifting rules with the incomplete-information Tullock contest model developed by Einy et al. (2015).

A Appendix: Proofs

This appendix contains the proofs.

Lemma 4 is a technical lemma which will be used to prove other Lemmas, Propositions and Corollaries.

Lemma 4. *On the subdomain $\mathbb{R}^2_{++}$, the success function $\theta(\cdot)$ satisfies the following properties:*

1. $\mu > 0.5$ *(respectively,* $= 0.5$, < 0.5*) and* $e_P = e_D$ *imply* $\theta > 0.5$ $(= 0.5, < 0.5)$.
2. $\frac{\partial}{\partial e_D}(1-\theta) > 0$, $\frac{\partial^2}{\partial e_D^2}(1-\theta) < 0$.
3. $$\frac{\frac{\partial^2}{\partial e_D^2}\left(\frac{1-\theta}{1-\lambda(1-\theta)}\right)}{\frac{\partial}{\partial e_D}\left(\frac{1-\theta}{1-\lambda(1-\theta)}\right)} < \frac{C''(e_D)}{C'(e_D)}. \tag{22}$$
4. $k = \lambda = 1 \Rightarrow \lim_{s\to+\infty} 1-\theta < 1$.
5. $$\frac{\partial s}{\partial e_P} = -\frac{s}{e_P}, \qquad \frac{\partial s}{\partial e_D} = \frac{s}{e_D}, \qquad \frac{\partial \theta}{\partial e_P} = -\frac{s\theta_s}{e_P}, \qquad \frac{\partial}{\partial e_D}(1-\theta) = -\frac{s\theta_s}{e_D},$$
$$\frac{\partial^2 \theta}{\partial e_P^2} = \frac{s^2\theta_{ss}}{e_P^2} + \frac{2s\theta_s}{e_P^2}, \qquad \frac{\partial^2}{\partial e_D^2}(1-\theta) = -\frac{s^2\theta_{ss}}{e_D^2}, \qquad \frac{\partial^2\theta}{\partial e_P \partial e_D} = -\frac{s(\theta_s + s\theta_{ss})}{e_P e_D}.$$

6. $\theta_s < 0$, $\theta_{ss} > 0$.
7. $s\theta_{ss} < -(k+1)\theta_s - \frac{2\lambda s\theta_s^2}{(1-\lambda\theta)}$.
8. $s\theta_{ss} > (k-1)\theta_s + \frac{2\lambda s\theta_s^2}{[1-\lambda(1-\theta)]}$.
9. $-\lambda(2-\lambda)s\theta_s < k(1-\lambda\theta)[1-\lambda(1-\theta)]$.
10. $\frac{\partial}{\partial s}(-s^2\theta_s) > 0$.

Proof of Lemma 4

Part 1

Let $e_P = e_D = e_1$ for some arbitrary $e_1 > 0$. First suppose $\mu = 0.5$. Then use Assumption 1 to obtain

$$\theta(e_1, e_1; 0.5) = 1 - \theta(e_1, e_1; 1 - 0.5) = 1 - \theta(e_1, e_1; 0.5)$$

which implies $\theta(e_1, e_1, \mu) = 0.5$. The results for $\mu > 0.5$ and $\mu < 0.5$ follow from Assumption 3.

Parts 2-4

Fix μ and $e_P = e_1$ for some arbitrary $e_1 > 0$. Then Assumption 1 implies $1 - \theta(e_1, e_D, \mu) = \theta(e_D, e_1; 1-\mu)$. Hence

$$\frac{\partial}{\partial e_D}(1 - \theta(e_1, e_D; \mu)) = \frac{\partial}{\partial e_D}\theta(e_D, e_1; 1-\mu) > 0$$

where the inequality follows from Assumption 4.

A similar approach establishes $\frac{\partial^2}{\partial e_D^2}(1 - \theta(e_P, e_D; \mu)) < 0$, condition (22), and part 4.

Part 5

The chain rule and some algebra will give

$$\frac{\partial s}{\partial e_P} = \frac{\partial}{\partial e_P}\left(\frac{e_D}{e_P}\right) = -\frac{e_D}{e_P^2} = -\frac{s}{e_P} \qquad \frac{\partial s}{\partial e_D} = \frac{\partial}{\partial e_D}\left(\frac{e_D}{e_P}\right) = \frac{1}{e_P} = \frac{s}{e_D}$$

$$\frac{\partial \theta}{\partial e_P} = \frac{\partial \theta}{\partial s}\frac{\partial s}{\partial e_P} = -\frac{\partial \theta}{\partial s}\frac{s}{e_P} \qquad \frac{\partial(1-\theta)}{\partial e_D} = -\frac{\partial \theta}{\partial s}\frac{\partial s}{\partial e_D} = -\frac{\partial \theta}{\partial s}\frac{s}{e_D}$$

$$\frac{\partial^2 \theta}{\partial e_P^2} = \frac{\partial}{\partial e_P}\left(\frac{\partial \theta}{\partial s}\frac{\partial s}{\partial e_P}\right) = \frac{\partial^2 \theta}{\partial s^2}\left(\frac{\partial s}{\partial e_P}\right)^2 + \frac{\partial \theta}{\partial s}\frac{\partial^2 s}{\partial e_P^2} = \frac{s^2}{e_P^2}\frac{\partial^2 \theta}{\partial s^2} + \frac{2s}{e_P^2}\frac{\partial \theta}{\partial s}$$

$$\frac{\partial^2(1-\theta)}{\partial e_D^2} = -\frac{\partial^2 \theta}{\partial s^2}\frac{1}{e_P^2} = -\frac{\partial^2 \theta}{\partial s^2}\frac{s^2}{e_D^2}$$

$$\frac{\partial^2 \theta}{\partial e_P \partial e_D} = \frac{\partial}{\partial e_D}\left(-\frac{\partial \theta}{\partial s}\frac{e_D}{e_P^2}\right) = -\frac{\partial^2 \theta}{\partial s^2}\frac{1}{e_P}\frac{e_D}{e_P^2} - \frac{\partial \theta}{\partial s}\frac{1}{e_P^2} = -\frac{s(\theta_s + s\theta_{ss})}{e_P e_D} = \frac{\partial^2 \theta}{\partial e_D \partial e_P},$$

where the last equality uses Young's Theorem.

Part 6

The chain rule and the properties $\partial\theta/\partial e_P > 0$, $s/e_P > 0$ imply $\theta_s < 0$.

Use the expression of $\frac{\partial^2}{\partial e_D^2}(1-\theta)$ in part 5 and the property $\frac{\partial^2}{\partial e_D^2}(1-\theta) < 0$ to obtain $\theta_{ss} > 0$.

Part 7

Apply Euler's theorem for homogeneous functions to obtain

$$C(e_P) = e_P^k C(1), \qquad C'(e_P) = \frac{k}{e_P} C(e_P) = k e_P^{k-1} C(1), \qquad C''(e_P) = k(k-1)e_P^{k-2} C(1).$$

Some algebra will reveal that condition (1) holds if and only if

$$\frac{(1-\lambda\theta)\partial^2\theta/\partial e_P^2 + 2\lambda(\partial\theta/\partial e_P)^2}{\partial\theta/\partial e_P} < \frac{(1-\lambda\theta)(k-1)}{e_P}.$$

Then use part 5 to obtain

$$\frac{(1-\lambda\theta)s\theta_{ss} + 2\theta_s(1-\lambda\theta) + 2\lambda s\theta_s^2}{-\theta_s} < (1-\lambda\theta)(k-1)$$

$$\Leftrightarrow s\theta_{ss} < -(k+1)\theta_s - \frac{2\lambda s\theta_s^2}{(1-\lambda\theta)}.$$

Part 8

Some algebra will reveal that condition (22) holds if and only if

$$\frac{-[1-\lambda(1-\theta)]\partial^2\theta/\partial e_D^2 + 2\lambda(\partial\theta/\partial e_D)^2}{-\partial\theta/\partial e_D} < \frac{[1-\lambda(1-\theta)](k-1)}{e_D}.$$

Then use part 5 to obtain

$$\frac{-[1-\lambda(1-\theta)]s\theta_{ss} + 2\lambda s\theta_s^2}{-\theta_s} < [1-\lambda(1-\theta)](k-1)$$

$$\Leftrightarrow (k-1)\theta_s + \frac{2\lambda s\theta_s^2}{[1-\lambda(1-\theta)]} < s\theta_{ss}.$$

Part 9

Using $\theta_s < 0$, some algebra will derive the results from parts 7 and 8.

Part 10

Take the partial derivative of $-s^2\theta_s$ with respect to s to obtain

$$\frac{\partial}{\partial s}(-s^2\theta_s) = -2s\theta_s - s^2\theta_{ss}.$$

Then use the expression of $\frac{\partial^2\theta}{\partial e_P^2}$ in part 5 and the property $\frac{\partial^2\theta}{\partial e_P^2} < 0$ to obtain the result. $\square$

Proof of Lemma 1

This proof establishes the result for Plaintiff. Defendant's result follows symmetric steps. This proof takes the following steps: (*i*) establish that if Plaintiff's FOC holds at a pair of efforts, then her SOC is negative at that pair; (*ii*) using the results established in step (*i*), a theorem by Diewert et al. (1981) proves that Plaintiff's payoff function is strictly quasiconcave in her own effort.

*Step (*i*)*

Take the partial derivatives of Plaintiff's payoff function in (2) with respect to her effort e_P to obtain

$$\frac{\partial u_P}{\partial e_P} = \frac{\partial\theta}{\partial e_P}[1 + \lambda C(e_P) + \lambda C(e_D)] - (1 - \lambda\theta)C'(e_P) \tag{23}$$

$$\frac{\partial^2 u_P}{\partial e_P^2} = \frac{\partial^2\theta}{\partial e_P^2}[1 + \lambda C(e_P) + \lambda C(e_D)] + 2\lambda\frac{\partial\theta}{\partial e_P}C'(e_P) - (1 - \lambda\theta)C''(e_P). \tag{24}$$

Suppose Plaintiff's FOC holds, then some algebra using equation (23) reveals

$$1 + \lambda C(e_P) + \lambda C(e_D) = \frac{(1 - \lambda\theta)C'(e_P)}{\partial\theta/\partial e_P}.$$

A substitution exercise using equation (24) gives

$$\frac{\partial^2 u_P}{\partial e_P^2} = \frac{\partial^2\theta}{\partial e_P^2}\left[\frac{(1 - \lambda\theta)C'(e_P)}{\partial\theta/\partial e_P}\right] + 2\lambda\frac{\partial\theta}{\partial e_P}C'(e_P) - (1 - \lambda\theta)C''(e_P)$$

$$= \frac{C'(e_P)}{(1 - \lambda\theta)}\left[\frac{(1 - \lambda\theta)\frac{\partial^2\theta}{\partial e_P^2} + 2\lambda\left(\frac{\partial\theta}{\partial e_P}\right)^2}{(1 - \lambda\theta)\frac{\partial\theta}{\partial e_P}} - \frac{C''(e_P)}{C'(e_P)}\right] < 0$$

where the last inequality uses condition (1) in Assumption (5).

*Step (*ii*)*

Corollary 9.3 of Diewert et al. (1981) holds that a twice continuously differentiable function f defined on an open S is strictly quasiconcave if and only if $x^0 \in S$, $v^T v = 1$ and $v^T\nabla f(x^0)v = 0$ implies $v^T\nabla^2 f(x^0)v < 0$; or $v^T\nabla^2 f(x^0)v = 0$ and $g(t) \equiv f(x^0+tv)$ does not attain a local minimum at $t = 0$. We apply their result.

Fix Defendant's effort $e_D = e_1$ for some arbitrary $e_1 > 0$, and consider Plaintiff's payoff function $u_P(\cdot)$. Suppose $e_P > 0$, $v^T v = 1$ and

$$0 = v^T\nabla u_P(e_P, e_1; \mu, \lambda, k)v = v^T\frac{\partial}{\partial e_P}u_P(e_P, e_1; \mu, \lambda, k)v.$$

That $v^T v = 1$ implies $v \neq 0$. Hence

$$\frac{\partial}{\partial e_P} u_P(e_P, e_1; \mu, \lambda, k) = 0.$$

Then step (i) proves:

$$\frac{\partial^2}{\partial e_P^2} u_P(e_P, e_1; \mu, \lambda, k) < 0$$

where

$$\frac{\partial^2}{\partial e_P^2} u_P(e_P, e_1; \mu, \lambda, k) = \nabla^2 u_P(e_P, e_1; \mu, \lambda, k).$$

That $v \neq 0$ implies $v^T \nabla^2 u_P(e_P, e_1) v < 0$. Hence an application of Corollary 9.3 of Diewert et al. (1981) proves Plaintiff's payoff function is strictly quasiconcave in her own effort. □

Proof of Lemma 2

Define a function $h : \mathbb{R}_{++} \to \mathbb{R}$ by:

$$h(s) = 1 - \lambda\theta - s^k[1 - \lambda(1 - \theta)]. \tag{25}$$

The first three steps of this proof establishes the existence of an s^* such that $h(s^*) = 0$, and its value relative to 0.5, in the following three cases: (i) $\mu = 0.5$ or $\lambda = 0$; (ii) $\mu < 0.5$ and $\lambda > 0$; and (iii) $\mu > 0.5$ and $\lambda > 0$. The forth step establishes uniqueness.

*Step (*i*)*

Suppose $\mu = 0.5$. Then part 1 of Lemma 4 implies that choosing $s^* = 1$ induces $\theta = 0.5 = 1 - \theta$. Hence $h(1) = 0$.

Now suppose $\lambda = 0$. Then choosing $s^* = 1$ induces $h(s^*) = 0$.

*Step (*ii*)*

Suppose $\mu < 0.5$ and $\lambda > 0$. Then part 1 of Lemma 4 implies that $s = 1$ induces $\theta(1; \mu) < 0.5 < 1 - \theta(1; \mu)$. Some algebra and the property $0 < \lambda \leq 1$ give

$$1 - \lambda\theta(1; \mu) > 1 - \lambda(1 - \theta(1; \mu)) \qquad \Leftrightarrow \qquad h(1) > 0.$$

Now consider the limit of $h(s)$ as s approaches $+\infty$. Some algebra reveals

$$\lim_{s\to+\infty} h(s) = \lim_{s\to+\infty} \left(1 - \lambda\theta - s^k[1 - \lambda(1 - \theta)]\right) = 1 - \lim_{s\to+\infty} (\lambda\theta) - \lim_{s\to+\infty} \left(\frac{1 - \lambda(1 - \theta)}{s^{-k}}\right).$$

Recall the property $\theta_s < 0$ from Lemma 4. Consider two scenarios:

1. Suppose $\lambda < 1$ or $\lim_{s\to+\infty} \theta > 0$. Then $0 < \lim_{s\to+\infty}(\lambda\theta) < 1$ and $\lim_{s\to+\infty}\left(\frac{1-\lambda(1-\theta)}{s^{-k}}\right) = +\infty$. These results imply $\lim_{s\to+\infty} h(s) = -\infty < 0$.
2. Suppose $\lambda = 1$ and $\lim_{s\to+\infty} \theta = 0$. Then part 4 of Lemma 4 implies $k > 1$. Some algebra

reveals $\lim_{s\to+\infty} \lambda\theta = 0$, $\lim_{s\to+\infty}[1-\lambda(1-\theta)] = 0$ and, by the L'Hospital's Rule:

$$\lim_{s\to+\infty}\left(\frac{1-\lambda(1-\theta)}{s^{-k}}\right) = \lim_{s\to+\infty}\left(\frac{\frac{\partial}{\partial s}[1-\lambda(1-\theta)]}{\frac{\partial}{\partial s}\left[s^{-k}\right]}\right) = \lim_{s\to+\infty}\left(\frac{\theta_s}{-ks^{-k-1}}\right)$$

$$= \lim_{s\to+\infty}\left(\frac{-s^2\theta_s}{ks^{1-k}}\right) = +\infty,$$

where the last inequality uses $\frac{\partial}{\partial s}(-s^2\theta_s) > 0$ from part 10 of Lemma 4 and the property $k > 1$. These results imply $\lim_{s\to+\infty} h(s) < 0$.

Hence using the results $h(1) > 0$ and $\lim_{s\to+\infty} h(s) < 0$, the intermediate value theorem implies that there exists some $1 < s^* < +\infty$ such that $h(s^*) = 0$.

*Step (*iii*)*

Suppose $\mu > 0.5$ and $\lambda > 0$. Define a new function $h_1(s)$ by:

$$h_1(s) = h(s)/s^k = \frac{1-\lambda\theta}{s^k} - [1-\lambda(1-\theta)]. \tag{26}$$

Part 1 of Lemma 4 implies that $s = 1$ induces $\theta(1;\mu) > 0.5 > 1-\theta(1;\mu)$. Some algebra and the property $0 < \lambda \leq 1$ give:

$$1-\lambda\theta(1;\mu) < 1-\lambda(1-\theta(1;\mu)) \qquad \Leftrightarrow \qquad h_1(1) < 0.$$

Now, consider the limit of $h_1(s)$ as s approaches 0

$$\lim_{s\to 0} h_1(s) = \lim_{s\to 0}\left(\frac{1-\lambda\theta}{s^k} - [1-\lambda(1-\theta)]\right) = \lim_{s\to 0}\left(\frac{1-\lambda\theta}{s^k}\right) - 1 + \lim_{s\to 0}[\lambda(1-\theta)].$$

Consider two scenarios:

1. Suppose $\lambda < 1$ or $\lim_{s\to 0}\theta < 1$. Then $\lim_{s\to 0}(\lambda\theta) < 1$, which implies $\lim_{s\to 0} h_1(s) = +\infty > 0$.
2. Suppose $\lambda = 1$ and $\lim_{s\to 0}\theta = 1$. Then Assumption 6 implies $k > 1$. Some algebra reveals $\lim_{s\to 0}(\lambda\theta) = 1$ and, by the L'Hospital's Rule:

$$\lim_{s\to 0}\left(\frac{1-\lambda\theta}{s^k}\right) = \lim_{s\to 0}\left(\frac{\frac{\partial}{\partial s}(1-\lambda\theta)}{\frac{\partial}{\partial s}(s^k)}\right) = \lim_{s\to 0}\left(\frac{-\theta_s}{ks^{k-1}}\right) = +\infty$$

   where the last equality uses the properties $k > 1$, $\theta_s < 0$ and $\theta_{ss} > 0$. These results imply $\lim_{s\to 0} h_1(s) = +\infty > 0$.

Using the results $h_1(1) < 0$ and $\lim_{s\to 0} h_1(s) > 0$, the intermediate value theorem implies that there exists some $0 < s^* < 1$ such that $h_1(s^*) = 0$. Then use the definition of $h_1(\cdot)$ in equation (26) to obtain $h(s^*) = 0$.

*Step (*iv*)*

The function $h(\cdot)$ is continuously differentiable. Differentiate it to obtain:

$$h'(s) = -\lambda(1+s^k)\theta_s - ks^{k-1}[1-\lambda(1-\theta)] = -\frac{ks^k[1-\lambda(1-\theta)] + \lambda(1+s^k)s\theta_s}{s}.$$

Steps (*i*)-(*iii*) prove the existence of some $s^* > 0$ that satisfies $h(s^*) = 0$. Choose one such s^* and consider $s = s^*$. Then some algebra gives

$$s^k = \frac{1-\lambda\theta}{1-\lambda(1-\theta)} \qquad\qquad (1+s^k) = \frac{2-\lambda}{1-\lambda(1-\theta)}$$

and a substitution exercise reveals

$$h'(s) = -\frac{k(1-\lambda\theta)[1-\lambda(1-\theta)] + \lambda(2-\lambda)s\theta_s}{s[1-\lambda(1-\theta)]} < 0$$

where the last inequality uses part 9 of Lemma 4. Hence $h'(s) < 0$ whenever $h(s) = 0$.

Now suppose, for a contradiction, that there exist two different $s^{*'} > s^* > 0$ satisfying: $h(s^{*'}) = h(s^*) = 0$; and $h(s') \neq 0$ for all $s^* < s' < s^{*'}$. Then that $h'(s^*), h'(s^*) < 0$ implies for some very small $\epsilon > 0$, we have $h(s^* + \epsilon) < 0$ and $h(s^{*'} - \epsilon) > 0$. Then the intermediate value theorem implies there exists some $s'' > 0$ satisfying $s^* < s'' < s^{*'}$ and $h(s'') = 0$, a contradiction. Hence there exists at most one s^* satisfying $h(s^*) = 0$. □

Proof of Proposition 1

This proof will first establish that the pair (e_P^*, e_D^*) satisfies both Plaintiff and Defendants' FOCs in system (4), thereby characterizing a Nash equilibrium. It will then prove the other direction. The uniqueness of Nash equilibrium follows from the property that the pair (e_P^*, e_D^*) is unique due to Lemma 2. An application of Lemma 2 gives the relative levels of e_P^*, e_D^*, and an application of Corollary 3 the size of $\theta(e_P^*, e_D^*; \mu)$ relative to 0.5.

*Step (*i*)*

Let $s = s^*$ and consider the pair of effort levels e_P^*, e_D^*. Use the expressions for e_P^*, e_D^* to obtain $e_D^*/e_P^* = s^* = s$. Then, from the expression for e_P^*, obtain

$$(1+s)^k e_P^{*k} = A^k = \frac{-s(1+s)^k\theta_s}{C(1)\left[ks^k[1-\lambda(1-\theta)] + \lambda s(1+s^k)\theta_s\right]}$$

$$= \frac{-s(1+s)^k\theta_s}{C(1)\left[k(1-\lambda\theta) + \lambda s(1+s^k)\theta_s\right]}$$

where the last equality uses the property $1-\lambda\theta = s^k[1-\lambda(1-\theta)]$ from Lemma 2. Then

$$\frac{-s(1+s)^k\theta_s}{C(1)\left[k(1-\lambda\theta) + \lambda s(1+s^k)\theta_s\right]} = (1+s)^k e_P^{*k}$$

$$\frac{-s\theta_s}{e_P^*} = C(1)\Big[k(1-\lambda\theta) + \lambda s(1+s^k)\theta_s\Big]e_P^{*k-1}$$

$$\frac{-s\theta_s}{e_P^*} = C(1)(1-\lambda\theta)ke_P^{*k-1} + C(1)\lambda s(1+s^k)\theta_s e_P^{*k-1}$$

$$\frac{-s\theta_s}{e_P^*} - C(1)\lambda s(1+s^k)\theta_s e_P^{*k-1} = C(1)(1-\lambda\theta)ke_P^{*k-1}$$

$$\frac{-s\theta_s}{e_P^*}\Big[1 + \lambda C(1)(1+s^k)e_P^{*k}\Big] = C(1)(1-\lambda\theta)ke_P^{*k-1}$$

$$\frac{-s\theta_s}{e_P^*}\Big[1 + \lambda C(1)e_P^{*k} + \lambda C(1)s^k e_P^{*k}\Big] = (1-\lambda\theta)C(1)ke_P^{*k-1}$$

$$\frac{\partial\theta}{\partial e_P}\Big[1 + \lambda C(e_P^*) + \lambda C(e_D^*)\Big] = (1-\lambda\theta)C'(e_P^*)$$

where the last equality uses the properties that $C(\cdot)$ is homogeneous of degree k, $s^k e_P^{*k} = e_D^{*k}$, and $\frac{\partial\theta}{\partial e_P} = \frac{-s\theta_s}{e_P}$ from Lemma 4. Hence the pair (e_P^*, e_D^*) satisfies Plaintiff's FOC.

Now consider the expression for e_D^*

$$(1+s)^k e_D^{*k} = s^k A^k = \frac{-s^{k+1}(1+s)^k\theta_s}{C(1)\left[ks^k[1-\lambda(1-\theta)] + \lambda s(1+s^k)\theta_s\right]}$$

a rearrangement of which gives:

$$\frac{-s^{k+1}\theta_s}{C(1)\left[ks^k[1-\lambda(1-\theta)] + \lambda s(1+s^k)\theta_s\right]} = e_D^{*k}$$

$$\frac{-s\theta_s}{e_D^*} = C(1)\Big[ks^k[1-\lambda(1-\theta)] + \lambda s(1+s^k)\theta_s\Big]\frac{e_D^{*k-1}}{s^k}$$

$$\frac{-s\theta_s}{e_D^*} = [1-\lambda(1-\theta)]C(1)ke_D^{*k-1} + \lambda C(1)s(1+s^k)\theta_s\frac{e_D^{*k-1}}{s^k}$$

$$\frac{-s\theta_s}{e_D^*} - \lambda C(1)s(1+s^k)\theta_s\frac{e_D^{*k-1}}{s^k} = [1-\lambda(1-\theta)]C(1)ke_D^{*k-1}$$

$$\frac{-s\theta_s}{e_D^*}\left[1 + \lambda C(1)\frac{(1+s^k)e_D^{*k}}{s^k}\right] = [1-\lambda(1-\theta)]C(1)ke_D^{*k-1}$$

$$\frac{-s\theta_s}{e_D^*}\left[1+\lambda C(1)\left(\frac{e_D^{*k}}{s^k}+e_D^{*k}\right)\right]=[1-\lambda(1-\theta)]C(1)ke_D^{*k-1}$$

$$\frac{\partial(1-\theta)}{\partial e_D}\left[1+\lambda C(e_P^*)+\lambda C(e_D^*)\right]=(1-\lambda\theta)C'(e_D^*)$$

where the last equality uses the properties that $C(\cdot)$ is homogeneous of degree k, $s^k e_P^{*k}=e_D^{*k}$, and $\frac{\partial(1-\theta)}{\partial e_D}=\frac{-s\theta_s}{e_D}$ from Lemma 4. Hence the pair (e_P^*, e_D^*) satisfies Defendant's FOC.

*Step (*ii*)*

Suppose $(e'_P, e'_D)\in\mathbb{R}^2_{++}$ is a Nash equilibrium with positive efforts. Denote $s'=e'_D/e'_P$. Some algebra reveals

$$e_P'^k=\frac{(e'_P+e'_D)^k}{(1+s')^k}\qquad\qquad e_D'^k=\frac{s'^k(e'_P+e'_D)^k}{(1+s')^k}.$$

Substituting these into Plaintiff and Defendant's FOCs in system (4), some algebra reveals:

$$\left.\frac{-s(1+s)^k\theta_s}{C(1)\left[k(1-\lambda\theta)+\lambda s(1+s^k)\theta_s\right]}\right|_{s=s'}=(e'_P+e'_D)^k$$

$$=\left.\frac{-s(1+s)^k\theta_s}{C(1)\left[ks^k[1-\lambda(1-\theta)]+\lambda s(1+s^k)\theta_s\right]}\right|_{s=s'}$$

where the first equality (respectively, second equality) is derived from Plaintiff's (Defendant's) FOC. Then some algebra using the equality of both sides will reveal that $s=s'$ induces $1-\lambda\theta=s^k[1-\lambda(1-\theta)]$. Hence the uniqueness limb of Lemma 2 implies $s'=s^*$.

Then obtain from the definition of A in (6)

$$e'_P+e'_D=\left.\left[\frac{-s(1+s)^k\theta_s}{C(1)\left[ks^k[1-\lambda(1-\theta)]+\lambda s(1+s^k)\theta_s\right]}\right]^{1/k}\right|_{s=s^*=s'}=A$$

where the properties $e'_P+e'_D=(1+s')e'_P$ and $s'=s^*$ imply $e'_P=A/(1+s^*)=e_P^*$. Similarly, use the properties $e'_P+e'_D=e'_D(1+s')/s'$ and $s'=s^*$ to obtain $e'_D=s^*A/(1+s^*)=e_D^*$. □

Lemmas 5 and 6 are technical lemmas on equilibrium properties, which will be used to prove subsequent propositions and corollaries.

Lemma 5. *Let $(e_P, e_D)=(e_P^*, e_D^*)$, the nontrivial Nash equilibrium characterized by Proposition 1. Let $s=s^*$ given by Lemma 2 and $\theta=\theta^*$ (Plaintiff's equilibrium probability of success). Denote $C^*=C(e_P^*)+C(e_D^*)$ and $\gamma=\frac{s^k}{(1+s^k)^2}$. The following properties hold:*

1.

$$\frac{\lambda(2\theta-1)}{2-\lambda}=\frac{1-s^k}{1+s^k}\qquad\qquad(1-\lambda\theta)[1-\lambda(1-\theta)]=(2-\lambda)^2\gamma.$$

2.

$$C^* = \frac{-(2-\lambda)s\theta_s}{k(1-\lambda\theta)[1-\lambda(1-\theta)] + \lambda(2-\lambda)s\theta_s} = -\left[\lambda + \frac{(2-\lambda)k\gamma}{s\theta_s}\right]^{-1}.$$

3.

$$\frac{d\gamma}{d\lambda} = \frac{k\gamma(1-s^k)}{s(1+s^k)}\frac{ds}{d\lambda} \qquad \frac{d\gamma}{d\mu} = \frac{k\gamma(1-s^k)}{s(1+s^k)}\frac{ds}{d\mu} \qquad \frac{d\gamma}{dk} = -\frac{\ln(s)s^k(1-s^k)}{(1+s^k)^3}.$$

Proof of Lemma 5

Part 1

Using Lemma 2, some algebra will give these results.

Part 2

Apply Lemma 4 to system (4) to obtain:

$$-\frac{s\theta_s}{e_P^*}[1+\lambda C^*] = k(1-\lambda\theta)C(1)e_P^{*k-1}, \qquad -\frac{s\theta_s}{e_D^*}[1+\lambda C^*] = k[1-\lambda(1-\theta)]C(1)e_D^{*k-1}.$$

Using the homogeneity of $C(\cdot)$ and Lemma 2, some algebra will give the result.

Part 3

some algebra reveals

$$\frac{d\gamma}{d\lambda} = \frac{ks^{k-1}(1-s^k)}{(1+s^k)^3}\frac{ds}{d\lambda} = \frac{k\gamma(1-s^k)}{s(1+s^k)}\frac{ds}{d\lambda}.$$

Similarly for $\frac{d\gamma}{d\mu}$ and $\frac{d\gamma}{dk}$. □

Lemma 6. *Consider two cases that differ only in respect of Plaintiff's prior probability of success; it is μ in one case and $\mu' = 1-\mu$ in the other case. Suppose (e_P^*, e_D^*) (respectively, $(e_P^{*'}, e_D^{*'})$) is the nontrivial Nash equilibrium in the case characterized by μ (respectively, μ'). Then Plaintiff's equilibrium effort in one case equals to Defendant's equilibrium effort in the other case. Formally, $e_P^* = e_D^{*'}$ and $e_D^* = e_P^{*'}$.*

Proof of Lemma 6

To facilitate presentation and for the purpose of this proof only, let u_{P1} (respectively, u_{P2}) denote the partial derivative of Plaintiff's payoff given by (2) with respect to its first (second) argument, namely, Plaintiff's effort (Defendant's effort). Similarly, let u_{D1} (respectively, u_{D2}) denote the partial derivative of Defendant's payoff given by (3) with respect to its first (second) argument, namely, Plaintiff's effort (Defendant's effort).

In respect of the case characterized by μ, fix arbitrary real numbers $e_1, e_2 > 0$ and consider generic efforts e_P, e_D taken by Plaintiff and Defendant respectively. Some algebra using Assumption

1 reveals

$$\begin{cases} u_P(e_P, e_2; \mu, \lambda, k) = u_D(e_2, e_P; \mu', \lambda, k) + 1 \\ u_D(e_1, e_D; \mu, \lambda, k) = u_P(e_D, e_1; \mu', \lambda, k) - 1, \end{cases}$$

which implies

$$\begin{cases} u_{P1}(e_P, e_2; \mu, \lambda, k) = u_{D2}(e_2, e_P; \mu', \lambda, k) \\ u_{D2}(e_1, e_D; \mu, \lambda, k) = u_{P1}(e_D, e_1; \mu', \lambda, k). \end{cases} \tag{27}$$

That the pair of positive real numbers (e_P^*, e_D^*) is the nontrivial Nash equilibrium in the case characterized by μ is equivalent to

$$\begin{cases} u_{P1}(e_P^*, e_D^*; \mu, \lambda, k) = 0 \\ u_{D2}(e_P^*, e_D^*; \mu, \lambda, k) = 0. \end{cases} \tag{28}$$

Then by choosing real numbers $e_1 = e_P^*$, $e_2 = e_D^*$ in system (27), a substitution exercise using systems (27) and (28) reveals

$$\begin{cases} u_{P1}(e_P^*, e_D^*; \mu, \lambda, k) = u_{D2}(e_D^*, e_P^*; \mu', \lambda, k) = 0 \\ u_{D2}(e_P^*, e_D^*; \mu, \lambda, k) = u_{P1}(e_D^*, e_P^*; \mu', \lambda, k) = 0. \end{cases}$$

Hence the pair of positive real numbers (e_D^*, e_P^*) is the nontrivial Nash equilibrium in the case characterized by μ'. Then the uniqueness limb of Proposition 1 implies $e_P^* = e_D^{*'}$ and $e_D^* = e_P^{*'}$, where $(e_P^{*'}, e_D^{*'})$ is the nontrivial Nash equilibrium in the case characterized by μ'. □

Proof of Corollary 1

This proof establishes that $\Theta(\{\bar{\lambda}\}, \{\underline{k}\}) \subset \Theta([0, \bar{\lambda}], [\underline{k}, +\infty))$; that is, if the success function θ satisfies Assumptions 1-6 for a cost function of homogeneous of degree $\underline{k}$ and the cost-shifting rule $\bar{\lambda}$, then θ also satisfies Assumptions 1-6 for any arbitrary pair of cost-shifting rule $\lambda \leq \bar{\lambda}$ and cost function $k \geq \underline{k}$. Then an application of Proposition 1 to the pair λ, k gives the result.

Suppose the success function $\theta \in \Theta(\{\bar{\lambda}\}, \{\underline{k}\})$ and choose an arbitrary pair of λ, k satisfying $\lambda \leq \bar{\lambda}$ and $k \geq \underline{k}$. Let $\underline{C}(\cdot)$ denote the homogeneous cost function characterized by $\underline{k}$. Satisfaction of Assumptions 1-4 does not depend on the values of λ, k. Assumption 6 is either satisfied if $\lambda = \bar{\lambda} = k = \underline{k} = 1$, or not applicable otherwise. It remains to check that θ satisfies Assumption 5 for the pair λ, k.

Some algebra using the property that θ satisfies Assumption 5 under the pair $\bar{\lambda}, \underline{k}$ reveals

$$\frac{\underline{C}''(e_P)}{\underline{C}'(e_P)} > \frac{\frac{\partial^2}{\partial e_P^2}\left(\frac{\theta}{1-\bar{\lambda}\theta}\right)}{\frac{\partial}{\partial e_P}\left(\frac{\theta}{1-\bar{\lambda}\theta}\right)} = \frac{(1-\bar{\lambda}\theta)\frac{\partial^2\theta}{\partial e_P^2} + 2\bar{\lambda}\left(\frac{\partial\theta}{\partial e_P}\right)^2}{\frac{\partial\theta}{\partial e_P}} \geq \frac{(1-\lambda\theta)\frac{\partial^2\theta}{\partial e_P^2} + 2\lambda\left(\frac{\partial\theta}{\partial e_P}\right)^2}{\frac{\partial\theta}{\partial e_P}}$$

where the last weak inequality uses the properties $\lambda \leq \bar{\lambda}$, $1 - \bar{\lambda}\theta \leq 1 - \lambda\theta$ and Assumption 4. Then the result that θ satisfies Assumption 5 under (λ, k) follows from some algebra revealing

$$\frac{\frac{\partial^2}{\partial e_P^2}\left(\frac{\theta}{1-\lambda\theta}\right)}{\frac{\partial}{\partial e_P}\left(\frac{\theta}{1-\lambda\theta}\right)} = \frac{(1-\lambda\theta)\frac{\partial^2\theta}{\partial e_P^2} + 2\lambda\left(\frac{\partial\theta}{\partial e_P}\right)^2}{\frac{\partial\theta}{\partial e_P}} \qquad \frac{\underline{C}''(e_P)}{\underline{C}'(e_P)} \leq \frac{C''(e_P)}{C'(e_P)}.$$

The choice of λ, k was arbitrary; hence θ satisfies Assumptions 1-6 for any pair of cost-shifting rule $\lambda \leq \bar{\lambda}$ and cost function $k \geq \underline{k}$. □

Proof of Corollary 2

This proof establishes the result for the case of $\mu > 0.5$. Similar steps establish the results for $\mu = 0.5$ and $\mu < 0.5$.

Let $s = s^*$ and $\theta = \theta^*$. Lemma 2 and Proposition 1 prove that in the nontrival Nash equilibrium

$$s^k = \frac{1-\lambda\theta}{1-\lambda(1-\theta)}.$$

Take the total derivative of both sides with respect to λ

$$ks^{k-1}\frac{\partial s}{\partial\lambda} = \frac{\left(-\theta - \lambda\theta_s\frac{\partial s}{\partial\lambda}\right)[1-\lambda(1-\theta)] - \left(-(1-\theta)+\lambda\theta_s\frac{\partial s}{\partial\lambda}\right)(1-\lambda\theta)}{[1-\lambda(1-\theta)]^2}$$

$$\begin{aligned}
ks^{k-1}[1-\lambda(1-\theta)]^2\frac{\partial s}{\partial\lambda} =& -\theta[1-\lambda(1-\theta)] + (1-\theta)(1-\lambda\theta)\\
&-\lambda[1-\lambda(1-\theta)+1-\lambda\theta]\theta_s\frac{\partial s}{\partial\lambda}\\
ks^{k}[1-\lambda(1-\theta)]^2\frac{\partial s}{\partial\lambda} =& s(1-2\theta) - \lambda(2-\lambda)s\theta_s\frac{\partial s}{\partial\lambda}\\
k(1-\lambda\theta)[1-\lambda(1-\theta)]\frac{\partial s}{\partial\lambda} =& s(1-2\theta) - \lambda(2-\lambda)s\theta_s\frac{\partial s}{\partial\lambda}
\end{aligned}$$

where the last step uses Lemma 2. Then some algebra reveals

$$\frac{\partial s}{\partial\lambda} = \frac{s(1-2\theta)}{k(1-\lambda\theta)[1-\lambda(1-\theta)]+\lambda(2-\lambda)s\theta_s} \tag{29}$$

where part 9 of Lemma 4 proves $k(1-\lambda\theta)[1-\lambda(1-\theta)]+\lambda(2-\lambda)s\theta_s > 0$.

Now, take the total derivative of θ with respect to λ

$$\frac{d\theta}{d\lambda} = \theta_s\frac{ds}{d\lambda} \tag{30}$$

where Lemma 4 proves $\theta_s < 0$.

Suppose $\mu > 0.5$. Then Proposition 1 proves $s \leq 1$. Part 1 of Lemma 4 and the property $\theta_s < 0$

together prove $\theta > 0.5$. From equation (29), that $\theta > 0.5$ implies $\frac{\partial s}{\partial \lambda} < 0$, which is equivalent to $\frac{\partial(1/s)}{\partial \lambda} > 0$. Then equation (30) implies $\frac{d\theta}{d\lambda} > 0$. □

Proof of Corollary 3

In this proof, let $s = s^*$ and $\theta = \theta^*$. Lemma 2 and Proposition 1 prove that in the nontrivial Nash equilibrium

$$s^k = \frac{1 - \lambda\theta}{1 - \lambda(1 - \theta)}.$$

Differentiate both sides respect to μ

$$ks^{k-1}\frac{ds}{d\mu} = \frac{-\lambda\frac{d\theta}{d\mu}[1 - \lambda(1 - \theta)] - \lambda\frac{d\theta}{d\mu}(1 - \lambda\theta)}{[1 - \lambda(1 - \theta)]^2} = -\frac{\lambda(2 - \lambda)}{[1 - \lambda(1 - \theta)]^2}\frac{d\theta}{d\mu}$$

where taking the total derivative of θ with respect to μ reveals

$$\frac{d\theta}{d\mu} = \theta_s\frac{ds}{d\mu} + \frac{\partial\theta}{\partial\mu}. \tag{31}$$

Then a substitution exercise reveals

$$\frac{ds}{d\mu}\left[ks^{k-1} + \frac{\lambda(2 - \lambda)\theta_s}{[1 - \lambda(1 - \theta)]^2}\right] = -\frac{\lambda(2 - \lambda)}{[1 - \lambda(1 - \theta)]^2}\frac{\partial\theta}{\partial\mu}$$

$$\frac{ds}{d\mu}\left[\frac{ks^k[1 - \lambda(1 - \theta)]^2 + \lambda(2 - \lambda)s\theta_s}{s[1 - \lambda(1 - \theta)]^2}\right] = -\frac{\lambda(2 - \lambda)}{[1 - \lambda(1 - \theta)]^2}\frac{\partial\theta}{\partial\mu}$$

$$\frac{ds}{d\mu}[k(1 - \lambda\theta)[1 - \lambda(1 - \theta)] + \lambda(2 - \lambda)s\theta_s] = -\lambda(2 - \lambda)s\frac{\partial\theta}{\partial\mu}$$

where the last step applies Lemma 2. Then some algebra reveals

$$\frac{ds}{d\mu} = \frac{-\lambda(2 - \lambda)s\frac{\partial\theta}{\partial\mu}}{k(1 - \lambda\theta)[1 - \lambda(1 - \theta)] + \lambda(2 - \lambda)s\theta_s} \tag{32}$$

where part 9 of Lemma 4 proves $k(1 - \lambda\theta)[1 - \lambda(1 - \theta)] + \lambda(2 - \lambda)s\theta_s > 0$ and Assumption 3 holds $\frac{\partial\theta}{\partial\mu} > 0$. Hence $\frac{ds}{d\mu} \leq 0$, holding strictly if $\lambda > 0$. Then an application of the chain rule gives the results with respect to s^*.

Now, using equations (31) and (32), some algebra reveals

$$\frac{d\theta}{d\mu} = \frac{\partial\theta}{\partial\mu} - \frac{\lambda(2 - \lambda)s\theta_s\frac{\partial\theta}{\partial\mu}}{k(1 - \lambda\theta)[1 - \lambda(1 - \theta)] + \lambda(2 - \lambda)s\theta_s}$$

$$= \frac{\partial\theta}{\partial\mu}\left(1 - \frac{\lambda(2 - \lambda)s\theta_s}{k(1 - \lambda\theta)[1 - \lambda(1 - \theta)] + \lambda(2 - \lambda)s\theta_s}\right)$$

$$= \frac{\partial \theta}{\partial \mu}\left(\frac{k(1-\lambda\theta)[1-\lambda(1-\theta)]}{k(1-\lambda\theta)[1-\lambda(1-\theta)]+\lambda(2-\lambda)s\theta_s}\right) \tag{33}$$

which implies $\frac{d\theta}{d\mu} > 0$. Then an application of the chain rule gives the results with respect to θ^*. □

Proof of Corollary 4

Consider the nontrivial Nash equilibrium, where $s = s^*$ given by Lemma 2 and $\theta = \theta^*$. Totally differentiate θ with respect to k to obtain

$$\frac{d\theta}{dk} = \theta_s \frac{ds}{dk}$$

where Lemma 4 proves $\theta_s < 0$ and some algebra differentiating both sides of equation (5) with respect to k reveals

$$\frac{d}{dk}\left(s^k\right) = \frac{d}{dk}\left(\frac{1-\lambda\theta}{1-\lambda(1-\theta)}\right)$$

$$\ln(s)s^k = \frac{-\lambda\theta_s\frac{ds}{dk}[1-\lambda(1-\theta)] - \lambda\theta_s\frac{ds}{dk}(1-\lambda\theta)}{[1-\lambda(1-\theta)]^2} = \frac{-\lambda(2-\lambda)\theta_s}{[1-\lambda(1-\theta)]^2}\frac{ds}{dk}.$$

There are three cases:

1. If $\lambda = 0$ or $\mu = 0.5$, then Proposition 1 proves $s = 1$ for all $k \geq 1$, implying $\ln(s) = 0$. Hence $\frac{ds}{dk} = 0$, which implies $\frac{d\theta}{dk} = 0$.
2. If $\lambda > 0$ and $\mu > 0.5$, then Proposition 1 proves $s < 1$, implying $\ln(s) < 0$. Then the property $\theta_s < 0$ (from Lemma 4) implies $\frac{ds}{dk} < 0$ and $\frac{d\theta}{dk} > 0$.
3. If $\lambda > 0$ and $\mu < 0.5$, then Proposition 1 proves $s > 1$, implying $\ln(s) > 0$. Then the property $\theta_s < 0$ implies $\frac{ds}{dk} > 0$ and $\frac{d\theta}{dk} < 0$. □

Proof of Corollary 5

In this proof, let $s = s^*$ given by Lemma 2. Part 2 of Lemma 5 reveals

$$C^* = -\left[\lambda + \frac{(2-\lambda)k\gamma}{s\theta_s}\right]^{-1}.$$

Differentiate both sides of C^* with respect to λ

$$\frac{dC^*}{d\lambda} = \left[1 + \frac{s\theta_s\left(-k\gamma + (2-\lambda)k\frac{d\gamma}{d\lambda}\right) - (2-\lambda)k\gamma(\theta_s + s\theta_{ss})\frac{ds}{d\lambda}}{s^2\theta_s^2}\right]\left(\lambda + \frac{(2-\lambda)k\gamma}{s\theta_s}\right)^{-2}$$

$$\frac{dC^*}{d\lambda} = \left[s^2\theta_s^2 + \left(-k\gamma + (2-\lambda)k\frac{d\gamma}{d\lambda}\right)s\theta_s - (\theta_s + s\theta_{ss})(2-\lambda)k\gamma\frac{ds}{d\lambda}\right]s^{-2}\theta_s^{-2}C^{*2}$$

$$\frac{s^2\theta_s^2}{C^{*2}}\frac{dC^*}{d\lambda} = s^2\theta_s^2 + \left(-k\gamma + (2-\lambda)k\frac{d\gamma}{d\lambda}\right)s\theta_s - (\theta_s + s\theta_{ss})(2-\lambda)k\gamma\frac{ds}{d\lambda}$$

$$\frac{s^2\theta_s^2}{C^{*2}}\frac{dC^*}{d\lambda} = s^2\theta_s^2 - k\gamma s\theta_s + \frac{(2-\lambda)(1-s^k)k^2\gamma\theta_s}{(1+s^k)}\frac{ds}{d\lambda} - (\theta_s + s\theta_{ss})(2-\lambda)k\gamma\frac{ds}{d\lambda}$$

$$\frac{s^2\theta_s^2}{C^{*2}}\frac{dC^*}{d\lambda} = s^2\theta_s^2 - k\gamma s\theta_s - k\gamma(2-\lambda)\frac{ds}{d\lambda}\left(\theta_s + s\theta_{ss} - \frac{k(1-s^k)\theta_s}{1+s^k}\right) \tag{34}$$

where the second last equality uses Lemma 5, and equation (29) in the proof of Corollary 2 reveals

$$\frac{\partial s}{\partial \lambda} = \frac{s(1-2\theta)}{k(1-\lambda\theta)[1-\lambda(1-\theta)] + \lambda(2-\lambda)s\theta_s}.$$

Then a substitution exercise using Lemma 5 and equation (34) gives the result. □

Proof of Corollary 6

To facilitate presentation, define a function $g(\mu, \lambda, k)$ by

$$g(\mu, \lambda, k) = s^2\theta_s^2 - k\gamma s\theta_s - k\gamma(2-\lambda)\frac{ds}{d\lambda}\left(\theta_s + s\theta_{ss} - \frac{k(1-s^k)\theta_s}{1+s^k}\right) \tag{35}$$

where $s = s^*$ given by Lemma 2. Using equation (34), some algebra reveals that condition (9) is equivalent to $g(\mu, \lambda, k) > 0$.

Corollary 1 proves the nontrivial Nash equilibrium exists for all $\lambda \in [0, \lambda_2]$. Hence Corollary 5 applies to all $\lambda \in [\lambda_1, \lambda_2] \subset [0, \lambda_2]$. This proof will establish that for an arbitrary $\lambda \in [\lambda_1, \lambda_2]$, $g(\mu, \lambda, k) > 0$ in each of the following cases: (*i*) $\mu = 0.5$; (*ii*) $0.5 < \mu \leq 0.5 + \sigma(\lambda_2, k)$; (*iii*) $0.5 - \sigma(\lambda_2, k) \leq \mu < 0.5$. Than an application of Corollary 5 gives the result.

*Case (*i*)*

Suppose $\mu = 0.5$ and consider the nontrivial Nash equilibrium, where $s = s^*$ given by Lemma 2 and $\theta = \theta^*$. Then Corollary 2 proves $\frac{ds}{d\lambda} = 0$. Hence the property $\theta_s < 0$ from Lemma 4 implies $g(\mu, \lambda, k) > 0$.

*Case (*ii*)*

Suppose $0.5 < \mu \leq 0.5 + \sigma(\lambda_2, k)$ and consider the nontrivial Nash equilibrium, where $s = s^*$ given by Lemma 2 and $\theta = \theta^*$. Use equation (29) from the proof of Corollary 2 and part 2 of Lemma 5 to obtain

$$-k\gamma(2-\lambda)\frac{\partial s}{\partial \lambda} = \frac{k\gamma(2-\lambda)s(2\theta-1)}{k(1-\lambda\theta)(1-\lambda(1-\theta)) + \lambda(2-\lambda)s\theta_s} = \frac{k\gamma(2-\lambda)s(2\theta-1)}{k(2-\lambda)^2\gamma + \lambda(2-\lambda)s\theta_s}$$

$$= \frac{s(2\theta-1)[k(2-\lambda)\gamma + \lambda s\theta_s - \lambda s\theta_s]}{(2-\lambda)[k\gamma(2-\lambda) + \lambda s\theta_s]} = \frac{s(2\theta-1)(1+\lambda C^*)}{2-\lambda}.$$

Hence

$$g(\mu,\lambda,k) = s^2\theta_s^2 - k\gamma s\theta_s + \frac{s(2\theta-1)(1+\lambda C^*)}{(2-\lambda)}\left(s\theta_{ss} + \left(1 - \frac{k(1-s^k)}{1+s^k}\right)\theta_s\right)$$

$$= s^2\theta_s^2 - k\gamma s\theta_s + \frac{(2\theta-1)(1+\lambda C^*)}{(2-\lambda)}\left(1 - \frac{k(1-s^k)}{1+s^k}\right)s\theta_s + \frac{(2\theta-1)(1+\lambda C^*)}{(2-\lambda)}s^2\theta_{ss}$$

where the properties $\theta > 0.5$ (from Proposition 1) and $s\theta_{ss} > (k-1)\theta_s + \frac{2\lambda s\theta_s^2}{[1-\lambda(1-\theta)]}$ (from Lemma 4) imply

$$\begin{aligned} g(\mu,\lambda,k) >& s^2\theta_s^2 - k\gamma s\theta_s + \frac{(2\theta-1)(1+\lambda C^*)}{(2-\lambda)}\left(1 - \frac{k(1-s^k)}{1+s^k}\right)s\theta_s \\ &+ \frac{(2\theta-1)(1+\lambda C^*)}{(2-\lambda)}\left((k-1)s\theta_s + \frac{2\lambda s^2\theta_s^2}{[1-\lambda(1-\theta)]}\right) \\ =& s^2\theta_s^2 - k\gamma s\theta_s + \frac{2ks^k(2\theta-1)(1+\lambda C^*)}{(2-\lambda)(1+s^k)}s\theta_s \\ &+ \frac{(2\theta-1)(1+\lambda C^*)}{(2-\lambda)}\left(\frac{2\lambda s^2\theta_s^2}{[1-\lambda(1-\theta)]}\right). \end{aligned}$$

Now, some algebra reveals

$$\begin{aligned} 1+\lambda C^* &= 1 - \frac{\lambda s\theta_s}{k\gamma(2-\lambda)+\lambda s\theta_s} = \frac{k\gamma(2-\lambda)}{k\gamma(2-\lambda)+\lambda s\theta_s} \\ &= \frac{-s\theta_s k\gamma(2-\lambda)}{-s\theta_s[k\gamma(2-\lambda)+\lambda s\theta_s]} = \frac{k\gamma(2-\lambda)C^*}{-s\theta_s}. \end{aligned}$$

Hence

$$\begin{aligned} g(\mu,\lambda,k) >& s^2\theta_s^2 - k\gamma s\theta_s \\ &+ \frac{2ks^k(2\theta-1)(1+\lambda C^*)}{(2-\lambda)(1+s^k)}s\theta_s + \frac{(2\theta-1)k\gamma(2-\lambda)C^*}{-s\theta_s(2-\lambda)}\left(\frac{2\lambda s^2\theta_s^2}{[1-\lambda(1-\theta)]}\right) \\ =& s^2\theta_s^2 - k\gamma s\theta_s + \frac{2ks^k(2\theta-1)(1+\lambda C^*)}{(2-\lambda)(1+s^k)}s\theta_s - \frac{2k\gamma(2\theta-1)\lambda C^*}{1-\lambda(1-\theta)}s\theta_s \\ =& s^2\theta_s^2 - k\gamma s\theta_s + \frac{2ks^k(2\theta-1)}{(2-\lambda)(1+s^k)}s\theta_s \end{aligned}$$

$$+ 2ks\theta_s\lambda C^*(2\theta - 1)\left[\frac{s^k}{(2-\lambda)(1+s^k)} - \frac{\gamma}{1-\lambda(1-\theta)}\right]$$

where some algebra using the property $s^k = \frac{1-\lambda\theta}{1-\lambda(1-\theta)}$ from Lemma 2 and the definition of γ (that $\gamma = s^k/(1+s^k)^2$) reveals

$$\frac{s^k}{(2-\lambda)(1+s^k)} = \frac{\gamma(1+s^k)}{(2-\lambda)} = \frac{\gamma(1-\lambda(1-\theta)+1-\lambda\theta)}{(2-\lambda)[1-\lambda(1-\theta)]} = \frac{\gamma}{1-\lambda(1-\theta)}.$$

Hence

$$\begin{aligned}
g(\mu,\lambda,k) >& s^2\theta_s^2 - k\gamma s\theta_s + \frac{2ks^k(2\theta-1)}{(2-\lambda)(1+s^k)}s\theta_s = s^2\theta_s^2 - ks\theta_s\left[\gamma - \frac{2s^k(2\theta-1)}{(2-\lambda)(1+s^k)}\right] \\
=& s^2\theta_s^2 - ks\theta_s\left[\frac{s^k}{(1+s^k)^2} - \frac{2s^k(2\theta-1)}{(2-\lambda)(1+s^k)}\right] \\
=& s^2\theta_s^2 - \frac{ks^{k+1}\theta_s}{(1+s^k)}\left[\frac{1}{1+s^k} - \frac{2(2\theta-1)}{2-\lambda}\right] \\
=& s^2\theta_s^2 - \frac{ks^{k+1}\theta_s}{(1+s^k)}\left[\frac{1-\lambda(1-\theta)}{2-\lambda} - \frac{2(2\theta-1)}{2-\lambda}\right] \\
=& s^2\theta_s^2 - \frac{ks^{k+1}\theta_s[3-\lambda-\theta(4-\lambda)]}{(1+s^k)(2-\lambda)} \\
=& -s\theta_s\left[-s\theta_s + \frac{ks^k[3-\lambda-\theta(4-\lambda)]}{(1+s^k)(2-\lambda)}\right] \\
\Rightarrow g(\mu,\lambda,k) >& -s\theta_s\left[-s\theta_s + \frac{k(1-\lambda\theta)}{(2-\lambda)^2(4-\lambda)}\left(\frac{3-\lambda}{4-\lambda} - \theta\right)\right].
\end{aligned} \tag{36}$$

Now, some algebra reveals

$$\frac{d}{d\lambda}\left(\frac{3-\lambda}{4-\lambda} - \theta\right) = -\frac{1}{(4-\lambda)^2} - \frac{d\theta}{d\lambda} < 0$$

where the last inequality uses part 1 of Corollary 2 and the assumption $\mu > 0.5$. Hence, for all $\lambda \in [\lambda_1, \lambda_2]$,

$$\frac{3-\lambda}{4-\lambda} - \theta \geq \frac{3-\lambda_2}{4-\lambda_2} - \theta|_{\lambda=\lambda_2} \geq 0$$

where the last inequality uses the definition of $\sigma(\lambda_2, k)$. Then the property $\theta_s < 0$ from Lemma 4 and inequality (36) imply $g(\mu,\lambda,k) > 0$ for all $\lambda \in [\lambda_1, \lambda_2]$.

*Case (*iii*)*

Suppose $0.5 - \sigma \leq \mu < 0.5$. Let (e_P^*, e_D^*) denote the nontrivial Nash equilibrium given Plaintiff's prior probability of success is μ. Consider another case that differs only in respect of Plaintiff's prior probability of success, which is given by $\mu' = 1 - \mu$ instead. Let $(e_P^{*'}, e_D^{*'})$ denote the nontrivial Nash equilibrium if Plaintiff's prior probability of success is μ'. Then Lemma 6 proves $e_P^* = e_D^{*'}$, $e_D^* = e_P^{*'}$. Hence

$$C(e_P^*) + C(e_D^*) = C(e_P^{*'}) + C(e_D^{*'}). \tag{37}$$

Some algebra will reveal that the properties $\mu' = 1 - \mu$ and $0.5 - \sigma \leq \mu < 0.5$ imply $0.5 < \mu' \leq 0.5 + \sigma$. Hence the proof for case (*ii*) establishes that the right hand side of equation (37), being the litigation expenditure given Plaintiff's prior probability of success is μ', is increasing with λ. Then the left hand hand side of equation (37), being the litigation expenditure given Plaintiff's prior probability of success is μ, is also increasing with λ. □

Proof of Corollary 7

Corollary 1 proves the nontrivial Nash equilibrium exist for all $\lambda \in [0, \lambda_2]$. Hence Corollary 5 applies to all $\lambda \in [\lambda_1, \lambda_2] \subset [0, \lambda_2]$.

Suppose the homogeneous cost function is of degree $k \geq 2$ and consider the nontrivial Nash equilibrium, where $s = s^*$ given by Lemma 2. Choose an arbitrary $\lambda \in [\lambda_1, \lambda_2]$. If $0.5 - \sigma(\lambda, k) \leq \mu \leq 0.5 + \sigma(\lambda, k)$, then inequality (36) in the proof of Corollary 6 proves the result. There are two remaining cases: $\mu > 0.5 + \sigma(\lambda, k)$; and $\mu < 0.5 - \sigma(\lambda, k)$. In respect of the case of $\mu > 0.5 + \sigma(\lambda, k)$, this will proof establish $C_2^* > C_1^*$ by showing that $\frac{dC^*}{d\lambda} > 0$ for $\lambda \in [\lambda_1, \lambda_2]$. Then the case of $\mu < 0.5 - \sigma(\lambda, k)$ follows from steps similar to those for case (*iii*) in the proof of Corollary 6.

Suppose $\mu > 0.5 + \sigma(\lambda, k)$. If $-(2 - \lambda)^2 s\theta_s + k(1 - \lambda\theta)[3 - \lambda - \theta(4 - \lambda)] \geq 0$, then some algebra using inequality (36) in the proof of Corollary 6 gives the result. The rest of this proof supposes $-(2 - \lambda)^2 s\theta_s + k(1 - \lambda\theta)[3 - \lambda - \theta(4 - \lambda)] < 0$; using Lemma 2, it is equivalent to

$$s\theta_s > k\gamma - \frac{2k(2\theta - 1)s^k}{(2 - \lambda)(1 + s^k)}. \tag{38}$$

Using equation (29) from the proof of Corollary 2 and part 2 of Lemma 5, some algebra reveals

$$(2 - \lambda)\frac{\partial s}{\partial \lambda} = \frac{(2 - \lambda)s(1 - 2\theta)}{k(2 - \lambda)^2\gamma + \lambda(2 - \lambda)s\theta_s} = \frac{s(1 - 2\theta)}{k\gamma(2 - \lambda) + \lambda s\theta_s}.$$

Then substitute into the definition of $g(\mu, \lambda, k)$ (given by (35) in the proof of Corollary 6) to obtain

$$g(\mu, \lambda, k) = s^2\theta_s^2 - k\gamma s\theta_s + \left(\frac{k\gamma(2\theta - 1)}{k\gamma(2 - \lambda) + \lambda s\theta_s}\right)\left(s\theta_s + s^2\theta_{ss} - \frac{k(1 - s^k)s\theta_s}{1 + s^k}\right).$$

Define an auxiliary variable $X_4 = k\gamma(2 - \lambda) + \lambda s\theta_s$. Some algebra using the property $\theta_s < 0$

from Lemma 4 and inequality (38) reveals

$$
\begin{aligned}
X_4 g(\mu,\lambda,k) =&[s^2\theta_s^2 - k\gamma s\theta_s][k\gamma(2-\lambda)+\lambda s\theta_s] + k\gamma(2\theta-1)\left(s\theta_s + s^2\theta_{ss} - \frac{k(1-s^k)s\theta_s}{1+s^k}\right)\\
\geq&(s^2\theta_s^2 - k\gamma s\theta_s)\left[k\gamma(2-\lambda)+\lambda k\gamma - \frac{2k\lambda(2\theta-1)s^k}{(2-\lambda)(1+s^k)}\right]\\
&+ k\gamma(2\theta-1)\left(s\theta_s + s^2\theta_{ss} - \frac{k(1-s^k)s\theta_s}{1+s^k}\right)
\end{aligned}
$$

where the inequality holds strictly if $\lambda > 0$. Then some algebra using the properties $\lambda(2\theta-1)/(2-\lambda) = (1-s^k)/(1+s^k)$ (from Lemma 5) and $\gamma = s^k/(1+s^k)^2$ (the definition of the auxiliary variable γ) reveals

$$
\begin{aligned}
X_4 g(\mu,\lambda,k) \geq&(s^2\theta_s^2 - k\gamma s\theta_s)\left[2k\gamma - \frac{2k(1-s^k)s^k}{(1+s^k)^2}\right]\\
&+ k\gamma(2\theta-1)\left(s\theta_s + s^2\theta_{ss} - \frac{k(1-s^k)s\theta_s}{1+s^k}\right)\\
=&(s^2\theta_s^2 - k\gamma s\theta_s)\left[2k\gamma - 2(1-s^k)k\gamma\right]\\
&+ k\gamma(2\theta-1)\left(s\theta_s + s^2\theta_{ss} - \frac{k(1-s^k)s\theta_s}{1+s^k}\right)\\
=&2k\gamma s^k(s^2\theta_s^2 - k\gamma s\theta_s) + k\gamma(2\theta-1)\left(s\theta_s + s^2\theta_{ss} - \frac{k(1-s^k)s\theta_s}{1+s^k}\right)\\
\Leftrightarrow \frac{X_4 g(\mu,\lambda,k)}{k\gamma} \geq&2s^{k+2}\theta_s^2 - \frac{2ks^{2k+1}\theta_s}{(1+s^k)^2} + (2\theta-1)\left(s\theta_s + s^2\theta_{ss} - \frac{k(1-s^k)s\theta_s}{1+s^k}\right)\\
=&2s^{k+2}\theta_s^2 + (2\theta-1)s^2\theta_{ss}\\
&- \frac{s\theta_s}{(1+s^k)^2}\left[2ks^{2k} + (2\theta-1)(1+s^k)^2\left(\frac{k(1-s^k)}{1+s^k} - 1\right)\right]
\end{aligned}
$$

where some algebra reveals $1 = 2\theta - 1 + 2(1-\theta)$. Then the property $s > 0$ from Proposition 1, and the properties $\theta_s < 0$ and $\theta_{ss} > 0$ from Lemma 4 imply

$$X_4 g(\mu,\lambda,k)$$

$$> -\frac{s\theta_s}{(1+s^k)^2}\left[4k(1-\theta)s^{2k} + 2(2\theta-1)ks^{2k} + (2\theta-1)(1+s^k)^2\left(\frac{k(1-s^k)}{1+s^k} - 1\right)\right]$$

$$= -\frac{s\theta_s}{(1+s^k)^2}\left\{4k(1-\theta)s^{2k} + (2\theta-1)\left[2ks^{2k} + k(1-s^k)(1+s^k) - (1+s^k)^2\right]\right\}$$

$$= -\frac{s\theta_s}{(1+s^k)^2}\left\{4k(1-\theta)s^{2k} + (2\theta-1)\left[2ks^{2k} + k\left(1-s^{2k}\right) - 1 - 2s^k - s^{2k}\right]\right\}$$

$$= -\frac{s\theta_s}{(1+s^k)^2}\left\{4k(1-\theta)s^{2k} + (2\theta-1)\left[(k-2)(1+s^{2k}) + (1-s^k)^2\right]\right\}.$$

Then the properties $\theta_s < 0$ (from Lemma 4) and $X_4 > 0$ (due to Lemma 5 and part 9 from Lemma 4) and $k\gamma > 0$, and the assumption $k \geq 2$ imply $g(\mu, \lambda, k) > 0$, which is equivalent to $\frac{dC^*}{d\lambda} > 0$. □

Proof of Proposition 2

Corollary 1 proves the nontrivial Nash equilibrium exist for all $\lambda \in [0, \lambda_2]$. Hence Corollary 5 applies to all $\lambda \in [\lambda_1, \lambda_2] \subset [0, \lambda_2]$.

Choose an arbitrary $\lambda \in [\lambda_1, \lambda_2]$ and consider the nontrivial Nash equilibrium, where $s = s^*$ given by Lemma 2. If $0.5 - \sigma(\lambda, k) \leq \mu \leq 0.5 + \sigma(\lambda, k)$, then inequality (36) in the proof of Corollary 6 proves the result. If $k \geq 2$, then Corollary 7 establishes the result. There are two remaining cases: $k < 2$ and $\mu > 0.5 + \sigma(\lambda, k)$; $k < 2$ and $\mu < 0.5 - \sigma(\lambda, k)$. In respect of the case of $k < 2$ and $\mu > 0.5 + \sigma(\lambda, k)$, this will proof establish $C_2^* > C_1^*$ by showing that $\frac{dC^*}{d\lambda} > 0$ for $\lambda \in [\lambda_1, \lambda_2]$. Then the case of $k < 2$ and $\mu < 0.5 - \sigma(\lambda, k)$ follows from steps similar to those for case (*iii*) in the proof of Corollary 6.

Suppose $k < 2$ and $\mu > 0.5 + \sigma(\lambda, k)$. Suppose condition (11) holds. Then some algebra using equation (34) in the proof of Corollary 5 proves the result.

Now suppose condition (10) holds. Then some algebra using Lemma 2 and equation (34) gives the result. □

Proof of Corollary 8

Corollary 1 proves the nontrivial Nash equilibrium exists for all $\lambda \in [0, \lambda_2]$. Hence Corollary 2 applies to all $\lambda \in [\lambda_1, \lambda_2] \subset [0, \lambda_2]$.

Consider the nontrivial Nash equilibrium, where $\theta = \theta^*$. Suppose $\mu > 0.5$ (respectively, $\mu < 0.5$), then Proposition 1 and part 1 (respectively, part 3) of Corollary 2 prove that $\theta^* > 0.5$ and $d\theta^*/d\lambda > 0$ (respectively, $\theta^* < 0.5$ and $d\theta^*/d\lambda < 0$). Hence that $\lambda_2 > \lambda_1$ implies $\theta_2 > \theta_1 > 0.5$ (respectively, $\theta_2 < \theta_1 < 0.5$). □

Proof of Proposition 3

In this proof, let $s = s^*$ given by Lemma 2. This proof will first prove the case of $\mu > 0.5$, and then the case of $\mu < 0.5$.

*Case (*i*)*

Suppose $\mu > 0.5$. Then Proposition 1 proves that $s^* \leq 1$, holding strictly if $\lambda > 0$. The property $\theta_s < 0$ (from Lemma 4) implies $\theta(1, \mu) \leq \theta(s^*, \mu) = \theta^*$, where the weak inequality holds strictly if $\lambda > 0$. Then use Assumption 8 to obtain:

$$\theta^* \geq \theta(1, \mu) = \mu$$

where the weak inequality holds strictly if $\lambda > 0$.

*Case (*ii*)*

Suppose $\mu < 0.5$. Use Assumption 8 to obtain:

$$1 - \mu = \theta(1; 1 - \mu) = 1 - \theta(1; \mu) \tag{39}$$

where the last equality applies Assumption 1.

Now, Proposition 1 proves that $s^* \geq 1$, holding strictly if $\lambda > 0$. The property $\theta_s < 0$ (from Lemma 4) implies $1 - \theta(1, \mu) \leq 1 - \theta(s^*, \mu) = 1 - \theta^*$, where the weak inequality holds strictly if $\lambda > 0$. Then use (39) to obtain

$$1 - \theta^* \geq 1 - \theta(1, \mu) = 1 - \mu$$

where the weak inequality holds strictly if $\lambda > 0$. □

Proof of Corollary 9

Corollary 1 proves the nontrivial Nash equilibrium exists for all $\lambda \in [0, \lambda_2]$. Hence Corollary 2 applies to all $\lambda \in [\lambda_1, \lambda_2] \subset [0, \lambda_2]$. Then use part 1 (respectively, part 2) of Proposition 3 and part 1 (respectively, part 3) of Corollary 2 to obtain that $\mu > 0.5$ (respectively, $\mu < 0.5$) implies $\theta_2 > \theta_1 \geq \mu$ (respectively, $\mu \geq \theta_1 > \theta_2$). □

Proof of Proposition 4

Part 1

Suppose $\mu > 0.5$ and consider the nontrivial Nash equilibrium, where $s = s^*$ given by Lemma 2 and $\theta = \theta^*$. Denote an auxiliary variable $\gamma = \frac{s^k}{(1+s^k)^2}$. Lemma 5 proves $(1 - \lambda\theta)[1 - \lambda(1 - \theta)] = (2 - \lambda)^2\gamma$. A substitution exercise using equation (33) in the proof of Corollary 3 reveals

$$\frac{d\theta}{d\mu} = \frac{\partial\theta}{\partial\mu}\left(\frac{k(1 - \lambda\theta)[1 - \lambda(1 - \theta)]}{k(1 - \lambda\theta)[1 - \lambda(1 - \theta)] + \lambda(2 - \lambda)s\theta_s}\right) = \frac{\partial\theta}{\partial\mu}\left(\frac{k(2 - \lambda)\gamma}{k(2 - \lambda)\gamma + \lambda s\theta_s}\right)$$

$$= \frac{\partial\theta}{\partial\mu}\left(1 + \frac{\lambda s\theta_s}{k(2 - \lambda)\gamma}\right)^{-1}.$$

Denote $\theta_\mu = \frac{\partial\theta}{\partial\mu}$, $\theta_{\mu\mu} = \frac{\partial^2\theta}{\partial\mu^2}$, $\theta_{ss} = \frac{\partial^2\theta}{\partial s^2}$ and $\theta_{s\mu} = \frac{\partial^2\theta}{\partial s\partial\mu}$. Differentiate both sides of $\frac{d\theta}{d\mu}$ with respect to μ to obtain

$$\frac{d^2\theta}{d\mu^2} = \left(\theta_{\mu\mu} + \theta_{s\mu}\frac{ds}{d\mu}\right)\left(1 + \frac{\lambda s\theta_s}{k(2 - \lambda)\gamma}\right)^{-1}$$

$$- \theta_\mu\left(1+\frac{\lambda s\theta_s}{k(2-\lambda)\gamma}\right)^{-2}\left[\frac{\lambda\left(\theta_s\frac{ds}{d\mu}+s\theta_{ss}\frac{ds}{d\mu}+s\theta_{s\mu}\right)k(2-\lambda)\gamma-\lambda s\theta_s k(2-\lambda)\frac{d\gamma}{d\mu}}{k^2(2-\lambda)^2\gamma^2}\right]$$

$$=\left(\theta_{\mu\mu}+\theta_{s\mu}\frac{ds}{d\mu}\right)\left(\frac{k(2-\lambda)\gamma}{k(2-\lambda)\gamma+\lambda s\theta_s}\right)$$

$$- \theta_\mu\left(\frac{k(2-\lambda)\gamma}{k(2-\lambda)\gamma+\lambda s\theta_s}\right)^{2}\left[\frac{\lambda\left(\theta_s\frac{ds}{d\mu}+s\theta_{ss}\frac{ds}{d\mu}+s\theta_{s\mu}\right)k(2-\lambda)\gamma-\lambda s\theta_s k(2-\lambda)\frac{d\gamma}{d\mu}}{k^2(2-\lambda)^2\gamma^2}\right]$$

$$=\left(\theta_{\mu\mu}+\theta_{s\mu}\frac{ds}{d\mu}\right)\left(\frac{k(2-\lambda)\gamma}{k(2-\lambda)\gamma+\lambda s\theta_s}\right)$$

$$- \frac{\lambda\theta_\mu\left[\left(\theta_s\frac{ds}{d\mu}+s\theta_{ss}\frac{ds}{d\mu}+s\theta_{s\mu}\right)k(2-\lambda)\gamma-s\theta_s k(2-\lambda)\frac{d\gamma}{d\mu}\right]}{[k(2-\lambda)\gamma+\lambda s\theta_s]^2}.$$

Denote an auxiliary variable $X_6 = k(2-\lambda)\gamma$. Then some algebra using equation (32) in the proof of Corollary 3 gives

$$\frac{ds}{d\mu} = -\frac{\lambda s\theta_\mu}{X_6+\lambda s\theta_s}. \tag{40}$$

A substitution exercise gives

$$\begin{aligned}(X_6+\lambda s\theta_s)^2\frac{d^2\theta}{d\mu^2} =&\left(\theta_{\mu\mu}+\theta_{s\mu}\frac{ds}{d\mu}\right)X_6(X_6+\lambda s\theta_s)\\ &-\lambda\theta_\mu\left[X_6 s\theta_{s\mu}+X_6(\theta_s+s\theta_{ss})\frac{ds}{d\mu}-s\theta_s k(2-\lambda)\frac{d\gamma}{d\mu}\right]\\ =&\left(\theta_{\mu\mu}-\frac{\lambda s\theta_{s\mu}\theta_\mu}{X_6+\lambda s\theta_s}\right)X_6(X_6+\lambda s\theta_s)\\ &-\lambda\theta_\mu\left[X_6 s\theta_{s\mu}-\frac{X_6(\theta_s+s\theta_{ss})\lambda s\theta_\mu}{X_6+\lambda s\theta_s}-\frac{s\theta_s k^2(2-\lambda)\gamma(1-s^k)}{s(1+s^k)}\frac{ds}{d\mu}\right]\end{aligned}$$

where the last step uses the property $\frac{d\gamma}{d\mu} = \frac{k\gamma(1-s^k)}{s(1+s^k)}\frac{ds}{d\mu}$ from Lemma 5 and equation (40). Hence

$$\begin{aligned}(X_6+\lambda s\theta_s)^2\frac{d^2\theta}{d\mu^2} =&\theta_{\mu\mu}X_6(X_6+\lambda s\theta_s)-X_6\lambda s\theta_\mu\theta_{s\mu}\\ &-\lambda\theta_\mu\left[X_6 s\theta_{s\mu}-\frac{X_6(\theta_s+s\theta_{ss})\lambda s\theta_\mu}{X_6+\lambda s\theta_s}+\frac{X_6 k(1-s^k)\theta_s\lambda s\theta_\mu}{(1+s^k)(X_6+\lambda s\theta_s)}\right]\end{aligned}$$

$$\frac{(X_6+\lambda s\theta_s)^2}{X_6}\frac{d^2\theta}{d\mu^2} = \theta_{\mu\mu}(X_6+\lambda s\theta_s) - 2\lambda s\theta_\mu\theta_{s\mu}$$

$$+ \frac{\lambda^2 s\theta_\mu^2}{X_6+\lambda s\theta_s}\left[\theta_s + s\theta_{ss} - \frac{k(1-s^k)\theta_s}{(1+s^k)}\right]$$

$$\frac{(2-\lambda)(X_6+\lambda s\theta_s)^2}{X_6}\frac{d^2\theta}{d\mu^2} = \theta_{\mu\mu}(k(2-\lambda)^2\gamma + \lambda(2-\lambda)s\theta_s) - 2\lambda(2-\lambda)s\theta_\mu\theta_{s\mu}$$

$$+ \frac{\lambda^2(2-\lambda)^2 s\theta_\mu^2}{k(2-\lambda)^2\gamma + \lambda(2-\lambda)s\theta_s}\left[\theta_s + s\theta_{ss} - \frac{k(1-s^k)\theta_s}{(1+s^k)}\right] \tag{41}$$

where the last step uses the definitions of γ and X_6. Then using Lemmas 2 and 5, some algebra reveals

$$\frac{(2-\lambda)(X_6+\lambda s\theta_s)^2\frac{d^2\theta}{d\mu^2}}{X_6}$$

$$= \theta_{\mu\mu}(k(1-\lambda\theta)[1-(1-\lambda)\theta] + \lambda(2-\lambda)s\theta_s) - 2\lambda(2-\lambda)s\theta_\mu\theta_{s\mu}$$

$$+ \frac{\lambda^2(2-\lambda)^2 s\theta_\mu^2}{k(1-\lambda\theta)[1-(1-\lambda)\theta] + \lambda(2-\lambda)s\theta_s}\left\{s\theta_{ss} + \left[1 - \frac{k\lambda(2\theta-1)}{2-\lambda}\right]\theta_s\right\} \tag{42}$$

where $X_6 > 0$ (due to the properties $\gamma > 0$ and $0 \le \lambda \le 1$), and some algebra using part 9 of Lemma 4 reveals

$$X_6 + \lambda s\theta_s = \frac{k(2-\lambda)^2\gamma + \lambda(2-\lambda)s\theta_s}{2-\lambda}$$

$$= \frac{k(1-\lambda\theta)[1-(1-\lambda)\theta] + \lambda(2-\lambda)s\theta_s}{2-\lambda} > 0.$$

Suppose condition (15) holds. Then $\frac{d^2\theta}{d\mu^2} \ge 0$, holding strictly if condition (15) holds strictly.

Now suppose condition (16) holds. Then some algebra using equation (41) reveals $\frac{d^2\theta}{d\mu^2} \ge 0$, holding strictly if condition (16) holds strictly.

Part 2

Suppose $\mu < 0.5$, and let (e_P^*, e_D^*) denote the nontrivial Nash equilibrium given Plaintiff's prior probability of success is μ. Consider another case that differs only in respect of Plaintiff's prior probability of success, which is given by $\mu' = 1-\mu$ instead. Let $(e_P^{*'}, e_D^{*'})$ denote the nontrivial Nash equilibrium if Plaintiff's prior probability of success is μ'. Then Lemma 6 proves $e_P^* = e_D^{*'}$, $e_D^* = e_P^{*'}$, and an application of Assumption 6 reveals

$$\theta(e_P^*, e_D^*; \mu) = 1 - \theta(e_P^{*'}, e_D^{*'}; \mu')$$

where given $\mu' > 0.5$, the proof for part 1 establishes $\theta(e_P^{*'}, e_D^{*'}; \mu')$ is weakly convex in the parameter representing Plaintiff's prior probability of success. Hence an application of the chain rule gives the result. $\square$

Proof of Corollary 10

Corollary 1 proves the nontrivial Nash equilibrium exists for all $\lambda \in [0, \lambda_2]$. Hence Corollary 2 applies to all $\lambda \in [\lambda_1, \lambda_2] \subset [0, \lambda_2]$.

Suppose $\mu > 0.5$. That θ is weakly convex in μ (from part 1 of Proposition 4) and the property $\theta \leq 1$ imply $\theta_2 \leq \mu$. Then use part 1 of Corollary 2 to obtain $\mu \geq \theta_2 > \theta_1$.

Now suppose $\mu < 0.5$. That θ is weakly concave in μ (from part 2 of Proposition 4) and the property $\theta \geq 0$ imply $\theta_2 \geq \mu$. Then use part 3 of Corollary 2 to obtain $\theta_1 > \theta_2 \geq \mu$. $\square$

Proof of Lemma 3

This proof applies the principle of finite induction. The result clearly holds for the base case $\theta = \theta_1$. The rest of this proof assumes the result holds for all integer value $j \leq n-1$, and proves that it holds for $j = n$. To facilitate presentation, define auxiliary variables $0 \leq a, \bar{a} \leq 1$ and a success function $\theta_I : \mathbb{R}_+^2 \to \mathbb{R}$ by

$$a = p_1 + p_2 +, \dots, +p_{n-1} \qquad \bar{a} = 1 - a \qquad \theta_I = \sum_{i}^{n-1} \left(\frac{p_i}{a} \theta_i \right)$$

where $p_1 + p_2 +, \dots, +p_{n-1}$ are weights assigned to $\theta_1, \theta_2, \dots, \theta_{n-1}$ respectively. The assumption that the result holds for $j = n-1$ implies $\theta_I \in \Theta(\{\lambda\}, \{k\})$.

Assumption 1

Consider three arbitrary real numbers $e_1, e_2 > 0$ and $0 < \mu_0 < 1$. The definition of θ and the property that θ_I, θ_n satisfy Assumption 1 imply

$$\begin{aligned}\theta(e_1, e_2; \mu_0) =& a\theta_I(e_1, e_2; \mu_0) + \bar{a}\theta_n(e_1, e_2; \mu_0) = a[1 - \theta_I(e_2, e_1; 1 - \mu_0)] \\ &+ \bar{a}[1 - \theta_n(e_2, e_1; 1 - \mu_0)] \\ =& 1 - [a\theta_I(e_2, e_1; 1 - \mu_0) + \bar{a}\theta_n(e_2, e_1; 1 - \mu_0)] = 1 - \theta(e_2, e_1; 1 - \mu_0).\end{aligned}$$

Assumption 2

For any $x > 0$, use the definition of θ and the property that θ_I, θ_n satisfy Assumption 2 to obtain

$$\begin{aligned}\theta(xe_P, xe_D, \mu) =& a\theta_I(xe_P, xe_D; \mu) + \bar{a}\theta_n(xe_P, xe_D; \mu) \\ =& a\theta_I(e_P, e_D; \mu) + \bar{a}\theta_n(e_P, e_D; \mu) = \theta(e_P, e_D; \mu).\end{aligned}$$

Assumptions 3-4

Using the linearity of differentiation, some algebra will establish the result.

Assumption 5

Using the property that θ_I, θ_n satisfy condition (1), some algebra reveals

$$a\frac{\partial^2\theta_I}{\partial e_P^2} + a\frac{2\lambda\left(\frac{\partial\theta_I}{\partial e_P}\right)^2}{(1-\lambda\theta_I)} < a\frac{\partial\theta_I}{\partial e_P}\frac{C''(e_P)}{C'(e_P)}, \qquad \bar{a}\frac{\partial^2\theta_n}{\partial e_P^2} + \bar{a}\frac{2\lambda\left(\frac{\partial\theta_n}{\partial e_P}\right)^2}{(1-\lambda\theta_n)} < \bar{a}\frac{\partial\theta_n}{\partial e_P}\frac{C''(e_P)}{C'(e_P)}.$$

Summing these inequalities give

$$a\frac{\partial^2\theta_I}{\partial e_P^2} + \bar{a}\frac{\partial^2\theta_n}{\partial e_P^2} + a\frac{2\lambda\left(\frac{\partial\theta_I}{\partial e_P}\right)^2}{(1-\lambda\theta_I)} + \bar{a}\frac{2\lambda\left(\frac{\partial\theta_n}{\partial e_P}\right)^2}{(1-\lambda\theta_n)} < a\frac{\partial\theta_I}{\partial e_P}\frac{C''(e_P)}{C'(e_P)} + \bar{a}\frac{\partial\theta_n}{\partial e_P}\frac{C''(e_P)}{C'(e_P)}$$

$$\Leftrightarrow \quad \frac{\partial^2\theta}{\partial e_P^2} + a\frac{2\lambda\left(\frac{\partial\theta_I}{\partial e_P}\right)^2}{(1-\lambda\theta_I)} + \bar{a}\frac{2\lambda\left(\frac{\partial\theta_n}{\partial e_P}\right)^2}{(1-\lambda\theta_n)} < \frac{\partial\theta}{\partial e_P}\frac{C''(e_P)}{C'(e_P)} \tag{43}$$

where the last step uses the definition of θ and the linearity of differentiation.

Now, some algebra reveals

$$0 \le a\bar{a}\left[\frac{\partial\theta_I}{\partial e_P}(1-\lambda\theta_n) - \frac{\partial\theta_n}{\partial e_P}(1-\lambda\theta_I)\right]^2$$

$$\Leftrightarrow 2a\bar{a}\frac{\partial\theta_I}{\partial e_P}\frac{\partial\theta_n}{\partial e_P}(1-\lambda\theta_I)(1-\lambda\theta_n) \le a\bar{a}\left(\frac{\partial\theta_I}{\partial e_P}\right)^2(1-\lambda\theta_n)^2 + a\bar{a}\left(\frac{\partial\theta_n}{\partial e_P}\right)^2(1-\lambda\theta_I)^2$$

$$\Leftrightarrow \; 2a\bar{a}\frac{\partial\theta_I}{\partial e_P}\frac{\partial\theta_n}{\partial e_P} + a^2\left(\frac{\partial\theta_I}{\partial e_P}\right)^2 + \bar{a}^2\left(\frac{\partial\theta_n}{\partial e_P}\right)^2 \le \frac{a\bar{a}\left(\frac{\partial\theta_I}{\partial e_P}\right)^2(1-\lambda\theta_n)^2 + a\bar{a}\left(\frac{\partial\theta_n}{\partial e_P}\right)^2(1-\lambda\theta_I)^2}{(1-\lambda\theta_I)(1-\lambda\theta_n)} + a^2\left(\frac{\partial\theta_I}{\partial e_P}\right)^2 + \bar{a}^2\left(\frac{\partial\theta_n}{\partial e_P}\right)^2$$

$$\Leftrightarrow \; \frac{a^2\left(\frac{\partial\theta_I}{\partial e_P}\right)^2 + 2a\bar{a}\frac{\partial\theta_I}{\partial e_P}\frac{\partial\theta_n}{\partial e_P} + \bar{a}^2\left(\frac{\partial\theta_n}{\partial e_P}\right)^2}{a(1-\lambda\theta_I) + \bar{a}(1-\lambda\theta_n)} \le \frac{a(1-\lambda\theta_n)\left(\frac{\partial\theta_I}{\partial e_P}\right)^2 + \bar{a}(1-\lambda\theta_I)\left(\frac{\partial\theta_n}{\partial e_P}\right)^2}{(1-\lambda\theta_I)(1-\lambda\theta_n)}$$

$$\Leftrightarrow \; \frac{\left[a\frac{\partial\theta_I}{\partial e_P} + \bar{a}\frac{\partial\theta_n}{\partial e_P}\right]^2}{1-\lambda(a\theta_I + \bar{a}\theta_n)} \le \frac{a(1-\lambda\theta_n)\left(\frac{\partial\theta_I}{\partial e_P}\right)^2 + \bar{a}(1-\lambda\theta_I)\left(\frac{\partial\theta_n}{\partial e_P}\right)^2}{(1-\lambda\theta_I)(1-\lambda\theta_n)}$$

$$\Leftrightarrow \; \frac{\left(\frac{\partial\theta}{\partial e_P}\right)^2}{1-\lambda\theta} \le \frac{a(1-\lambda\theta_n)\left(\frac{\partial\theta_I}{\partial e_P}\right)^2 + \bar{a}(1-\lambda\theta_I)\left(\frac{\partial\theta_n}{\partial e_P}\right)^2}{(1-\lambda\theta_I)(1-\lambda\theta_n)}$$

where the last step uses the definition of θ and the linearity of differentiation. Hence

$$\frac{\left(\frac{\partial\theta}{\partial e_P}\right)^2}{1-\lambda\theta} \leq \frac{a(1-\lambda\theta_n)\left(\frac{\partial\theta_I}{\partial e_P}\right)^2 + \bar{a}(1-\lambda\theta_I)\left(\frac{\partial\theta_n}{\partial e_P}\right)^2}{(1-\lambda\theta_I)(1-\lambda\theta_n)} = a\frac{\left(\frac{\partial\theta_I}{\partial e_P}\right)^2}{1-\lambda\theta_I} + \bar{a}\frac{\left(\frac{\partial\theta_n}{\partial e_P}\right)^2}{1-\lambda\theta_n}$$

which implies

$$\frac{2\lambda\left(\frac{\partial\theta}{\partial e_P}\right)^2}{(1-\lambda\theta)} \leq a\frac{2\lambda\left(\frac{\partial\theta_I}{\partial e_P}\right)^2}{(1-\lambda\theta_I)} + \bar{a}\frac{2\lambda\left(\frac{\partial\theta_n}{\partial e_P}\right)^2}{(1-\lambda\theta_n)}.$$

Then use inequality (43) to obtain

$$\frac{\partial^2\theta}{\partial e_P^2} + \frac{2\lambda\left(\frac{\partial\theta}{\partial e_P}\right)^2}{(1-\lambda\theta)} < \frac{\partial\theta}{\partial e_P}\frac{C''(e_P)}{C'(e_P)}$$

where some algebra will reveal that it is equivalent to θ satisfying condition (1).

Assumption 6

Suppose $k = \lambda = 1$. Use the definition of θ and the properties of limits to obtain

$$\lim_{s\to 0}\theta = \lim_{s\to 0}(a\theta_I + \bar{a}\theta_n) = a\left(\lim_{s\to 0}\theta_I\right) + \bar{a}\left(\lim_{s\to 0}\theta_n\right) < a + \bar{a} = 1$$

where the last inequality uses the property that θ_I, θ_n satisfy Assumption 6. □

Proof of Proposition 5

Part 1

Suppose $\mu > 0.5$. Using part 2 of Lemma 5, some algebra reveals

$$\frac{dC^*}{d\mu} = \left\{s\theta_s(2-\lambda)k\frac{d\gamma}{d\mu} - (2-\lambda)k\gamma\left[(\theta_s + s\theta_{ss})\frac{ds}{d\mu} + s\theta_{s\mu}\right]\right\}s^{-2}\theta_s^{-2}\left[\lambda + \frac{(2-\lambda)k\gamma}{s\theta_s}\right]^{-2}$$

where Lemma 5 gives

$$\frac{d\gamma}{d\mu} = \frac{k\gamma(1-s^k)}{s(1+s^k)}\frac{ds}{d\mu}.$$

Then a substitution exercise reveals

$$[(2-\lambda)k\gamma + \lambda s\theta_s]^2\frac{dC^*}{d\mu} = \frac{\theta_s(2-\lambda)k^2\gamma(1-s^k)}{(1+s^k)}\frac{ds}{d\mu} - (2-\lambda)k\gamma\left[(\theta_s + s\theta_{ss})\frac{ds}{d\mu} + s\theta_{s\mu}\right]$$

$$\frac{[(2-\lambda)k\gamma + \lambda s\theta_s]^2\frac{dC^*}{d\mu}}{(2-\lambda)k\gamma} = \frac{k(1-s^k)\theta_s}{(1+s^k)}\frac{ds}{d\mu} - (\theta_s + s\theta_{ss})\frac{ds}{d\mu} - s\theta_{s\mu}$$

$$= \frac{ds}{d\mu}\left[\frac{k(1-s^k)\theta_s}{(1+s^k)} - \theta_s - s\theta_{ss}\right] - s\theta_{s\mu}$$

$$= -\frac{\lambda s\theta_\mu}{k(2-\lambda)\gamma + \lambda s\theta_s}\left[\frac{k(1-s^k)\theta_s}{(1+s^k)} - \theta_s - s\theta_{ss}\right] - s\theta_{s\mu} \tag{44}$$

where the last step uses equation (40) in the proof of Proposition 4.

Suppose condition (18) holds. Then using the properties $(2-\lambda)k\gamma + \lambda s\theta_s > 0$ (due to part 9 of Lemma (4) and $\gamma > 0$, equation (44) reveals $\frac{dC^*}{d\mu} \geq 0$, holding strictly if condition (18) holds strictly.

Now suppose condition (19) hold. Then using Lemma (5), some algebra will reveal that $\frac{dC^*}{d\mu} \geq 0$, holding strictly if condition (19) holds strictly.

Part 2

Suppose $\mu < 0.5$. Let (e_P^*, e_D^*) denote the nontrivial Nash equilibrium given Plaintiff's prior probability of success is μ. Consider another case that differs only in respect of Plaintiff's prior probability of success, which is given by $\mu' = 1 - \mu$ instead. Let $(e_P^{*'}, e_D^{*'})$ denote the nontrivial Nash equilibrium if Plaintiff's prior probability of success is μ'. Then Lemma 6 proves $e_P^* = e_D^{*'}$, $e_D^* = e_P^{*'}$. Hence

$$C(e_P^*) + C(e_D^*) = C(e_P^{*'}) + C(e_D^{*'})$$

where the proof for part 1 establishes that the right hand side is convex in Plaintiff's prior probability of success μ'. Then an application of the chain rule gives the result. □

Proof of Proposition 6

Using equation (44) in the proof of Proposition 5, some algebra will give the result. □

B Appendix: Derivatives of Illustrative Success Functions

This Appendix calculates the partial, second partial and cross derivatives for the illustrative success functions θ_T in (13) and θ_L in (17).

The Tullock success function θ_T satisfies the following properties:

$$\frac{\partial\theta_T}{\partial e_P} = \frac{\mu(1-\mu)e_D}{[\mu e_P + (1-\mu)e_D]^2} \qquad \frac{\partial^2\theta_T}{\partial e_P^2} = \frac{-2\mu^2(1-\mu)e_D}{[\mu e_P + (1-\mu)e_D]^3}$$

$$\frac{\partial\theta_T}{\partial\mu} = \frac{s}{[\mu + (1-\mu)s]^2} \qquad \frac{\partial^2\theta_T}{\partial\mu^2} = \frac{2(s-1)s}{[\mu + (1-\mu)s]^3}$$

$$\frac{\partial\theta_T}{\partial s} = \frac{-\mu(1-\mu)}{[\mu + (1-\mu)s]^2} \qquad \frac{\partial^2\theta_T}{\partial s^2} = \frac{2\mu(1-\mu)^2}{[\mu + (1-\mu)s]^3}$$

$$\frac{\partial^2\theta_T}{\partial\mu\partial s} = \frac{\mu-(1-\mu)s}{[\mu+(1-\mu)s]^3} \qquad \frac{\partial\theta_T}{\partial s} + s\frac{\partial^2\theta_T}{\partial s^2} = \frac{\mu(1-\mu)[(1-\mu)s-\mu]}{[\mu+(1-\mu)s]^3}.$$

The linear success function θ_L satisfies the following properties:

$$\frac{\partial\theta_L}{\partial e_P} = \frac{(1-\eta)e_D}{[e_P+e_D]^2} \qquad \frac{\partial^2\theta_L}{\partial e_P^2} = \frac{-2(1-\eta)e_D}{[e_P+e_D]^3}$$

$$\frac{\partial\theta_L}{\partial\mu} = \eta \qquad \frac{\partial^2\theta_L}{\partial\mu^2} = 0$$

$$\frac{\partial\theta_L}{\partial s} = \frac{-(1-\eta)}{(1+s)^2} \qquad \frac{\partial^2\theta_L}{\partial s^2} = \frac{2(1-\eta)}{(1+s)^3}$$

$$\frac{\partial\theta_L}{\partial\mu\partial s} = 0 \qquad \frac{\partial\theta_L}{\partial s} + s\frac{\partial^2\theta_L}{\partial s^2} = \frac{(s-1)(1-\eta)}{(1+s)^3}.$$

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