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Fiscal Space under Demographic Shift*

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Abstract

In this paper we quantitatively explore implications of ageing demographic structure for government revenue raising capacity through lens of fiscal space. We base our analysis on dynamic general equilibrium, overlapping generations model calibrated to data from Japan and USA. We first map out fundamental-based fiscal limit using the Laffer curve approach, and then compute fiscal space in terms of budgetary room between the current revenue and the maximum revenue defined by the peak of Laffer curves. We demonstrate that the evolution of underlying demographic structures plays an important role in shaping a country’s fiscal limit and fiscal space. There will be significant contractions in fiscal space in Japan and USA when the two countries enter their late stage of demographic transition in 2040. In particular, the results from the model calibrated to Japan indicates that an increase in old-age dependency ratio to over 70 percent can reduce Japan’s fiscal space by 36 percent. When factoring in the increased fiscal cost of existing commitments to the age pension program, the net fiscal space for Japan turns negative.

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Keywords: Population Ageing, Laffer Curve, Fiscal Limit, Sustainability, Heterogeneity, Dynamic General Equilibrium.

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1 Introduction

The issue of fiscal sustainability has taken on special importance in the aftermath of the global financial crisis in advanced economies. There have been sharp increases in debt to GDP ratio because of the worldwide financial crisis and the prolonged and deep recession that generated automatic budget deficits and induced many countries to implement sizeable fiscal stimulus packages. Unfortunately, the fiscal situation in advanced economies are heading to even more trouble. The long-term projections of debt to GDP ratio indicates the fiscal situation will be even worse with rising public debt is a prominent worry in all advanced economies (e.g., see Figure 15 and Cecchetti, Mohanty and Zampolli (2010)). Yet, providing funding to meet the need of age-related government spending will be even more of a pressing challenge in years to come. Many advanced economies that have committed to generous benefits of ageing-related public programs are soon heading into a phrase of fiscal unsustainability. It now becomes clear that many governments are not able to raise enough revenue to fulfill all existing fiscal commitments (e.g., see IMF (2010)). There is no doubt that every government is grounded by its own revenue raising capacity. However, we are far from fully understanding of how such fiscal capacity evolves with population ageing.

In this paper we quantify the effects of ageing demographic structure on a government’s fiscal capacity, using a model-based framework featured with demographic factors and economic fundamentals. We base our analysis on a new approach. We first measure government revenue raising capacity through lens of fiscal space defined by in terms of budgetary room between the current tax revenue and the maximum tax revenue (fiscal limit). We then quantitatively assess how the size and shape of fiscal space are affected by the ageing demographic structure.

More specifically, we formulate an overlapping generations model with uninsurable idiosyncratic risk and incomplete markets. The model consists of heterogeneous households, a perfect competitive representative firm and government. In our fundamentals-based model, Laffer curves to define government revenue raising capacity. Basically, the peak of Laffer curves defines maximum revenue to that government can raise in order to cover its spending programs and payback government debt, i.e. fiscal limit. We argue that Laffer curve fiscal limit is country-specific and determined by model fundamentals including preferences, endowments, technologies and market structure. Demographic factors as a part of the underlying model fundamentals affect household behaviors and equilibrium conditions and play a role in defining a country’s fiscal limit and fiscal space.

We calibrate our benchmark model to data from Japan in 2010. Our model is capable to match key patterns of life-cycle behavior and essential features of the Japanese
Next, we use the benchmark model to conduct a quantitative analysis. Our main results are summarized as follows.

First, we construct Laffer curves for labor and capital income taxes, and consumption taxes. We find that Laffer curves for labor and capital taxes have a single peak, while Laffer curve for consumption tax has no peak and monotonically increases as the tax rate increases. For both labor and capital taxes, the benchmark economy lies to the left of the peaks of the Laffer curves. The Japanese government can increase one of the two taxes to raise revenue. Specifically, we use a gap between the 2010 tax revenue and the maximum revenues defined by Laffer curves to measure the fiscal space for Japan. We find that the government can, by altering either labor income tax or capital income tax while keeping the other constant, increase the fiscal spaces for labor and capital income taxation by 43 percent and 17 percent, respectively. Moreover, when the labor and capital income taxes are allowed to vary at the same time we find that the fiscal space for Japan, measured in terms of Laffer hill, can be expanded further up to 45 percent.

Next, we quantitatively characterize how changing demographic structure affect the size and shape of fiscal space. We employ two alternative demographic structures: younger one in 1980 and older one in 2040. We find that aging demographic structure shifts the Laffer curves downward and causes a significant contraction in fiscal space for Japan by 2040. Specifically, the demographic shift leads to a contraction in the fiscal space for capital taxation by 65 percent, compared to the 2010 level. Meanwhile, the fiscal space for the labor income tax is contracted by 38 percent and the fiscal space for the consumption tax is decreased by 11 percent. Even though the fiscal space is diminished significantly, the government still has some room to raise tax revenues. The government can increase tax revenues by an additional 51 percent, when the labor income tax rate is set to 0.68. By raising labor and capital tax rates simultaneously, the government can raise revenue by an additional 54 percent, compared to the benchmark level of 2010 tax revenue.

In addition, we construct the net fiscal space where we account for the increased government outlay due to commitments to the age pension program. Expectedly, the net fiscal spaces will shrink even more. When the government can only alter one tax instrument at a time, the net fiscal spaces for labor and consumption taxes will be contracted by 59.0 percent and 13 percent, respectively. Most notably, the net fiscal space for the capital income tax will disappear completely, and only 96 percent of the 2010 fiscal space can be maintained at the peak of Laffer curve for the capital income tax.

Moreover, we identify the relative importance of the fertility and mortality rates in contributing to the decrease in fiscal space. We find that over the 1980 to 2010 period, the
increase in survival probabilities and decrease in fertility rate have contributed equally in ageing the population and decreasing fiscal space. However, the contribution will be asymmetrical over the 2010 to 2040 period, with fertility as the main driver for further decreases in the fiscal space. We also note that a purely fertility rate driven change in the dependency ratio will shift the fiscal space to a greater extent than a purely mortality rate driven change in the dependency ratio of the same size.

In an extension, we explore the US economy’s fiscal limit and fiscal space under demographic shift. We re-calibrate our benchmark model to match the US data in 2010. We conduct similar experiments and find that the effects of demographic shift on fiscal space in the US are quite different. Unlike Japan, there was a large expansion in fiscal space in the US from 1980 to 2010. This expansion is driven mainly by increase in the population share of workers in their 40s. However, as the population ageing is accelerated, the fiscal space for the US will be contracted by 40 percent in 2040. These results highlight that fiscal limits and fiscal spaces vary greatly across countries and over time, depending on the evolution of underlying demographic factors.

**Related literature.** Our paper is related to the recent literature attempting to quantify fiscal limit and space. Ostry, Ghosh, Kim and Qureshi (2010) use the notion of the debt limits above which the debt becomes unsustainable to define fiscal limit. They estimate the debt limits using a reduced-form model for policy reaction function. They construct fiscal space in terms of a distance between the current debt levels and the debt limits. Similarly, Ghosh, Kim, Mendoza, Ostry and Qureshi (2013) estimate a country’s debt limit using a stochastic ability-to-pay model of sovereign default. They use data from 23 advanced economies to estimate the responses of primary surpluses to debt levels and compute a debt limit for each country that is fully determined by the risk-free interest rate, the recovery rate, and the support of the shock to primary balances. They define fiscal space in terms of the difference between the long-run average debt ratio and the debt limit. Notice that, the empirically-based approach to calculating fiscal limit and space is grounded in the historical data. Their calculations rely on the underlying assumption that the government always follows its historically estimated rule as well as there is no structural change in the economic environment. Any change in policy rules as well as economic fundamentals would alter the country’s fiscal limit, destabilizing the backward measure of fiscal space.

Bi and Leeper (2012) argue that a country’s fiscal limit varies systematically with the economic environment, including the specification of policy behavior. They formulate a real business cycle model and maps the economic environment into a distribution for the maximum sustainable debt-GDP ratio. They calibrate their model to Greek and Swedish data to illustrate how the framework can be used to study actual fiscal re-
forms undertaken by developed economies facing sovereign risk pressures. Bi and Traum (2014) use Bayesian methods to estimate the fiscal limit distribution for Greece, using a real business cycle model that allows for interactions among fiscal policy instruments, the stochastic fiscal limit and sovereign default. Richter (2015) uses a perpetual youth model to examine how intergenerational redistributions of wealth, the average duration of government debt, and entitlement reform impact the consequences of explosive government transfers. These studies focus on interactions between monetary and fiscal policies. They point out that future projected fiscal deficits and fiscal stress are driven by the growth in entitlement spending. Note that, these studies abstract from demographic factors that switch promised government transfers from a stable to an explosive are not included in the economic environment. They are not able to map out an explicit link between demographic structures and fiscal limit, which the main focus of our paper.

Our paper contributes to a growing body of literature using the neoclassical growth models to quantify Laffer curves. Trabandt and Uhlig (2011) construct Laffer curves for the US and the EU 14 in a infinitely lived representative agent model. They find that the classic Laffer curve shape exists for labor and capital income taxes. Park (2012) applies the Trabandt and Uhlig representative framework to examine the effects of ageing on the revenue generating capacities of the G-7 nations. However, Park abstracts from foundations of demographic structure and model population ageing as an exogenous shift in the dis-utility from supplying labor over time. Mendoza, Tesar and Zhang (2014) formulate a two-country representative agent model with cross-country tax externality to quantify Laffer curves for eurozone countries. They focus on the positive and normative effects of alternative tax strategies that countries could follow to restore fiscal solvency in response to debt shocks. Different from these studies, we develop a full life-cycle model that explicitly accounts for demographic factors that drive changes in demographic structures. This modelling approach allows us to transparently model the ageing process and its impact on household behavior, tax bases and fiscal limits; and disaggregate the effects of changes in the survival probabilities and fertility rate on fiscal capacity.

Feve, Matheron and Sahuc (2013) develop an infinitely lived heterogeneous agent model with government debt and show that the Laffer curve depends on the level of outstanding government debt. Holter, Krueger and Stepanchuk (2014) extends Trabandt and Uhlig (2011)’s analysis to a life cycle model with realistically calibrated wage heterogeneity and risk, extensive margin labor supply choice as well as endogenous human capital accumulation. They find that household heterogeneity and the degree of tax progressivity matter for the level and location of the peak of the Laffer curve. Note that, Holter et al (2014) do not consider the impact of changing demographic structure on Laffer curves. We have a similar modelling approach, but focus on implications of
demographic shift for fiscal limit and fiscal space.

Since Auerbach and Kotlikoff (1987), there is a vast literature that uses overlapping generations (OLG) models featured with inter- and intra-generational heterogeneity and population dynamics study the dynamic effects of policy policy. Recently, that literature is extended to quantifying the consequences of population ageing and fiscal adjustment in advanced economies (e.g., see Kitao (2015), Braun and Joines (2015), Nishiyama (2015) and Kudrna, Tran and Woodland (2015)). Notice that, most of these studies focus on quantifying the expenditure-side effects of population ageing as well as the effects of counterfactual fiscal adjustments. In this paper, we connect that ageing literature to the Laffer curves literature (e.g., see Trabandt and Uhlig (2011), Feve, Matheron and Sahuc (2013), Mendoza, Tesar and Zhang (2014) and Holter, Krueger and Stepanchuk (2014)). That is, we rely on the Laffer curve approach to quantify the effects of changing demographic factors on fiscal sustainability through lens of fiscal limit and fiscal space.

The remaining paper is organized as follows. Section 2 provides an overview of the model. Section 3 outlines the calibration process to match for the benchmark Japanese economy in 2010. Section 4 presents the quantitative analysis and results. Section 5 offers a sensitivity analysis and extension. Lastly, section 6 offers a conclusion. The additional information, tables and graphs are included in Appendix.

2 Model

The model is a stochastic dynamic general equilibrium model, which consists of overlapping households, a perfect competitive representative firm, and a government with full commitment technology.

2.1 Demographics

In each discrete time period $t$, the economy is populated by $J$ overlapping generations of households of generations $j = 1, ..., J$. Each period, a new cohort of households of the generation is born. Each period, households of a particular generation share a common chance of dying before reaching the next period. The probability of surviving to generation $j + 1$ conditional on belonging to generation $j$ is denoted by $sp_j$. We denote the size of the generation $j$ cohort at the beginning time $t$ as $P_j$. The cohort share of the generation $j$ households at time $t$ is given by $\mu_j = \frac{P_j}{\sum_{j=1}^{J} P_j}$. Demographic structure is driven by two factors: (i) the age-dependent survival probability ($sp_j$), and (ii) the growth rate of the new generation ($g^n$). When the demographic pattern is stationary, as
assumed here, the population share of the cohort age \( j \) is constant at any point in time and can be recursively defined as \( \mu_j = \mu_{j-1}sp_j/(1+g^n) \). The share of agents who do not survive to age \( j \) is \( \tilde{\mu}_j = \mu_{j-1}(1-sp_j)/(1+g^n) \).

2.2 Preferences

All households have identical lifetime preferences over consumption \( c_j \geq 0 \) and leisure \( l_j \), where household leisure time per period for household \( j \) is constrained by \( 0 \leq l_j \leq 1 \). Preferences are time-separable with a constant subjective discount factor \( \beta \) and are given by the expected utility function

\[
E \left[ \sum_{j=1}^{J} \beta^j u(c_j, l_j) \right].
\]  

(1)

2.3 Endowments

In each period of life households are endowed with 1 unit of labor time that has labor efficiency (or working ability) denoted by \( e_j \). The efficiency unit \( e_j \) is skill and age dependent and follows a Markov switching process with \( \pi_j(e_{j+1}|e_j) \) denoting the conditional probability that an individual of working ability \( e_j \) at age \( j \) will have working ability \( e_{j+1} \) when at age \( j+1 \). According to this specification, agents have working abilities that vary by age and change stochastically over the life cycle; they therefore face idiosyncratic earnings risk, which is assumed to be non-insurable.

Households devote \( l_j \) units of time to leisure and supply \( n_j = (1-l_j) \) units of time to the labor market. The quantity of effective labor supplied is given by \( h_j = (1-l_j)e_j = n_je_j \), and labor earnings are \( w_th_j \). For the retired periods \( j = J^w+1, ..., J \), households are out of the labor force and consume \( l_j = 1 \) units of leisure time, and do not have labor earnings.

We let \( a_j \) denote asset holdings of a typical agent at age \( j \). We assume households enter the economy with no assets, and \( a_1 = 0 \). We let \( x_j = \{a_j, e_j\} \) denote the state variable of a typical household at age \( j \) and \( \mu(x_j) \) denote the measure of households in state \( x_j \).

2.4 Technology

The production sector is made up of a large number of competitive firms, and can be proxied by a single producer that maximizes profits. It produces a single output \( (Y_t) \) each period from two inputs, capital \( (K_t) \) and effective labor \( (H_t) \), based on a constant returns
to scale production function \( Y_t = A_t F(K_t, H_t) \) where \( A_t \) is the total factor productivity and grows at a constant rate \( g \). The firm is a price taker in the input markets for capital and labor, and aims to maximize its profit given the rental rate \( q_t \) and market wage rate \( w_t \) by choosing \( K_t \) and \( H_t \) such that

\[
\max_{K_t, H_t} \{ A_t F(K_t, H_t) - q_t K_t - w_t H_t \} \tag{2}
\]

### 2.5 Fiscal policy

The government runs a social security system and two other spending programs.

**Social security system.** The government provides the retirement benefits \( p_j \) to all retiring households at age \( j = J^w + 1, \ldots, J \). The social security benefits are given by a replacement rate \( \Psi \) and an average life-time labor earning \( w H_{j^w} \) at time \( t \), so that \( p_j = \Psi w H_{j^w} \). The total social security payment for all retirees at time \( t \) is \( SS_t = \sum_{j=J^w+1}^{J} \sum_{x_j} p_j \mu_j (x_j) \). The social security system is partially funded by social security tax revenue and partially funded by the general government budget. Let \( \theta^{ss} \) denote a fraction of the total social security payment funded by the general government budget. Social security tax \( \tau^{ss} \) adjusts to clear the rest of the social security payment

\[
\tau^{ss} \sum_{j=1}^{J^w} \sum_{x_j} w_t h_j \mu_j (x_j) = (1 - \theta^{ss}) SS_t, \tag{3}
\]

with \( \theta^{ss} \in [0, 1] \). There are two special cases. When \( \theta^{ss} = 0 \), the social security system is fully self-financed. When \( \theta^{ss} = 1 \), the social security system is a part of overall government operations.

**Government budget.** The government has two spending programs: general government purchases, \( G_t \), and transfers to the households, \( Tr_t = \sum_{j=1}^{J} \sum_{x_j} tr_j \mu_j (x_j) \), where \( tr_j \) is individual lump-sum transfers received by households. In order to finance its expenditures the government collects revenues through taxes on consumption \( (\tau^c) \), labor income \( (\tau^l) \) and capital income \( (\tau^k) \). The total tax revenue is given by

\[
\text{Tax}_t = \tau^c \sum_{j=1}^{J} \sum_{x_j} c_j \mu_j (x_j) + \tau^l \sum_{j=1}^{J^w} \sum_{x_j} w_t h_j \mu_j (x_j) + \tau^k \sum_{j=1}^{J} \sum_{x_j} r_t a_j \mu_j (x_j),
\]

where \( w_t \) is the market wage rate and \( r_t \) is the market interest rate.

The government maintains budget balance each period, using a combination of the taxation revenue and issuance of new debt \( D_{t+1} \) to fund interest and principle payments.
on existing debt \((1 + r^d_t)D_t\) with \(r^d_t\) is the interest rate for government debt. The government inter-temporal budget is given by

\[(1 + g^n)(1 + g)D_{t+1} + Tax_t = (1 + r^d_t)D_t + G_t + \theta^{ss}SS_t + Tr_t. \tag{4}\]

### 2.6 Market structure

Markets are incomplete and households cannot insure against the idiosyncratic labor income and mortality risks by trading state contingent assets. They can, however, hold one-period riskless assets to imperfectly self-insure against idiosyncratic risks. We assume that households are not allowed to borrow against future income, implying asset holdings are non-negative, i.e., \(a_j \geq 0\) for all \(j\).

The economy is closed, and the domestic interest rate is determined endogenously. It is related to the rental price of capital by \(r_t = q_t - \delta\), where \(q_t\) is determined by the demand and supply for capital in the economy, and \(\delta\) is the depreciation rate of capital.

### 2.7 Household problem

In this model, households are heterogeneous with respect to their age, working ability and asset holdings. Over their lifetime, households have different sources of income. First, as a worker with age between 1 and \(J^w\), households supply labor to the firm in return for wage income, \(w_t h_j\). Labor income is subjected to a social security tax \((\tau^{ss})\) and labor income tax \((\tau^l)\). Households exit the labor market and retire at age \(j > J^w\). Households’ savings are rented out to the firms in the form of capital the following period. It earns interest at the rate \(r_t\) and is taxed at the rate \(\tau^k\). Henceforth, the post-tax return is \(R^k_t = 1 + (1 - \tau^k)r_t\). During the retirement time, households receive a public pension benefit \((p_j)\) from the government. Households also receive an equal share of lump-sum transfers from the government \((tr_j)\). Finally, as there are no annuity markets, the savings (including interest return) of households who die each period are shared out equally amongst the remaining households as accidental bequests \((b_j)\).

At the beginning of age \(j\) the household realizes its individual state \(x_j\) and chooses its optimal consumption, \(c_j\), leisure time, \(l_j\), or working hours, \((1 - l_j)\), and the end-of-period asset holdings, \(a_{j+1}\), taking the transition law for working ability, \(\pi_j(c_{j+1}|c_j)\), conditional survival probabilities, \(sp_j\), the wage and interest rates, and government tax and pension policies as given. Formally, the Bellman equation for a household of age \(j\) is given by

\[V_j(x_j) = \max_{c_j, l_j, a_{j+1}} \{u(c_j, l_j) + sp_j \beta E[V_{j+1}(x_{j+1})]\} \tag{5}\]
subject to the budget constraint

\[(1 + g) a_{j+1} + (1 + \tau^c) c_j = \begin{cases} R_k^k a_j + (1 - \tau^l - \tau^{ss}) w_j h_j + tr_j + b_j & \text{if } j = 1, \ldots, J^w \\ R_k^k a_j + p_j + tr_j + b_j & \text{if } j > J^w, \end{cases}\]

where \( V_j(x_j) \), the value function of a household at age \( j \) conditional on the given state variable \( x_j \) and \( E[V_{j+1}(x_{j+1})] \) is the expected value function. Additional constraints are \( a_1 = 0, a_{J+1} = 0, a_j \geq 0 \) and \( 0 < l_j \leq 1 \).

### 2.8 Equilibrium

Given a set of exogenous for demographic parameters \( \{sp_j\}_{j=1}^J \) and \( \{\gamma^n\} \), exogenous growth rate \( \{g\} \) and fiscal policy variables \( \{\Psi, \theta^{ss}, \tau^{ss}, \tau^k, \tau^l, \tau^c, G, D, tr, p, r^d\} \), a competitive equilibrium consists of a collection of household decisions \( \{c_j(x_j), l_j(x_j), a_{j+1}(x_j)\}_{j=1}^J \) for each state vector \( x_j \), factor prices \( \{w, r\} \), consumption tax \( \{\tau^l\} \), the measure of individual state \( \{\mu(x_j)\} \) such that

(a) the households solve the household problem (5);

(b) the firm chooses labor and capital inputs to solve the profit maximization problem (2);

(c) factor prices are determined competitively, i.e., \( w = F_L(K, L), q = F_K(K, L) \) and \( r = q - \delta \); and the domestic markets for capital and labor clear

\[
K = \sum_{j \in J} \sum_{x_j} a_j(x_j) \mu_j(x_j) + B - D,
\]

\[
H = \sum_{j \in J} \sum_{x_j} (1 - l_j) e_j(x_j) \mu_j(x_j),
\]

where \( B = \sum_{j \in J} \sum_{x_j} a_j(x_j) \tilde{\mu}_j(x_j) \) is the total amount of assets left by all the deceased agents;

(d) the labor tax \( \{\tau^l\} \) adjusts, so that government budget constraint defined in Eq. (4) is satisfied;

(f) the aggregate resource constraint is given by \( Y = C + I + G \), where

\[
C = \sum_{j \in J} \sum_{x_j} c_j(x_j) \mu_j(x_j) \quad \text{and} \quad I = \sum_{j \in J} \sum_{x_j} a_{j+1}(x_j) \mu_j(x_j).
\]
3 Calibration

We choose Japan as a benchmark case for quantitative analysis. We calibrate our benchmark model to match the Japanese economy in an artificial steady state in 2010. In this section we describe the parameterization and calibration of the benchmark model.

We source the values of model parameters from the previous literature and the macro data on government tax and fiscal policy, and population dynamics. We calibrate some structural parameters and fiscal policy variables to replicate life-cycle profiles of labor supply and asset holdings and targeted macroeconomic aggregates in the base year. The values of key parameters of the benchmark model in Table 1.

3.1 Demographics

The model economy is populated by 16 overlapping generations of households in each discrete time period $t$, with each period lasting for 5 years. Households become economically active at age 20, and the $j = 1$ generation correspond to ages 20 to 24, $j = 2$ to 25 to 29 and so forth, with $J = 16$ – the oldest generation – corresponding to ages 95 to 99. We base on the data from the Japanese National Institute of Population and Social Security Research (IPSS) to construct the demographic structures for Japan. In our benchmark calibration, we use the actual age-distribution in 2010. In addition, we use actual 5 x 5 (age interval × year interval) life tables (for both sexes) from IPSS mortality database to construct the conditional survival probabilities. We use the demographic structure of 2010 to solve the optimization problem and to compute aggregate statistics.

3.2 Preferences

The instantaneous utility from consumption and leisure is given by

$$u(c_j, l_j) = \frac{1}{1-\sigma} \left\{ [c_j]^{1-\sigma} \left[ 1 - \kappa(1-\sigma)(1-l_j)^{1+\frac{1}{\sigma}} \right]^\sigma - 1 \right\},$$

where $\kappa$ represents the dis-utility from work and $1/\sigma$ is the inter-temporal elasticity of substitution. This functional form of Constant Frisch Elasticity (CFE) preferences is specified in Trabandt and Uhlig (2011).

Liboshi, Nishiyama and Watanabe (2006) estimated a mean of $\sigma$ at 2.041 in their dynamic stochastic general equilibrium model of the Japanese economy. Kuroda and Yamamoto (2007) estimated the Frisch elasticity ($\phi$) for Japan on the extensive and intensive margins combined at between 0.7 and 1.0 for both sexes over the 1990 period. These estimates are in the value range used in the literature (e.g., see Hall (2009) and
Trabandt and Uhlig (2011)).

We calibrate the value of parameter \( \kappa \) so that the average hours worked per working age person (\( \bar{n} \)) as a fraction of total time is 0.30. The value of parameter \( \beta \) is set to match capital output ratio \( \left( \frac{K}{Y} \right) \) of 3.

### 3.3 Endowments

The labor productivity, \( e_j \), of an age \( j \) household in the model economy evolves over the life cycle according to \( \ln e_j = \ln \tau_j + \ln \tilde{z}_j \) for \( j = 21, ..., 65 \). The evolution of labor productivity has two components: deterministic one \( \tau_j \) and stochastic one \( \tilde{z}_j \).

![Age-dependent labor productivities](image.png)

**Figure 1: Age-specific Labour Productivities in the Benchmark Model**

The deterministic component \( \{\tau_j\} \) is based on the estimates of the age-profile for Japan in Braun, Ikeda and Joines (2009). That age-profile of time-invariant labor productivity is constructed from Japanese data on employment, wages, and weekly hours from 1990 to 2000. The age-specific labor productivities are values over five-year age groups, starting from age 20−24, to ages 65 and over. We set \( \tau_j = 0 \) for \( j = J^w + 1, ..., J \). The results plotted in Figure 1. The life-cycle profile is hump-shaped, reflecting the productivity gains as households gain experience, before declining at the end of the working life.

The idiosyncratic component \( \tilde{z}_j \) of labor productivity is specified as a first-order autoregressive process in log as

\[
\ln \tilde{z}_j = \rho \ln \tilde{z}_{j-1} + \epsilon_j,
\]
where the temporary shock, \( \epsilon_j \), is normally distributed. We set the persistence parameter \( \rho = 0.97 \) and the variance of the white noise \( \sigma^2 \epsilon = 0.03 \), which lie in the range of estimates in Lise et al. (2014). We approximate this continuous process with a three-state, first-order discrete Markov process.

### 3.4 Technology

We assume the production function has the Cobb-Douglas functional form

\[
Y = AK^\alpha H^{1-\alpha}. \tag{7}
\]

We set the capital share \( \alpha \) at 0.4 and the depreciation \( \delta \) at 0.082, using estimates from Muto, Oda and Sudo (2012). These are also close to the values from Hayashi and Prescott (2002). We set \( A \) grow at a constant rate.

### 3.5 Fiscal policy

**Social security.** The replacement rate \( \Psi \) is set at 0.33 to match the size of the social security system as a proportion of output in 2010. The fraction of social security payment contributed by the general government budget is determined by

\[
\theta^{ss} = 1 - \frac{\text{total pension contribution}}{\text{total pension transfers}} \tag{8}
\]

As in Muto, Oda and Sudo (2012) we choose \( \theta^{ss} = 0.413 \). The social security tax rate is adjusted to keep the social security fund in balance. In our benchmark model, the equilibrium social security tax rate is around 10%.

**Other government expenditures and debt.** Government expenditures including the spending for healthcare and long-term care are 20% of aggregate output according to the National Accounts of Japan (SNA) in 2010. We set \( G \) at 20% to match that size.

The net government debt to GDP ratio \( \frac{D}{Y} \) is calculated using the net debt to GDP ratio from the IMF. The government debt to GDP is set at 110%. The average number of years to maturity of outstanding government bonds is about 7 years and the average real interest rate on 7 year government bond is 1.0% in 2000 – 2010. The interest rates on government bonds \( (r^d) \) is set at 0.01, matching the implied yield on 10-year Japanese government bonds. This is set exogenously as they are significantly lower compared to the endogenous interest rate on private capital \( (r) \).

**Taxes.** The consumption tax is set at 5% in the initial steady state. Capital income tax is set at 30%, which is in the range of estimates of effective tax rates on capital
income, for example, in Braun and Joines (2015). The labor income tax rate that clears the government budget constraint is 26% in the benchmark calibration. The combining rate of the labor income tax rate and the social security tax rate is around 36%, which is very close to the labor tax rate in Kitao (2015).

3.6 Benchmark model performance

In this section, we present the calibration results of the benchmark model based on 2010 demographics, and discuss how well the model matches the data in describing the Japanese economy.

The results for household assets, wages and labor supply are summarized in Figure 2. The first panel depicts the life-cycle asset holdings by households, relative to the asset holdings at age 50. The hump-shape is consistent the life-cycle hypothesis, where households are expected to build up their assets over their working life and run down the savings over retirement. We discipline the household sector to data from the 2009 National Survey of Family Income and Expenditure (NSFIE). Our model is able to replicate the accumulation of asset holdings over the households’ working life, with households maximising their asset holdings at age 60.

The second panel presents the average number of labor hours supplied by households of different ages per week. We compare this to data from the 2010 Labor Force Survey, using the average hours of hours worked by employed persons of different age groups across all industries. As we do not distinguish between employed and unemployed households in our model, the hours worked are adjusted by the employment rate within
each particular age group. Our model is able to generate the lifecycle shape of labor supply, with households supplying less hours at the younger ages, supplying more during the middle ages, and supplying less again as they approach retirement age.

The last panel presents the labor earning profile of different age groups, relative to the age 50 group. Our model can produce the life cycle pattern of average labor income for workers from the 2009 NSFIE. Specifically, the model can generate the hump-shape of the labor incomes. However, whereas labor income is maximised for households at age 50 in the data, it is maximised at age 45 in the model. Furthermore, the drop off in labor income is both earlier and steeper in the model. This is especially apparent for ages 65 and over where we assumed exogenous retirement and the labor income is zero. This is in contrast to the data, where the surveyed households by definition continue to work in those periods and earn wages income.

4 Quantitative analysis

In this section, we first discuss how we quantify fiscal limit and measure fiscal space. We next analyze how evolution of demographic structures over time affects the size and shape of fiscal space.

4.1 Fiscal space

Laffer curve. In a fundamental-based model, higher distorting taxes diminish incentives to work and save, that subsequently shrinks tax bases. Indeed, there is a trade off that government has to face when raising tax revenues. In such setting, there are some tax rates that maximize revenues. The Laffer curve reasoning imposes a natural limit on the government’s tax revenue generating capacity. The maximum tax revenue that can be collected is when the economy is placed at the peak of Laffer curve.

Since there are three taxes in the benchmark model we can construct three Laffer curves. To do so, we vary one tax rate of interest at a time, while holding the other two tax rates constant at the benchmark level. Specifically, we can construct the Laffer curves for the labor and capital income taxes, and the consumption tax. In our general equilibrium model, there are several options to keep the government budget in balance when raising taxes: decreasing general government purchases ($G$) and government debt ($D$), or increasing transfers ($Tr$) and public pension ($p$).

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1. Average hours worked per week per household (per generation) = \( \frac{\text{Total number employed}}{\text{Total population}} \times \frac{\text{Average hours worked per week for each employed person}}{14} \times 7 \), where 14 is the number of total free hours we assumed households have per day from the calibration section.
In our baseline analysis, we assume that the government adjusts general government purchases \((G)\) to keep its budget in balance. We call it the \(g\)-Laffer curve.\(^2\) Figure 3 presents the \(g\)-Laffer curves. To ease our comparison we normalize the tax revenue in the benchmark model to 100.

![Graph of three different Laffer curves: Labour, Capital, and Consumption](image)

**Figure 3:** Benchmark \(g\)-Laffer Curves, Fiscal Limit and Fiscal Space. Note that, the total tax revenue in the benchmark model is normalized to 100.

The Laffer curves for labor and capital income taxes have the classic single-peaked concave shape, while the Laffer curve for consumption tax does not have a peak and is an increasing function of the tax rate. The differences in the shape of three Laffer curves are mainly driven by interactions between the tax rates and tax bases.\(^3\) The graph for the decomposition can be found in Figure 4. As the labor income tax rate increases, the labor income tax base decreases almost linearly as the hours worked decrease. The multiplication of two linear factors with opposing signs gives rise to the inverse-U shape. For the capital income taxation, the tax base is mostly flat at the lower tax rates, before decreasing abruptly at the higher rates. This gives rise to the asymmetrical shape. Interestingly, the shape of the consumption Laffer curve arises as both labor and capital tax bases are unaffected as the consumption tax rate increases.

Overall, the shape of Laffer curves resulted from our heterogeneous agents model are quite similar to ones resulted from representative agent models (e.g., see Trabandt and

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\(^2\)We will consider the other types of Laffer curve in section 5.

\(^3\)The labour tax, capital and consumption tax bases are given by \((1 - \tau^s(\tau)) \times w(\tau) \times H(\tau), r(\tau) \times K(\tau), \text{ and } C(\tau)\), respectively, where \(\tau = \{\tau^l, \tau^c, \tau^s\}\).
Uhlig (2011)). However, there are some small differences. The capital income tax Laffer curve is not flat to the left of the peak. Whereas the labor income tax revenue decreases in the representative agent model, it is flatter in our overlapping generations model as households supply is more elastic to decreases in wage rates in responding to an increase in the capital income tax rate.

![Graphs showing tax revenue decomposition and tax bases for benchmark g-Laffer Curve](image)

Figure 4: Tax Revenue Decomposition and Tax Bases for Benchmark g-Laffer Curve

The benchmark economy lies to the left of the peaks of the Laffer curves for labor and capital income taxes. The maximum tax revenues are reached when setting labor income tax rate ($\tau_l$) at 63 percent and capital income tax rate ($\tau_k$) at 82 percent. On other hand, the Laffer curve for consumption tax does not have a peak, we note that by setting $\tau_c = 0.95$, the government can raise tax revenue up to 206 percent of the current revenue.

Qualitatively, our results from a heterogenous agents model differ from the previous studies. The results based on a representative model (e.g., see Park (2012)) indicate that tax revenues can be increased by raising the labor income tax rates while lowering the capital income tax rates in the US and other G7 countries. Differently, our results based on a heterogenous agents model suggest that both taxes should be increased to maximise revenue. This discrepancy likely arises from the different shape of the capital income tax Laffer curve. In a representative agent model, the capital income tax Laffer curve is mostly flat to the left of the peak. Therefore, lowering capital income tax would have minimal impact on tax revenue; meanwhile, it significantly reduces tax distortion and expands the tax bases for labor and consumption in the economy. This in return enables
the government to raise more revenue at a lower capital income tax rate. By contrast, lowering the capital income tax rate in our model would decrease revenue more, as the slope is steeper to the left of the peak.

**Fiscal space.** We base on the Laffer curves to formulate our concept of fiscal space in terms of budgetary room between the current (benchmark) tax revenue level and the maximum tax revenue level. Our fiscal space consists of fiscal limit and the current tax revenue. Intuitively, it is a relative measure of how flexible government can be in terms of generating more tax revenue. Holding government spending constant, fiscal space describes the government’s fiscal flexibility to raise revenue to meet its spending commitments without compromising fiscal sustainability or the level of debt.

In our benchmark model, the current tax revenue ($\overline{Tax}$) is computed directly from this equation:

$$\overline{Tax} = \overline{\tau} w H + \overline{\tau} k K + \overline{\tau} c C,$$

where $\overline{\tau}_l$, $\overline{\tau}_k$, and $\overline{\tau}_c$ are the labor, capital, and consumption tax rates, respectively, $w$ is the wage rate, $r$ is the interest rate, and $H$, $K$ and $C$ are aggregate human capital, physical capital, and consumption, respectively.

Let $Tax^{\text{max}}_{\tau}$ denote the maximum tax revenue for the labor and capital income taxes and consumption tax with $\tau = \{\overline{\tau}_l, \overline{\tau}_k, \overline{\tau}_c\}$, respectively. The fiscal space ($FS_{\tau}$) is defined by a gap between the maximum tax revenue ($Tax^{\text{max}}_{\tau}$) and the benchmark tax revenue ($\overline{Tax}$) according to $FS_{\tau} = Tax^{\text{max}}_{\tau} - \overline{Tax}$.

Figure 3 depicts the fiscal spaces for labor and capital income taxes and consumption tax. We find that the fiscal space for the labor income tax is 43 percent of the current level of tax revenue. The government can raise revenue by increasing the labor income tax rate up to 63 percent and collect a maximum revenue of 143 percent of the current revenue. The government can also generate more tax revenue through raising the capital income tax rate. However, the fiscal space for the capital income tax is much smaller. The additional revenue is only 11.8 percent of the current revenue, achieved when increasing the capital income tax rate to $\tau_k = 0.82$.

**Laffer hill and fiscal space.** We now consider a new measure of fiscal space using Laffer hill for the joint labor-capital income taxation. We construct Laffer hill by jointly varying the labor and capital income tax rates, while keeping the consumption tax rate unchanged at the benchmark level. The peak of Laffer hill gives us all a combination of the labor income tax rate ($\tau_l$) and the capital income tax rate ($\tau_k$) that results in a maximum tax revenue. We quantify the Laffer hill for Japan in 2010 and present it in Figure 5.

The contour lines present the different tax rate combinations that raise the same
level of revenue, relative to the benchmark revenue level that is set at 100. The dotted lines mark the benchmark taxation rates. As the peak of the hill lies in the north-east quadrant, the government can maximise the level of tax revenue by increasing both the capital and labor tax rates. We find that the peak of Laffer hill is achieved when setting $\tau^l = 0.63$ and $\tau^k = 0.55$.

We compute a two-dimensional fiscal space for Japan using the Laffer hill approach. The fiscal space for joint capital and labor income taxation is measured by a gap between the maximum tax revenue (peak of Laffer hill) and the current tax revenue, $FS(\tau^l, \tau^k) = Tax_{\text{max}}(\tau^l, \tau^k) - Tax$.

As shown in Figure 5, the Laffer hill fiscal space is about 45 percent of the current revenue in 2010. That is, when the government is allowed to change both labor and capital income taxes it can raise up to 45 percent of the current revenue. Compared to the Laffer curve fiscal space, the Laffer hill fiscal space is slightly larger by around 2 percent.
In this experiment, we keep the consumption tax rate unchanged at the level in the benchmark economy. Figures 16 and 17 in Appendix demonstrate how the Laffer hill fiscal space can be expanded further when the government is allowed to raise the consumption tax rate to 10 percent and 20 percent.

4.2 Demographic shift and fiscal space

In this section, we quantify the effects of demographic shift on Japan’s fiscal space. We do so by varying the demographic structure, while keeping other non-demographic variables constant at their benchmark levels.

**Demographics.** We use two alternative demographic structures for Japan: younger one in 1980 and older one in 2040. We base on the survival and fertility rates from demographic data from IPSS to construct the corresponding demographic structures. Panels 1 and 2 of Figure 6 present the age distributions and conditional survival probabilities that we extract from the data and use in our experiments. Panel 3 of Figure 6 demonstrates how the old-age dependency ratio in the model fits the data.

![Figure 6: Demographic Shift in Japan: Model and Data](image)

**Behavioral responses and aggregate effects.** We first examine the impacts of the demographic shift on the household and macroeconomic variables. The aggregate results are summarised in Table 2. The 2010 results from the model are given, while most of the 1980 and 2040 figures are reported as percentage deviations from their 2010 (per capita) levels, with the exception of the tax rate changes, which are expressed in...
percentage point changes instead.

Household behavior changes as a result of the improvement in survival probabilities over the 1980 to 2040 period. Despite the decrease in overall levels, savings and asset holdings remain high for longer and dissaving in retirement occurs at a slower rate than in 1980. In 2010, household savings surpass the 1980 levels at age 80, while in 2040, household savings surpass the 1980 levels at age 85. As survival probability improvements continue to occur at the higher ages between 2010 and 2040, dissaving for households aged 85 and above occurs even slower, and catches up to 2010 levels by age 95.

Household labor supply is affected by the decrease in disposable wages. Households increase their labor supply, especially at the later working years. From 1980 to 2010, the labor supply falls during the middle periods from ages 30 to 39, before rising for the remainder working periods from ages 40 to 64. This can be explained by the interaction of the income and substitution effects triggered by the fall in disposable wages as a result of ageing. In the younger periods when the decrease in effective wages is relatively larger once we account for the lower age-specific labor productivity, the substitution effect dominates over the income effect, and households consequently work less. In the older periods when the decrease in effective wages is relatively smaller due to the higher age-specific productivity, the income effect dominates and households are willing to work more. From 2010 to 2040, as the disposable wages fall further, the substitution effect dominates, and the labor supply falls for all ages. Overall, we find that households aged of 40 to 64 still supply more labor than in 1980.

The labor tax base was 17.67 percent higher in 1980, and will be 18.48 percent lower in 2040, compared to the benchmark level in 2010. This arises from the decreases in the labor supply ($N_t$) and effective labor supply ($H_t$). As labor supply is mostly unchanged on the individual level, this decrease is mostly driven by the smaller share of working age cohorts as the dependency ratio rises. The per capita labor supply was 19.02 percent higher in 1980, and will be 15.24 percent lower in 2040, compared to 2010 levels. The effective labor supply changes to a greater extent, arising from demographic and behavioral changes. The age-specific labor productivity is asymmetrical, peaking at age 50 with the decrease over ages 50 to 64 sharper than the rise from ages 20 to 50. While ageing leads to more labor supply at age 50, it also increases labor supply at the beginning and end of working ages (20 to 25, and 50 to 65) where households are relatively less productive, compared ages 30 to 40. As the cohort share is concentrated between ages 20 to 50 in 1980, it captures the additional effective labor supply of the households aged between 30 to 40. In 2040, as cohort shifts to the 50 to 80 age range, it captures the lower effective supply of the 50 to 65 age range.

The capital tax base shrinks as a result of demographic shift, with it higher by
20.27 percent in 1980, and lower by 15.66 percent in 2040. This arises from the fall in capital stock resultant from both behavioral and level changes. This is consistent with the life-cycle hypothesis, with the proportion of working age savers decreasing, and retired age dissavers increasing as a result of ageing. This is compounded by the decrease in disposable wages as the population ages, and the fall of household savings. The asymmetrical decrease from 1980 to 2010 and 2010 to 2040 arises from the different levels of asset held over the life cycle. From 1980 to 2010, the effects of ageing was masked as in 2010, the most populous cohorts were concentrated in the high-assets age range between ages 50 and 70. However, with the shift to the 2040 demographic structure, the 60 to 80 age range will be more populous, and as they lie to the right of the asset holdings peak, total capital stock within the economy will fall to a greater extent. While the labor supply decreases proportionally less than the labor tax base, the capital tax base decreases more than the capital stock. This arises from the simultaneous fall in interest rates (and rise in wage rates) driven by capital deepening.

Consumption is negatively affected by ageing, with the consumption tax base 20.27 percent higher in 1980, and 15.66 percent lower in 2040. The asymmetrical decrease results from the life-cycle behavior of households, with the 1980 to 2010 effects softened by the increase in households with high disposable incomes. As the proportion of low income retired households continue to increase, the consumption tax base will decrease at a faster rate.

Despite a decrease in the pension benefits received by each retired household in 2040, the effects of the demographic shift will dominate, and the social security system will expand by 29.48 percent as a result of ageing. To support the growing social security system, both \( \tau^{ss} \) and \( \tau^l \) have to be increased for debt and government spending levels to remain constant. At the aggregate level, the ageing population has a negative effect on output, and GDP per capita will be 13.80 percent lower in 2040.

Next, we examine how the demographic shift affects Japan’s fiscal limit and fiscal space.

**Laffer curve fiscal space.** We construct the g-Laffer curves for the labor and capital income taxes, and consumption tax with alternative demographic structures in 1980 and 2040. Panels 1 to 3 of Figures 7 present the corresponding Laffer curves.

Japan’s fiscal limit measured by the peak of the Laffer curves decreases significantly since 1980. In absolute terms, the Laffer curve for the labor income tax is affected most by the population ageing in Japan. The maximum tax revenue raised by the labor income tax decreases from 156 percent of the baseline tax revenue in 1980 to 143 percent and 128 percent in 2010 and 2040, respectively. There is a smaller decrease in the maximum revenue raised by the capital income tax. The government could raise at most
Figure 7: Demographic Shift and Fiscal Spaces

127 percent of the baseline tax revenue at $\tau^k = 0.83$ in 1980. However, that maximum revenue decreases to 106 percent at $\tau^k = 0.86$ in 2040. The tax revenue raised by the consumption tax at the rate of 95 percent is also decreased from 210 percent in 1980 to 192 percent in 2040. Thus, all three peaks of Laffer curves shift down significantly due to the decrease in the tax bases caused by ageing population.

Next, we construct the fiscal spaces for the three taxes and calculate the percentage changes in the fiscal spaces caused by the changes in the underlying demographic structure. To ease comparison, we normalize the fiscal space in 2010 to 100. Panel 4 of Figure 7 presents the contraction in Japan’s fiscal space since 1980 as old-age dependency ratio increases.

When comparing the relative changes in the fiscal space, we find that the fiscal space for the capital income tax is the most sensitive to increase in old-age dependency ratio, with the fiscal space contracted by 64 percent by 2040. This is followed closely by the labor income tax, with the fiscal space lower by 34 percent. The fiscal space for the consumption tax is the least sensitive since it is only contracted by 13 percent by 2040. The sensitivity of the fiscal space for the capital income tax is again driven by the compounded fall of both the capital stock and interest rate.

Analyzing the slopes of changes in the fiscal spaces, we can quantify the relative movements of the fiscal space over the next 30 year period, compared to the last 30 year period. While the fiscal spaces for the labor and capital income taxation are linear, the consumption fiscal space is kinked and will decrease at a faster rate over the next 30
Lastly, we note that while the fiscal space will shrink as a result of the population ageing, it will remain positive for all three taxes. This implies that the Japanese government still has some room to raise tax revenue by increasing either one of the labor, capital, and consumption tax rates.

**Laffer hill fiscal space.** We now analyze the impact of the demographic shift on the g-Laffer hill. Panels 1 – 3 of Figure 8 depict the g-Laffer hills for 1980, 2010 and 2040, respectively.

![Laffer hill graphs](image)

**Figure 8: Demographic Shift and Fiscal Space: Labor-Capital Laffer Hill**

The contour lines present all combinations of capital and labor income tax rates that result in a similar level of tax revenue. We find that the contour lines shift north-east as a result of population ageing. This indicates that higher labor and capital tax rates are needed to generate the same level of income as before. For instance, at the benchmark labor and capital tax rates, 116 percent of the current revenue could be raised in 1980, and only 98 percent will be raised in 2040. The maximum tax revenue decreases from 160 percent in 1980 to 145 percent in 2010 and to 129 percent in 2040.

We compute the fiscal space based on the g-Laffer hill approach and present it in the last panel of Figure 8. There is a significant contraction in revenue generating capacity of the government since 1980. Compared to the 2010 level, the Laffer hill fiscal space was 34 percent larger in 1980, but will be 36 percent smaller in 2040. The slope of the dependency ratio-fiscal space curve implies how elastic the fiscal space is as the population is ageing. It appears that there is an almost linear relationship between the
dependency ratio and the change in fiscal space. Approximately, a 10 percentage point increase in the dependency ratio is associated with an 12 percentage point decrease in the g-Laffer hill fiscal space.

4.3 Role of each demographic factor

In our analysis so far, we have assumed both the age-specific survival probabilities and fertility rate change according to either their actual or projected paths. This allowed us to quantify the total effect of the two demographic factors on the fiscal space. In order to isolate the quantitative importance of each demographic factor, we perform the following decomposition exercise.

We keep the 2010 economy based on the actual time series of survival probabilities and fertility rates up until 2010 as the baseline scenario. We consider four counter-scenarios: (i) a “high fertility” scenario, where the survival probabilities are kept at the benchmark values, and the fertility rate revert to their time sequence up until 1980; (ii) a “low fertility” scenario with 2010 survival probabilities and fertility rates at their 2040 levels; (iii) a “high mortality” scenario with benchmark fertility rates and 1980 survival probabilities; and (iv) a “low mortality” scenario with benchmark fertility rates and 2040 survival probabilities.

The effects of the alternative demographic assumptions on the population composition are plotted in Figure 9. For ease of comparison, we have included the benchmark 2010 as the baseline.

Both high fertility and high mortality rates increase the proportion of young households relative to the baseline, while low fertility and mortality rates decrease the proportion. Over the 1980 to 2040 period, the mortality rate decrease is the single biggest driver of the demographic composition, with the dependency ratio at 0.33 in the high mortality scenario, compared to 0.51 in the baseline. This is also followed closely by a decrease in fertility rate. The dependency ratio is 0.37 in the high fertility scenario. As the demographic changes slow down over the 2010 to 2040 period, so does the relative change in cohort sizes. This is especially true for the mortality rate (see Figure 9), and the dependency ratio will only rise to 0.60 in the low mortality scenario. This compares to 0.64 for the low fertility scenario.

To isolate the effects of changes in the fertility rate only on the gross fiscal space, we compare the g-Laffer curve under the baseline economy with the high fertility and low fertility cases. Similarly, we compare the g-Laffer curve under the baseline economy with the high mortality and low mortality cases to isolate the effects of the changes in survival probabilities. The results are summarised in Figure 10 (a) to (c), and (d) to
Figure 9: Demographic Composition under Different Fertility and Mortality Assumptions. Note that, the low mortality case and low fertility case uses 2040 survival probabilities and fertility rates from the IPSS’s medium-fertility medium-mortality population projections.

\((f)\) respectively.

All three Laffer curves shift down as either the fertility rate or mortality rate decreases. This is consistent with our intuition. Using the size of the effect on the fiscal space as a measure of the relative sizes of the demographic trends, we can compare the relative sizes of the fertility trends with the mortality trends. Over the 1980 to 2010 period, the size effect of the isolate fertility and mortality changes on the gross fiscal spaces are similar. In the high fertility counter-scenario where the fertility rates decrease does not occur, the gross labor fiscal space would be 95.33 percent of current revenue levels, compared to 94.58 percent for the high mortality counter-scenario where the survival probability improvements do not occur. Interestingly, we find that the change in fertility has a bigger effect on the fiscal space despite affecting the population composition (through the dependency ratio) to a lesser degree. This is reversed for the capital Laffer curve, with the fiscal space under the high mortality scenario slightly higher than the high fertility scenario, at 64.38 and 63.73 percent respectively.

Over the 2010 to 2040 period, the changes in both the fertility and mortality rates will slow down, as measured by a smaller shift in the Laffer curves between the high fertility and the baseline scenarios, compared to the baseline and low fertility scenario. The fiscal space for the labor income tax under the high fertility scenario would have
been 23 percent larger, compared to 20 percent lower for the low fertility scenario when comparing to the baseline fiscal space. Similarly, the fiscal spaces for the capital income and consumption taxes under the low fertility scenario will decrease less, by 21 percent and 5.5 percent, respectively. This compares to an increase of 23 percent and 6.5 percent under the high fertility scenario.

Further, the relative scale of the fertility and mortality changes were similar over the 1980 to 2010 period. However, this is not the case for the 2010 to 2040 period. Improvements in mortality rate contribute less to the fiscal space contraction experienced by the economy in 2040. Under the low mortality case, the fiscal spaces for the labor, capital and consumption taxes are 9.7 percent, 11.9 percent, and 2.8 percent lower than the baseline levels, respectively. As fertility rates are projected to remain depressed, we expect that future decreases in the fiscal space will be increasingly driven by the fertility rate.

It is expected that a larger change in the scale of the demographic factors would lead to a corresponding larger movement in the Laffer curves. In order to control for the size effect, we quantify the relative importance of the two demographic factors on a per unit basis, with the dependency ratio used as a general measure of the level of ageing. We plot the fiscal space of the various scenarios against the dependency ratio that they produce, with the results summarised in Figure 11. The “fertility only” case refers to the low and high fertility scenarios, the “mortality only” case refers to the low and high fertility scenarios, and the “total” case refers to the actual population dynamics we
originally used, with 1980 survival probabilities matched with 1980 fertility rates, and 2040 survival probabilities matched with 2040 fertility rates.

Figure 11: Demographic Factors and Fiscal Space

There are some minor qualitative differences on how the mortality and fertility rates affect the Laffer curves on a per unit basis. The Laffer curve for the labor income tax is relative more sensitive to changes in the fertility rate. This is depicted in the first panel by the position of the “fertility only” points relative to the benchmark 2010 point and the trend line. For the high fertility point to the left of the benchmark, the point lies above the trend line, indicating that for a given decrease in the dependency ratio driven purely by a fertility rate increase, the fiscal space would increases more than the trend suggests. Similarly for the low fertility point to the right of the benchmark, the point lies below the trend line, indicating that for a given increase in the dependency ratio driven by purely by a fertility rate decrease, the fiscal space decreases more than the trend suggests. In comparison, the low mortality point lies above the trend line, while the high mortality point lies below it, indicating a lesser impact on the fiscal space than suggested by the movements in the dependency ratio. Looking at the fiscal spaces for the capital income tax and consumption tax, the relative importance of fertility on a per unit basis also holds true, abiet to a lesser extent.
5  Sensitivity analysis and extension

In this section, we conduct sensitivity analysis on the results by altering the specifications and parameter values for the preferences.

5.1 Alternative Laffer curves

In our model, the government has other options to balance its budget when constructing Laffer curves. In this section, we consider two alternative options: transfers to households and public debt reduction.

\textbf{tr-Laffer curve.} For the tr-Laffer curve, we assume that additional revenue is redistributed evenly to all households in the form of a lump-sum transfer ($tr$). For a certain combination of tax rates $\bar{\tau}_c, \bar{\tau}_k, \bar{\tau}_l$, the transfer would be characterised by

\begin{equation}
tr = \frac{\bar{\tau}_c C + \bar{\tau}_k rK + \bar{\tau}_l(1 - \tau^{ss})wH - \bar{G} - r^d \bar{D} - \bar{\theta}^{ss} SS}{\sum_j \sum_{x_j} \mu_j(x_j)}
\end{equation}

\textbf{d-Laffer curve.} For the d-Laffer curve, we relax the assumption that debt is fixed at $\bar{D}$ and the government uses the additional tax revenue to repay outstanding government debt. The new maximum sustainable level of debt in period $t$ is given by

\begin{equation}
D = \frac{\bar{\tau}_c C + \bar{\tau}_k rK + \bar{\tau}_l(1 - \tau^{ss})wH - \bar{G} - \bar{\theta}^{ss} SS}{r^d}
\end{equation}

Notice that, the d-Laffer and g-Laffer curves are equivalent in our model as the interest rate on government bonds ($r^d$) is exogenous, and either the debt or government spending is fixed at the benchmark levels. It becomes a matter of balancing the government’s budget condition by adjusting either the sustainable debt level or government spending, and revenue levels are unaffected.

We present tr-Laffer curves for the labor, capital and consumption taxes in Figure 12. We also present the d-Laffer/g-Laffer for comparison.

Compared to the g-Laffer curves, the tr-Laffer curves have lower peaks in both the labor and capital income tax curves, while a higher peak for the consumption tax Laffer curve. The difference arises from the additional revenue that is fed back into the economy through transfers. The transfers increase household incomes and result in positive income effects, which subsequently induces households to consume more leisure and consumption, while saving less. The labor and capital tax bases are contracted, and the entire curve shifts down as a result. Conversely, the increase in the consumption base expands consumption tax revenue under the tr-Laffer specification. The effects become
5.2 Alternative preferences

We examine whether our results are robust to different functional forms of preferences. Following Trabandt and Uhlig (2011), we consider two alternative functional forms. In the first case, we set the inverse of the IES ($\sigma$) to 1 as this reflects the recent shift in the consensus view and estimation. For instance, Sugo and Ueda (2008) estimates the mean of the IES $1/\sigma = 1.25$ for Japan, in contrast to the higher value found by Liboshi, Nishiyama and Watanabe (2006). We set the Frisch labor supply elasticity $\phi = 3$ and labor supply is more elastic to changes in wage rate. With these alternative CFE parameters, the preferences has a form of

$$u(c_j, l_j) = \log(c_j) - \kappa (1 - l_j)^{1+1/\phi}$$  \hspace{1cm} (11)

In the second case, we use Cobb-Douglas (C-D) preferences as it is one of the most commonly used functional forms and an appropriate benchmark. In particular, we use the following preferences:

$$u_{c_j, l_j} = \rho \log(c_j) + (1 - \rho) \log(l_j)$$  \hspace{1cm} (12)

where $\rho$ is the leisure-consumption preference parameter. Under Cobb-Douglas prefer-
ences, the Frisch elasticity is given by $1/(n_j) - 1$, where $n_j = 1 - l_j$ and is no longer constant. Instead, the elasticity decreases as the labor supply increases (leisure decreases).

We calibrate and compare the benchmark preferences specified above with two alternative specifications to test the robustness of our results. The budget constraints and equilibrium conditions of the model hold as before. The values of deep parameters under both the original and the alternative specifications are given in Table 3. All other parameters for demographics, endowments, technology, and fiscal policy are kept the same as in Table 1.

The common parameters for the alternative CFE and Cobb-Douglas cases are $\beta = 0.962$ and $\psi = 0.33$. This similarity stems from the separability of the consumption and leisure in the utility function, and the common $\log(c_j)$ term from $\sigma = 1$. When we compare the alternative CFE case with the benchmark CFE case, the time discount factor is lower, while the disutility from work is higher, with $\kappa = 3.53$.

We compute the g-Laffer curves in under the two alternative preference specifications. The results are plotted below in Figure 13. The benchmark revenue level under the CFE preferences is normalized to 100.

![Laffer Curves](image)

Figure 13: Comparing the 2010 g-Laffer curves under alternative preferences

We compare the results of the alternative CFE and Cobb-Douglas preferences. We find that the Laffer curves for all three taxes are quite similar. However, there are differences at the high tax rates for labor and capital incomes. In the labor Laffer curve, the slope of the Laffer curve under Cobb-Douglas preferences is greater than in both CFE preferences.
preferences at the higher tax rates. This results from the changing Frisch elasticity, so that households decrease their labor supply less when increasing tax rates at the higher end. Consequently, the total tax revenue falls slower in the Cobb-Douglas preference case.

For the Laffer curve for the labor income tax, the maximum revenue is higher with the alternative CFE and Cobb-Douglas preferences, compared to the benchmark CFE preference case. This is qualitatively different to the result produced in Trabandt and Uhlig (2011) representative-agent framework, where the labor Laffer curves under the benchmark CFE preferences had a higher peak for labor. This results from the difference in \( \beta \) value needed to calibrate the economy in the OLG framework. As \( \beta \) increases, the competitive interest rate decreases. This means that the tax revenue collected from capital is lower at every level, leading to a lower peak for the benchmark CFE case.

In the capital Laffer curve case, the benchmark CFE raises less at lower capital tax rates, compared to the Cobb-Douglas case. However, it peaks at a higher tax rate. In contrast, the capital curves under the benchmark and alternative preferences in the representative-agent model are almost identical. The difference arises again from the calibrated \( \beta \) value, with a higher \( \beta \) implying households are patient and willing save more even as the return on capital decreases. This leads to a slower decrease in the tax base and the the total taxation revenue level is higher. The consumption Laffer curves for all three specifications are very similar, especially between the two CFE specifications. The Cobb-Douglas preferences produces the highest taxation revenue at a higher tax rate.

5.3 Net fiscal space

In our previous analysis, we demonstrate how population ageing undermines the revenue generating capacity. However, that gross measure of fiscal space overestimate the true degree to which fiscal manoeuvrability can be compromised in the context of ageing. In this section, we take into account the fiscal cost of the social security program caused by population ageing.\(^5\)

In particular, we consider a new measure of fiscal space in which we take the gross fiscal space as measured before and subtract the government contributions to the social security system. We first compute the pb-Laffer hill for the joint capital and capital

\(^5\)In this paper, we abstract from the fiscal costs of other age-related government spending programs such health care and long-term care.
Figure 14: Demographic Shift and Net Fiscal Space

We then compute net fiscal space according to this formula

\[
NFS\{\tau_l, \tau_k\} = \text{Tax}_{\text{max}}\{\tau_l, \tau_k\} - \overline{\text{Tax}} - \theta_{ss} \sum_{j=J_w+1}^{J} P_{j,t},
\]

where \(\text{Tax}_{\text{max}}\{\tau_l, \tau_k\}\) is the maximum tax revenue when allowing both labor and capital income taxes to vary, \(\overline{\text{Tax}}\) is the current tax revenue in 2010, \(\theta_{ss}\) and \(\theta_{ss}\) are the level of pension benefits and the policy variable that determines the government contribution to the social security program in 2010. In our framework, the net fiscal space is interpreted as the “uncommitted” tax revenue.\(^6\)

Figure 14 depicts \(p_b\)-Laffer hills for 1980, 2010 and 2040, respectively. Notice that, the total tax revenue minus the government contribution to the social security system in 2010 is the benchmark (net) tax revenue in this experiment.

As seen in the first three panels of Figure 14, the Laffer hills shift down as Japan moves toward an ageing society over time. We find that accounting for the expenditure side effect leads to a further downward shift of the peaks of the \(p_b\)-Laffer hills, compared to the \(g\)-Laffer counterparts. This arises due to two reasons: as in the 1980 pre-ageing economy, there is a smaller proportion of retired households; and the government’s contribution to the social security system is relatively smaller than the benchmark levels because the

\(^6\)We also compute the \(p_b\)-Laffer curves for the labor and capital income taxes and consumption tax. These results are available upon request.
The contribution rate is fixed at the 2010 level, which is higher than was needed in the 1980 economy to maintain budget balances. As a direct consequence, the net fiscal space is relatively larger than the gross fiscal space in 1980.

The net fiscal space will be smaller in 2040 as the government has to contribute more to the social security system to meet the pension payment commitments to a larger retiring population. More importantly, if the government keeps the same level of pension benefits committed in 2010, the net fiscal space for Japan will be negative in 2040. That is, the Japanese government would not be able to raise enough tax revenue to finance all its current spending commitments in 2040.

Thus, we conclude that if the Japanese government budget is fiscally unsustainable. Structural reforms will be required to keep Japan away from fiscal insolvency by 2040.

5.4 Cross-country comparison

In this section we examine the Laffer curves and fiscal space for the U.S. We choose the US because the differences in the age structure of the tax and transfer system between the U.S. and Japan are substantial. We demonstrate that timing of demographic shift are the key drivers of differences in fiscal limit and fiscal spaces across two countries after accounting for cross-country differences in endowments, preferences, technologies and tax-transfer systems.

**Calibration.** We calibrate our benchmark model to match the US economy in an artificial steady state in 2010. We source the values of model parameters from (i) the previous literature for specifying preferences; (ii) the macro data on government tax and fiscal policy, and population dynamics. We calibrate some structural parameters and fiscal policy variables to replicate life-cycle profiles of labor supply and asset holdings and targeted macroeconomic aggregates in the base year. The values of key parameters of the benchmark model in Table 4.

**Demographics.** We again consider two alternative demographic structures: one in 1980 and one in 2040. Figure 18 presents the US age distributions and conditional survival probabilities in our experiments.

**Experiments.** We conduct a similar experiment in which we vary the demographic structure and quantify the effects of demographic shift on the US’s fiscal limit and fiscal space.

Figure 19 presents the labor, capital and consumption g-Laffer curves in the benchmark economy in 2010 and two alternative cases in 1980 and 2040. Note that, the values of non-demographic parameters for the US economy are kept constant at their levels in 2010. To easy our comparison we normalize the benchmark tax revenue level to 100.
The percentage changes in the fiscal spaces for labor, capital and consumption taxes are computed accordingly.

We find that the US benchmark economy lies to the left of the peaks of the Laffer curves for the labor and capital income taxes. This indicates that the US government still has room to raise tax revenue by increasing labor and capital tax rates. The peak of g-Laffer curve for labor income taxation is achieved at the tax rate of 63 percent, $\tau_l = 0.63$. The peak of g-Laffer curve for capital income taxation is achieved when the capital income tax rate is set at 86 percent, $\tau_k = 0.86$. The g-Laffer curve for the consumption tax does not have a peak.

Interestingly, we find that evolution of the fiscal spaces are quite different in the US. The changes in the US demographic structure drives an large expansion in the fiscal space between 1980 and 2010, but causes a sharp contraction between 2010 and 2040. The logic is clear. The changing age structure causes an expansion/contraction in the labor force, capital accumulation and consumption bases over time, which subsequently expands/contracts the tax bases and fiscal spaces. More specifically, as seen in panel 1 of Figure 18 there is a relatively larger fraction of Americans aged 40s and 50s in 2010, compared to that in 1980. Since households in that age groups are on top of their labor productivities, that shift in the US demographic structure causes a significant expansion of the tax bases. The US government stands on a better position to raise tax revenue in 2010, compared to 1980. That is, the main driver of the fiscal space expansion is a shift in the underlying age structure. Conversely, as the US enters the late phrase of population aging the fiscal position is reversed in 2040. Indeed, the aging demographic structure shifts the peaks of Laffer curves down significantly and results in a sharp contraction in the fiscal space in 2040. Precisely, the fiscal spaces for the labor, capital and consumption taxes are decreased by 36%, 41% and 13%, respectively.

6 Conclusion

In this paper we analyze the implications of demographic shift for fiscal sustainability through lens of fiscal space. We quantify fiscal space using a stochastic dynamic general equilibrium, overlapping generations models calibrated to Japan and US data. We then assess quantitatively the government capacity to raise tax revenue when accounting for population ageing.

We demonstrate that the size of fiscal space varies greatly over time and across countries, depending on the timing of demographic transition. The fiscal spaces for Japan and USA will be contracted sharply by 2040, when the two countries enter their late stage of demographic transition. Our results also point out that contraction in fiscal
space will be predominantly driven by the fertility rate, as mortality improvements are slowing down while fertility rates remain below replacement rate. When we account for the size of the changes in the demographic variables, we find that there is almost linear negative relationship between dependency ratio and fiscal space.

In particular, our model calibrated to Japan indicates that the increase in dependency ratio from around 40 percent in 2010 to over 70 percent in 2040 will lead to a contraction in the capital-labor fiscal space by around 36 percent. The net fiscal limit will be gone when factoring in the increased fiscal cost of age-related spending commitments. The fiscal space for capital income taxation is the most sensitive to ageing, with a 10 percentage point increase in the dependency ratio is associated with a 14 percentage point decrease in the gross fiscal space relative to the benchmark fiscal space. This sensitivity arises as ageing causes both a decrease in the capital stock and a decrease in the return on capital.

There are several matters that we abstract from in this analysis. First, we have kept policies constant in the simulations and altered the population dynamics only. This does not account for policy reaction to the ageing trend – including increasing the retirement age, labor force participation and pension reform. Second, the peak of the Laffer curve does not necessarily maximise the welfare of the population, and the results do not imply that raising labor and capital tax rates would be optimal. Third, while the peaks of the Laffer curves offer the theoretical maximum revenue that can be raised, the “effective” maximum revenue that is politically feasible is likely much lower than that economic level. Other factors that matter for government capacity to raise taxes include size of informal sector, international tax competition and tax haven, are assumed away in this analysis. We leave these issues for future research.
References


7 Appendix

7.1 An algorithm to solve the model

We use dynamic programming to solve the model in MATLAB. The general procedure can be summarised as follows:

1. Discretise the state space of assets as \([a_0, ..., a_{\text{max}}]\), the grids for leisure as \([l_{\text{min}}, ..., l_{\text{max}}]\) (where \(l_{\text{min}} \geq 0\) and \(l_{\text{max}} \leq 1\)), and the taxation rate as \([0, ..., \tau_{\text{max}}]\) (where \(\tau_{\text{max}} \leq 1\)).

2. Choose a combination of \(\tau^l\), \(\tau^k\), and \(\tau^c\) by either setting 2 constant and looping 1 over the taxation grid, or setting 1 constant and looping 2 over the taxation grid.

3. Guess for initial wage rate \(w\), post-tax interest rate \(R^k\), pension benefit \(\text{pb}\), bequest \(b\) and capital stock \(K\).

4. Work backwards from \(J\) to period 1 to obtain optimal decision rules for consumption, savings, labor supply, and the value and marginal value functions of the household.

5. Iterate forwards to obtain the vector of optimal consumption, savings, and labor supply choices for the households across different generations, using \(a_1 = 0\) and following the optimal decision rules.

6. Work out the distribution of households within the economy \(\mu_j\) using the exogenous fertility rates and survival probabilities, before aggregating for the capital and effective labor in the economy; Work out the new wage rate and interest rates from the market clearing conditions, new bequest, and pension values; Balance government and social security budgets to determine endogenous fiscal variables.

7. Check the relative changes in the aggregate variables after each iteration and stop the algorithm when the change is sufficiently small. Otherwise, repeat from steps 3 to 6.

7.2 Tables and Figures
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Comments/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Demographics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J = 16$</td>
<td>Maximum lifetime periods</td>
<td>Equivalent to 80 years</td>
</tr>
<tr>
<td>$J^w = 9$</td>
<td>Maximum working periods</td>
<td>Equivalent to 45 years</td>
</tr>
<tr>
<td><strong>Preferences</strong></td>
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<td></td>
</tr>
<tr>
<td>$\sigma = 2$</td>
<td>Inverse of IES</td>
<td>Literature</td>
</tr>
<tr>
<td>$\phi = 1$</td>
<td>Frisch labour supply elasticity</td>
<td>Literature</td>
</tr>
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<tr>
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<tr>
<td><strong>Endowments</strong></td>
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<td></td>
</tr>
<tr>
<td>$\pi$</td>
<td>Age-specific labour productivity</td>
<td>Braun (2008)</td>
</tr>
<tr>
<td>$\tilde{z}$</td>
<td>Stochastic component of labor productivity</td>
<td>Lise et al (2014)</td>
</tr>
<tr>
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<td></td>
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<td>Consumption tax</td>
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<tr>
<td>$\tau^i = 0.25$</td>
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<td>Data (14%)</td>
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<td>$\tau^k = 0.40$</td>
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<tr>
<td>$r^d = 0.01$</td>
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Table 1: The parameter values for demographics, preferences, technology and fiscal policy for the benchmark economy calibrated to Japan
<table>
<thead>
<tr>
<th>Variable</th>
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<th>2010</th>
<th>2040</th>
</tr>
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<tbody>
<tr>
<td>Labour tax base %</td>
<td>17.67</td>
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<td>-18.48</td>
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<tr>
<td>Labour supply %</td>
<td>19.02</td>
<td>0.00</td>
<td>-15.24</td>
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<tr>
<td>Effective labour supply %</td>
<td>20.10</td>
<td>0.00</td>
<td>-15.66</td>
</tr>
<tr>
<td>Wage rate %</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Capital tax base %</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Capital stock %</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest rate %</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption (tax base) %</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Social security system %</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pension benefit %</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Pension contribution rate</td>
<td>percentage point</td>
<td>-5.10</td>
<td>0.00</td>
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<td>Labour income tax rate %</td>
<td>percentage point</td>
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<td>Output (GDP) %</td>
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<td>-13.80</td>
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Table 2: The effects of demographic shift on macroeconomic variables. Note that, all variables other than the wage and interest rates, and pension contribution and labour income tax rates, are expressed in per capita terms. The changes are calculated in terms of percentage deviations from the 2010 benchmark levels.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Comments</th>
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</thead>
<tbody>
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<tr>
<td>$\phi = 1$</td>
<td>Frisch labour supply elasticity</td>
<td>–</td>
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<tr>
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<td>Weight of labour</td>
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<td>CFE preferences (Alternative)</td>
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Table 3: Model parameter values for alternative preferences
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Table 4: Model parameter values for the US
Figure 15: Ageing and Fiscal Costs from Cecchetti et. al. (2010)

Figure 16: The g-Laffer Hill for Labor and Capital Income Taxation in Japan with the consumption tax is set at \( \tau_c = 0.1 \).
Figure 17: The g-Laffer Hill for Labor and Capital Income Taxation in Japan with the consumption tax is set at $\tau^c = 0.2$.

Figure 18: Demographic Shift in the US: Model and Data
Figure 19: Laffer Curves and Fiscal Spaces in the US