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Private Externalities and Environmental Public Goods: *Politico-economic Consequences*★

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ABSTRACT

We study an overlapping-generations model with private externalities on a public good (e.g. the environment). Emergent politico-economic equilibria, depending on model primitives (e.g. the degree of externality), imply that average income and environmental outcomes may be related positively or negatively, or not at all. Qualitatively, these equilibria provide a cross-country interpretation for existing disagreements in empirical findings on average income and environment. Normatively, inefficiently excessive environmental outcomes may emerge. These are partly explained by a politico-economic redistributive tension along a taxation Laffer curve. However, with externalities, this tension is further modified, resulting in these excesses being non-monotonic in the degree of externality.

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KEYWORDS: Income, Environment; Markovian Voting; Externality

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1. INTRODUCTION

In this paper, we consider a public good that can be affected by externalities from private decisions. A natural interpretation of such public goods would be the environment. For example, many national parks, marine reserves, or animal habitats are a direct consequence of government intervention—i.e. through expending taxpayer resources on creating and maintaining these public goods. However, the sustainability of these resources also depend on externalities arising from private decisions—e.g. the use of "clean" versus "dirty" technologies. These effects are externalities because individual decision makers do not internalize the impact of their individual choices on the aggregate public good. For lack of a better term, in this paper, we shall interpret this public good as an environmental public good.

In our model based on Hassler et al. (2007), tax-funded public expenditure on an environment-improving technology is perfectly substitutable with (net positive) externality from private production. The latter depends positively on private agents' use of a "clean" technology and negatively on their use of a "dirty" technology. However, there is a redistributive effect to consider, when a government is limited to a small number of tax instruments. On one hand, more tax cuts could mean less public expenditure devoted to maintaining the public good. On the other hand, there are two countervailing forces to consider. First, there is a usual opposing Laffer-curve effect—a lower present value of taxes induces a larger tax base and hence a larger public contribution to the environmental good. Second, with net positive externalities, lower taxes additionally promote a larger positive externality effect on the environmental good as the measure of clean-technology users increases. In other words, there is a double-dividend channel of taxes in this model (e.g. Glomm et al., 2008). A Ramsey planner in our model exploits this additional externality channel more, the larger is the externality effect. However, as we will show, in the politico-economic equilibria, this double-dividend channel is under-exploited (relative to Ramsey). How this trade-off is resolved depends on the nature of the policy or political regime in place, which in turn, will depend on the size of the externality channel.

Given such an environment, we study allocations and welfare arising under alternative politico-economic equilibria that may emerge. We contrast these with a benchmark Ramsey planner's allocation. We consider two questions in this paper. First, from a positive perspective, we ask what are the properties of equilibria that may arise; and as a consequence, how average income (or living standard) and environmental outcomes are related. Second, we ask if democracy generates better environmental outcomes; and, from a normative aspect, we ask if these outcomes are desirable in a welfare sense. The second question relates to recent political rhetoric on whether a country should modify income taxes to include funding for improving the environment.[1]

In relation to the first question, we show that the relationship between income and environment can be negatively related, positively related or even unrelated under different politico-economic equilibrium regimes. The emergence of these regimes depend on model primitives,

[1] Australia, for example, is a country that has recently (since July 1, 2012) introduced "carbon taxes" that are effectively income-contingent taxes on households. Broadly described, this system imposes a common tax (per unit of pollution emission) on major industrial emitters of carbon (which ultimately gets passed on to energy-consuming households); but then there is a reduction in income taxes, and an increase in welfare payments to certain income groups. While we do not explicitly model such income-contingent tax instruments, the age-dependent income tax scheme in which the poor in the model can be interpreted as exempt from taxes, in our model, approximates such a reality to some extent.

and in particular, on the size of the externality effect. We show that a continuum of equilibria with politically manipulative voters can emerge, when the externality channel is small, given all else equal. Also, another unique equilibrium, where majority voters have no room to be manipulative, can arise when either agents are more patient, or, when young agents face a low probability of drawing high individual productivities. These equilibria reflect known results in Hassler et al. (2007), as our model builds upon the authors'. However, our politico-economic equilibria nest their equilibria as special cases. In contrast to Hassler et al. (2007), another new unique equilibrium with manipulative voters, can emerge when the externality effect and proportion of high-productivity young agents are sufficiently large.[2] Moreover, because of the net positive externality effect, most of our politico-economic equilibria feature a tax on the current young that is lower than the equilibria's respective counterparts in an economy without the externality—i.e. the corresponding Hassler et al. (2007) economies.

Our result, with respect to this first question, may also relate to a broader empirical question: Do countries with higher income necessarily have better environmental outcomes? The well-known "Environmental Kuznets Curve" hypothesis, suggests that there ought to be an inverted U-shaped (or U-shaped) relationship between income per capita and pollution (or environmental quality). Various empirical studies have attempted to test this hypothesis (see e.g. Grossman and Krueger, 1995; Torras and Boyce, 1998; Stern and Common, 2001; Harbaugh, Levinson and Wilson, 2002; Dijkgraaf and Vollebergh, 2005).[3] However, there is no conclusive panel-data evidence in favor of the hypothesis. Specifically, the literature has produced a wide variety of estimated polynomial relationships (see e.g. Harbaugh et al., 2002). In terms of the cross-sectional interpretation of these disagreeing estimates, it implies that the direction of the predicted income-environment relationship has no particularly conclusive order with respect to increasingly richer countries. As Harbaugh et al. (2002) pointed out, the empirical relationship between income and the environment estimated from data is sensitive to small variations in the data, econometric specifications, omitted variables, or country-specific effects. The ambiguity in the cross-country empirical evidence on average income and environment, may thus be rationalizable, in our model, as outcomes from various politico-economic equilibria.[4]

With respect to the second question, we show how politico-economic conflicts of interest distort environmental outcomes relative to a benchmark Ramsey allocation. We provide some examples in which the environment can be either inefficiently over-allocated (under-allocated)

[2] In comparison to Hassler et al. (2007), having a larger proportion of "rich" agents in the economy yields a "double dividend" on ex ante welfare. That is, it results in larger net positive externality on the environmental good, and increases the consumption of the environmental good for all agents. At the same time, it also delivers a larger tax base for the purposes of income redistribution. However, having a larger proportion of high-income agents implies lower tax rates, that in turn, may reduce tax revenue and welfare gains from redistribution. These trade-offs are evaluated by a policymaker who is either some far-sighted Ramsey planner, or some myopic democratically elected politician who implements the preference of a median voter of the day. We show that the resulting equilibrium policies, and also, politico-economic equilibrium regimes, crucially depend on the magnitude of this environmental externality, given other intrinsic features of an economy.

[3] Also see Dinda (2004) for a survey of this literature.

[4] To address the empirical income-pollution relationship, many other theoretical explanations with different perspectives have been advanced (see Dinda, 2004, for a survey). Among those perspectives, Jones and Manuelli (2001) provide a growth model where environmental policy is chosen endogenously through "voting" over time, by agents who are within-period homogenous. Their model can be taken to be one possible account of the dynamic aspect of the empirical relationship between average income and the environment, for a single country. The approach taken here is silent on issues of growth, but as a complement, it provides for an account of the conundrum arising in the same empirical relationship, when viewed from a cross-country perspective.

under all the politico-economic equilibria, relative to their respective Ramsey prescriptions, when the externality effect is large (small) enough. These are partly explained by a politico-economic redistributive tension along a taxation Laffer curve, given the policy trade-off described previously. However, with externalities, this tension is further modified, resulting in these excesses being non-monotonic in the degree of externality.

Not surprisingly, all the politico-economic equilibria considered in the paper are (weakly) dominated by their respective Ramsey allocation. From a normative perspective, political rhetoric and debates on introducing income taxes that are specifically targeted at improving the environment—e.g. carbon taxes—should account for the possibilities that such policies may be endogenously constrained by various politico-economic regimes, resulting in possibly an over- or even under-allocation of the environmental public good. In other words, our results suggest that proper measurements of the degree of potential private externalities, and therefore, the double-dividend channel, should be taken into account in informed policy debates.

The remainder of the paper is as follows. In section 2, we describe the economic model. In section 3, we provide an analytical characterization of the benchmark Ramsey optimal policy and its outcomes. In section 4 we study the same economy in which policy is to be determined by state-dependent majority voting. In section 5 we study the income-environment relationships induced by the different politico-economic equilibria and compare their welfare, income and environmental-good outcomes with those from each corresponding Ramsey equilibrium outcome. Finally we conclude in section 6.

2. Model

Our model builds upon Hassler et al. (2007), with the exception that the public good is partly determined by a net positive externality, in addition to a usual public expenditure component. Time is indexed by $t \in \mathbb{N} := \{0, 1, 2, ...\}$.

2.1. **Agent types.**

Each agent lives for two periods, denoted by "Y" for when they are young, and "O", for when they are old. The agents exist on a continuum and the constant population of young agents is normalized to unity. Every agent is partially indexed by his age type $a \in \{Y, O\}$. An agent is also described by his age-dependent productivity state z_a.

At the beginning of each period t, a Y agent realizes an idosyncratic productivity shock $z_Y \in \{H, L\}$, where H and L, respectively, refer to "high productivity" and "low productivity" agent types. For each Y agent at time t, z_Y is independently and identically distributed as $(\mu, 1-\mu)$, where $\mu \in (0, 1)$, on the support $\{H, L\}$.

Given the realization $z_Y = H$, a (Y, H)-type agent gets to make an investment decision $i_t \in [0, 1]$, which induces an effort cost $(i_t)^2$, in utility terms. The decision i_t maps linearly into a probability that the agent succeeds in attaining the "clean" technology, which in turn, yields the per-period income (also utility flow) of $\overline{R} \equiv 1$. This agent, when old ($a = O$) attains productivity status S for being successful in her investment when young – i.e. the agent is of type (O, S). Conversely, the same (Y, H)-type agent faces the probability of $(1 - i_t)$ that her investment choice is unsuccessful (type U), and thus, yielding the lower per-period income of $\underline{R} = 0$. This agent, when old is re-labelled as (O, U), in terms of her old-age and old-age productivity status. If a Y agent draws $z_Y = L$, then his per-period income is also $\underline{R} \equiv 0$; and when his age is $a = O$, his type is labelled (O, L).

Thus each agent, at the beginning of time t, has individual state (a, z_a), where $a \in \{Y, O\}$, and,

$$z_a \in \begin{cases} \{H, L\}, & \text{if } a = Y \\ \{U, S, L\}, & \text{if } a = O \end{cases},$$

respectively, refer to current age, and, current-age contingent productivity status.

2.2. **Ex ante preference representations.** Existing agents may have their income flows taxed. Denote their respective age-contingent income tax rates at each time t as τ_t^Y and τ_t^O.[5] Generally, we may allow for a uniform lump-sum transfer of s_t to all agent types. Denote $b_t \geq 0$ as a possible one-off rebate, at time t, to (Y, H) agents who succeed in attaining $\overline{R} \equiv 1$ given their investment i_t. That is, these are the agents who will turn out to become type (O, S) in time $t+1$, and also to succeed in attaining the "clean technology".

Generically, let $\tau^a \in \{\tau^Y, \tau^O\}$ and $R \in \{\underline{R}, \overline{R}\}$. Agents' common per-period utility function is linear in private consumption $C(\tau^a, R, s)$ and the environmental good E:

$$C(\tau_t^a, R, s_t) + \lambda E_t;$$

where $\lambda > 0$ is per-period marginal utility of the environmental good E_t; or equivalently in this model, the marginal rate of substitution between C and E. Let $V_t^{(a,z_a)}$ denote the lifetime payoff of an agent, after realizing their idosyncratic type (a, z_a), at time t. Denote $\beta \in (0, 1)$ as the common subjective discount factor. The expected lifetime payoffs of each realized type of agent at time t are described below. First, consider the (O, S)-agent with budget constraint $C(\tau_t^O, \overline{R}, s_t) = (1 - \tau_t^O)\overline{R} + s_t$. This says that the agent's private-good consumption is financed by his after-tax income and (potential) lump-sum transfer. The (O, S)-agent has the following payoff:

$$V_t^{O,S} := V^{O,S}(s_t, E_t, \tau_t^O; \overline{R}) = (1 - \tau_t^O) + s_t + \lambda E_t. \tag{2.1a}$$

Second, the (O, U)-agent, whose budget constraint is $C(\tau_t^O, \underline{R}, s_t) = (1 - \tau_t^O)\underline{R} + s_t$ has the following payoff,

$$V_t^{O,U} := V^{O,U}(s_t, E_t, \tau_t^O; \underline{R}) = s_t + \lambda E_t, \tag{2.1b}$$

since we normalized $\underline{R} = 0$. Third, the (O, L)-agent has identical payoff to the (O, U) agent:

$$V_t^{O,L} := V^{O,L}(s_t, E_t, \tau_t^O; \underline{R}) = V_t^{O,U}. \tag{2.1c}$$

Fourth, if a young agent realizes type (Y, H), this agent incurs effort cost $(i_t)^2$ to obtain immediate expected payoff $i_t(1 - \tau_t^Y + b_t)\overline{R} + (1 - i_t)(1 - \tau_t^Y)\underline{R}$, and, a resulting continuation payoff $\beta\left[i_t V_{t+1}^{O,S} + (1 - i_t)V_{t+1}^{O,U}\right]$. The (Y, H)-agent's total payoff thus reduces to the expression:

$$\begin{aligned} V_t^{Y,H} &:= V^{Y,H}(s_t, s_{t+1}, E_t, E_{t+1}, \tau_t^Y, b_t, \tau_t^O; \underline{R}, \overline{R}) \\ &= -(i_t)^2 + i_t\left[(1 - \tau_t^Y + b_t) + \beta(1 - \tau_{t+1}^O)\right] + (s_t + \lambda E_t) + \beta\left(s_{t+1} + \lambda E_{t+1}\right). \end{aligned} \tag{2.1d}$$

[5]The assumption of age-dependent taxes follows Hassler et al. (2007), and is a simplification for tractability. This can be interpreted as part of a simplified tax structure approximating more real-world policy instruments such as progressive income tax rates and pay-as-you-go social security systems.

Fifth, the current (Y, L)-agent, whose young-age budget constraint is $C(\tau_t^Y, \underline{R}, s_t) = (1-\tau_t^Y)\underline{R} + s_t$, knows that his continuation payoff will be $V_t^{O,L}$, so the total discounted payoff for him is:

$$V_t^{Y,L} := V^{Y,L}(s_t, s_{t+1}, E_t, E_{t+1}, \tau_t^Y, 0, \tau_t^O; \underline{R}, \overline{R}) = (s_t + \lambda E_t) + \beta\,(s_{t+1} + \lambda E_{t+1})\,. \qquad (2.1e)$$

2.3. **Optimal investment.** The investment decision, i_t, is the only decision available to be made by private agents, and is only made by agents of type (Y, H). Given some policy outcomes $(\tau_t^Y - b_t, \tau_{t+1}^O) \equiv (\hat{\tau}_t^Y, \tau_{t+1}^O)$, the μ-measure of (Y, H) agents are identical and thus choose the same optimal investment function:[6]

$$i_t^* := i^*(\hat{\tau}_t^Y, \tau_{t+1}^O) = \max\left\{0, \min\left[\frac{(1-\hat{\tau}_t^Y) + \beta\,(1-\tau_{t+1}^O)}{2}, 1\right]\right\}. \qquad (2.2)$$

Note that the optimal decision rule (2.2) is derived with E_t taken as parametric by the agents.

2.4. **Government budget constraint.** Given the proportion μ of (Y, H)-type agents at time t, the proportion of old agents who are of type (O, S) in $t+1$ will be $\mu\pi_{t+1}$, where

$$\pi_{t+1} = i_t^*, \qquad \forall t \geq 1; \text{ and } \pi_0 \text{ given.} \qquad (2.3)$$

Thus the model has a single natural state variable given by π_t.

The government is assumed to balance its budget period by period, so that total expenditure on the environment and general transfers must equal the tax revenue accruing from the current population of (O, S) agents and (Y, L) agents:

$$A_t + 2s_t = \tau_t^O \mu \pi_t + \hat{\tau}_t^Y \mu i_t^*. \qquad (2.4)$$

2.5. **Nature's law.** The environment co-state E_t is determined by two sources: (i) public expenditure A_t on improving it; and (ii) externality from the output of clean and dirty technology generated by the current young.[7] The law governing this feedback process on nature is given by

$$E_t = A_t + \epsilon\left[\mu i_t^* \overline{R} - (1-\mu i_t^*)\underline{R}\right] \equiv A_t + \epsilon\mu i_t^*, \qquad (2.5)$$

where $\epsilon \in (0, 1)$ is the degree of externality on the environment from aggregate private outputs. The second term on the right of the equality in (2.5) describes the aggregate positive externality induced by the μi_t^* measure of (Y, H)-agents producing output $\overline{R}$. The last term of the right of the equality in (2.5) is the negative externality effect generated by the $(1-\mu i_t^*)$ measure of (Y, L)-agents producing output $\underline{R}$. It is without loss of generality that we shall focus on just the "positive externality" channel, since this can be interpreted as a "net positive externality" effect in the model. As a result of this normalization, we have the last equivalence term in (2.5).

Observe that public expenditure on environmental improvement, A_t, increases E_t directly. Furthermore, E_t is also increasing in the aggregate level of private investment in the "clean" technology, μi_t^*. Note that the positive externality on E_t in the model arises only from the current use of the clean technology by a fraction i_t^* of the (Y, H)-type agents. While this may

[6] Note that without loss of generality, we shall henceforth make use of $\hat{\tau}_t^Y := \tau_t^Y - b_t$, the net tax rate on the (Y, H) agents.

[7] We abstract from the externality effects associated with the output flows that go to the various old agents. Introducing additional externalities associated with the old agents introduces history dependence in E_t, which means that we will not have analytical tractability. This additional complication would not add much to the understanding of how externalities from certain agents' decisions would matter for politico-economic outcomes.

not be completely realistic, we think it does capture some aspects of clean or environmentally-friendly technology that are related to wealthier segments of the population demographic.[8]

2.6. **Economic policy trade-offs.** The model setup implies a key trade-off for a policy maker, which in our case, will either be a Ramsey planner or an elected politician. This trade-off is decomposable into a "policy intensive margin" (i.e. the direct effect of the tax rates) and an "extensive margin" (i.e. the indirect effects of the policy on the proportion of high-income agents). The latter extensive margin acts through two channel, one a typical Laffer curve effect, and the other is our new double-dividend (net positive externality) channel.

First, the Laffer curve effect is as follows. Whereas raising the present value of taxes (i.e. $\hat{\tau}_t^Y + \beta\tau_{t+1}^O$) in the model will directly increase the public contribution (A_t) toward improving E_t (i.e. the "policy intensive margin"), it lowers the incentive for (Y, H)-type agents to invest (i_t^*); and this results in a lower current tax base (via i_t^*) and future tax base (via π_{t+1}) for providing A_t and A_{t+1}, and thus E_t and E_{t+1}.[9] Second, raising the present value of taxes, and therefore lowering i_t^*, will now also result in a smaller total externality effect ($\epsilon\mu i_t^*$) on E_t in the description of the environmental feedback equation (2.5). This second effect is new in the model and will play a vital role in inducing the possibility of a new class of politico-economic equilibrium (see Proposition 5). Moreover, the degree of externality effects in this channel, ϵ, will also matter in determining the relative sizes of government intervention in, and therefore allocation of, the environmental public good (see section 5.3).

The overall impact of (equilibrium) tax rates on welfare and the environmental co-state will depend on how a particular policy maker (Ramsey or decisive voter) evaluates this "intensive-versus-extensive" margin underlying her policy trade-off. Therefore, primitive parameters such as the discount factor (β), the degree of positive externality on the environment (ϵ), agents' common marginal utility of the environmental good (λ), and the probability of young agents drawing a high productivity state (μ) will appear in the characterizations of these trade-offs. In the following section, we first resolve the trade-off in terms of a benchmark Ramsey planner problem and an optimal allocation. Then in Section 4 we study the alternative situation where the resolution of the policy trade-off is determined by a decisive voter.

3. Benchmark Second-best Allocations

The model in general allows for government expenditure to be composed of general transfers $s_t \geq 0$, type-specific transfer $b_t \geq 0$, and a publicly provided environmental good $A_t \geq 0$. Similar to Hassler et al. (2007), the following knife-edge type result tells us that we can focus only on extreme allocations.

[8]For example, currently, in some countries where energy companies offer consumers a menu of choices over sources of energy (e.g. electricity), often the environment-friendly option costs more. Other examples include the option to purchase hybrid/electric cars, the option to build "smart" environment-friendly buildings, and etc. These "greener" options tend to be more expensive and are typically chosen by wealthy consumers or firms. We do not model an additional clean-versus-dirty technology decision margin by agents to maintain tractability. Thus the assumption that Nature decides who gets to use which technology can be interpreted as an approximation to more complicated decision margins. For the purposes of understanding the degree of such externalities on politico-economic equilibria, the added complication does not yield more insight.

[9]Note that all the equilibria induced by the various policy mechanisms considered later will turn out to be stationary equilibria, so that $i_t^* = \pi_{t+1} = \pi_t$, resulting in $E_{t+1} = E_t$, for all $t \geq 1$.

Lemma 1. *For any feasible allocation* $\{s_t, \hat{\tau}_t^Y, \tau_t^O, A_t\}_{t=0}^{\infty}$*, there exists another Pareto-dominating allocation* $\{\tilde{s}_t, \hat{\tau}_t^Y, \tau_t^O, \tilde{A}_t\}_{t=0}^{\infty}$ *such that for all* $t \in \mathbb{N}$*,*

$$(\tilde{s}_t, \tilde{A}_t) \begin{cases} = (0, A_t + 2s_t), & \text{if } \lambda > \frac{1}{2} \\ = (s_t + \frac{1}{2}A_t, 0), & \text{if } \lambda < \frac{1}{2} \\ \in \{(s_t, A_t) \in \mathbb{R}_+^2 | (2.4) \text{ holds} \}, & \text{if } \lambda = \frac{1}{2} \end{cases}.$$

The proof of Lemma 1 is immediate, as a consequence of the observation that for each given $(\hat{\tau}_t^Y, \tau_{t+1}^O)$ and therefore $i^*(\hat{\tau}_t^Y, \tau_{t+1}^O)$, the allocation process over (s_t, A_t) is a linear program. Observe that from the government budget constraint (2.4) the social rate of transformation between s_t and A_t is $-1/2$. The social marginal rate of substitution between s_t and A_t is $-\lambda$. In the knife-edge case, where $\lambda = 1/2$, the planner is indifferent between alternative compositions of (s_t, A_t).

From now on we will assume that $\lambda > 1/2$, the more interesting case. Also, for tractability, we will assume that the net tax faced by (Y, H)-type agents is bounded.

Assumption 1. $0 \leq b_t \leq \tau_t^Y \Leftrightarrow \hat{\tau}_t^Y \in [0, 1]$.

This assumption implies that the "clean-technology" rebate to each (Y, H) agent can be no larger than his income tax paid.[10]

Definition 1. A sequence of outcomes $\{A_t, \hat{\tau}_t^Y, \tau_{t+1}^O\}_{t=0}^{\infty}$ is a *feasible allocation* if it satisfies, for all $t \in \mathbb{N}$:

(1) Young agents best response (2.2);
(2) the evolution of the aggregate state (2.3); and
(3) government fiscal solvency (2.4);
(4) the environmental feedback law (2.5);
(5) feasibility:

$$A_t \geq 0, \tau_{t+1}^O \in [0, 1], \text{ and}, \hat{\tau}_t^Y \in [0, 1]. \tag{3.1}$$

Define

$$\mathcal{F}^\dagger := \left\{ (\hat{\tau}_t^Y, \tau_{t+1}^O) | (2.2) - (2.5), \text{ and } (3.1) \text{ hold } \forall t \in \mathbb{N} \right\},$$

as the set of feasible allocations. Note that $\mathcal{F}^\dagger$ is non-empty and does not depend on the state variable π_t.

3.1. **Benchmark Ramsey planning problem.** The Ramsey planner maximizes a weighted sum of each generation's total payoff subject to the requirement that the planner's allocation is feasible.[11] More generally, suppose the planner may also attach a different (relative) weight $J \geq 1$ to the payoffs of agents of types (Y, L) and (O, L). That is, a larger J measures a larger bias of the planner towards low-productivity agents; and $J = 1$ implies that the planner assigns the

[10] One would expect to see this sort of feature on tax rebates in most institutions. See the Supplementary Appendix for an alternative characterization, when an upper bound on b_t is not imposed a priori.

[11] It is well-known that in OLG models, there is actually a continuum of planners indexed by their different weighting functions on each generation's lifetime payoff. We consider, as is conventionally done, a particular planner who discounts the total payoff of each date-t generation using the same subjective discount factor as the agents themselves, i.e. β^t.

same weight to the current payoffs of all types of agents within the same date. The value to the planner beginning from a given initial state π_0 is

$$V(\pi_0) = \max_{\{\hat{\tau}_t^Y, \tau_t^O\}_{t=0}^\infty} \Bigg\{ \mu \left[\pi_0 V_0^{O,S} + (1-\pi_0) V_0^{O,U} \right] + (1-\mu) J V_0^{O,L} + \sum_{t=0}^{\infty} \beta^t \left[\mu V_t^{Y,H} + (1-\mu) J V_t^{Y,L} \right] \Bigg| \; \tau_0^O \in [0,1] \text{ and } \left(\hat{\tau}_t^Y, \tau_{t+1}^O\right) \in \mathcal{F}^\dagger \Bigg\},$$

subject to $(V_0^{O,S}, V_0^{O,U}, V_0^{O,L}, V_t^{Y,H}, V_t^{Y,L})$ satisfying (2.1a)-(2.1e). That is the planner maximizes the weighted ($J \geq 1$) sum of all agents' lifetime payoffs, subject to the requirement of feasibility given in Definition 1.

Let $B := [\mu + (1-\mu)J]\lambda$. After some algebra, it can be shown that the Ramsey (dual) problem is equivalently given by:

$$V(\pi_0) = \mu \left\{ \max_{\tau_0^O} \left[\pi_0 \left(1 - \tau_0^O + 2B\tau_0^O\right) + \tilde{V}(\pi_0; \tau_0^O) \right] \right\}, \tag{3.2a}$$

where

$$\tilde{V}(\pi_0; \tau_0^O) = \max_{\{\hat{\tau}_t^Y, \tau_{t+1}^O\}_{t=0}^\infty} \left\{ \sum_{t=0}^{\infty} \beta^t U(\hat{\tau}_t^Y, \tau_{t+1}^O) \middle| \left(\hat{\tau}_t^Y, \tau_{t+1}^O\right) \in \mathcal{F}^\dagger \right\}, \tag{3.2b}$$

$$U(\hat{\tau}_t^Y, \tau_{t+1}^O) = \left[\left(1 - \hat{\tau}_t^Y\right) + \beta\left(1 - \hat{\tau}_{t+1}^O\right) - i_t^* + 2B\left(\hat{\tau}_t^Y + \epsilon + \beta\tau_{t+1}^O\right) \right] i_t^*, \tag{3.2c}$$

and, i_t^* is defined in (2.2).

There are two things to note about the Ramsey planning problem. First, the constrained optimization problem in (3.2a)-(3.2c), has two parts. The first term on the right of (3.2a) is a static problem involving τ_0^O—i.e. it is independent of any continuation strategy $\sigma_R|_{(\pi_0;\tau_0^O)} := \{\hat{\tau}_t^Y(\pi_0; \tau_0^O), \tau_{t+1}^O(\pi_0; \tau_0^O)\}_{t=0}^\infty$. Second, the problem beginning from any given $(\pi_0; \tau_0^O)$ summarized by $\tilde{V}(\pi_0; \tau_0^O)$—i.e. the second term on the right of (3.2a)—is independent of $(\pi_0; \tau_0^O)$. This is simply a result of the fact that the planner's set of feasible choices $\mathcal{F}^\dagger$ is independent of the state π_t, similar to Hassler et al. (2007). Therefore, the continuation problem is equivalent to one of maximizing an infinite series of static payoffs:

$$\tilde{V}(\pi_0; \tau_0^O) = \max_{\tau_t^Y, \tau_{t+1}^O} \left\{ \frac{U\left(\hat{\tau}_t^Y, \tau_{t+1}^O\right)}{(1-\beta)} : (\hat{\tau}_t^Y, \tau_{t+1}^O) \in \mathcal{F}^\dagger \right\}. \tag{3.3}$$

Before we characterize the constrained-efficient Ramsey plan, we make a mild regularity assumption on parameters. The presence of externality effects $\epsilon \in (0,1)$ requires the following assumption, which will imply that the present value of the Ramsey planner's taxes is strictly positive.

Assumption 2. $(1-2B)(1+\beta) + 2B\epsilon < 0$.

Since $J \geq 1$, then, $B \geq \lambda$. Moreover, since we will consider the interesting case where $\lambda > 1/2$, then this implies that we will be working with $B > 1/2$. Given this, Assumption 2 admits most values of β and ϵ in $(0,1)$.

Now we consider the first observation regarding the Ramsey problem. The initial period problem can be solved for separately as stated in the following Lemma.

Lemma 2. *The Ramsey optimal tax on the initial old is*

$$\tau_0^O = \begin{cases} 1, & \text{if } B := [\mu + (1-\mu)J]\lambda > \frac{1}{2} \\ 0, & \text{otherwise.} \end{cases} \tag{3.4}$$

Proof. By inspection of the objective function $\pi_0 \left(1 - \tau_0^O + 2B\pi_0\tau_0^O\right)$ in (3.2a), the optimal tax function in (3.4) obtains. □

Since we assumed $\lambda > 1/2$, given $J \geq 1$, then $B > 1/2$; and the optimal tax on the initial old will be $\tau_0^O = 1$.

Proposition 1 (Optimal Ramsey tax plan). *An interior optimal Ramsey tax plan, denoted by σ_R^*, exists and is characterized by (3.4) and*

$$\hat{\tau}_t^Y(\sigma_R^*) + \beta\tau_{t+1}^O(\sigma_R^*) = \frac{(1+\beta)(2B-1) - 2B\epsilon}{4B-1} \equiv K > 0, \tag{3.5}$$

and $(\hat{\tau}_t^Y(\sigma_R^*), \tau_{t+1}^O(\sigma_R^*)) \in \mathcal{F}^\dagger$, for all $t \in \mathbb{N}$.

Proof. $\mathcal{F}^\dagger$ is a non-empty, closed and bounded set, and therefore compact in Euclidean space. Also, the payoff function $U : \mathcal{F}^\dagger \to \mathbb{R}$ is continuous. Therefore existence follows from an application of Weierstrass' Theorem to the problem (3.3). The characterization for the continuation optimal plan $\sigma^*(\pi_0; \tau_0^O)$ follows immediately from solving the problem (3.3). Finally, the time-0 optimal policy function follows from Lemma 2. □

Note that the Ramsey equilibrium present value of tax rates is independent of the proportion of (Y, H)-type agents, μ, if and only if $J = 1$. That is, the distribution of the exogenous idiosyncratic productivity shock does not directly matter for the optimal Ramsey plan, when the planner has no bias towards the "low-productivity" agents.

3.2. **Positive implications of optimal Ramsey tax plan.** We now discuss the properties of the optimal Ramsey tax path σ_R^* described in Proposition 1.

3.2.1. *Indeterminate policy and determinate allocations.* First, note that the optimal continuation plan $\sigma_R^*|_{\tau_0^O}$ is described by equation (3.5), which characterizes two unknowns. In other words, the composition of selections $(\hat{\tau}_t^Y, \tau_{t+1}^O)$ from the optimal tax strategy $\sigma_R^*|_{\tau_0^O}$ is indeterminate, but the present values of these taxes, $(\hat{\tau}_t^Y + \beta\tau_{t+1}^O)$, is. Since the optimal investment best response (2.2) depends only on the present value of taxes, the equilibrium path $\{(A_t, E_t, \pi_{t+1}, i_t^*)(\sigma_R^*)\}_{t\in\mathbb{N}}$, given π_0, under the optimal Ramsey tax path σ_R^* is still unique.

3.2.2. *A measure for average income.* Second, we consider the implications of σ_R^* on average income. The cross-sectional income distribution for each period $t \in \mathbb{N}$ has a finite support $\{\underline{R}, \overline{R}\} := \{0, 1\}$. Therefore, the measure for average income is calculated as

$$\bar{y}_t = \frac{\mu(i_t^* + \pi_t)}{2}. \tag{3.6}$$

Note that $\bar{y}_t$ is also the proportion of high-income agents in the economy. Hence, the larger the proportion is, the higher average income is.

The analysis is greatly simplified by the time-invariant optimal Ramsey policy in Proposition 1. The latter implies a constant investment (i_t) level. Specifically, given the optimal policy path (3.5), we have that

$$i_t^* = i_R^* := \frac{B(1+\beta+\epsilon)}{4B-1}, \tag{3.7}$$

for all $t \in \mathbb{N}$. Since $\pi_{t+1} = i_t^*$, given π_0, we also have a constant proportion of high-income agents under the Ramsey optimal plan. Therefore the average income measure is constant over time, $\bar{y}_t = \bar{y}^* := \mu i_R^*$, for all $t \geq 1$. Given π_t is fixed at each $t \geq 1$, if a policy ensures higher probability of attaining high income, then the wealthier the society will become. However, from (3.7), it can be seen that under the Ramsey policy here, this will only depend ultimately on model primitives.

3.2.3. *Ramsey environmental outcomes.* While the environmental state, $E_t = E_R := \mu(\tau^O + \hat{\tau}^Y + \epsilon)i_R^*$, is a time invariant function of Ramsey policy choices, its outcome is indeterminate due to the indeterminacy in terms of any pair $(\tau^O, \hat{\tau}^Y)$ that satisfies the optimal Ramsey policy, as characterized by (3.4) and (3.5). Furthermore, it can be shown that the environmental state is increasing (or decreasing) in τ^O (or $\hat{\tau}^Y$) given the policy combination of $(\hat{\tau}_t^Y, \tau_t^O)$ is kept on the Ramsey optimal policy path. To see this, using (2.4), (2.5) and (3.5), we can rewrite E_t as

$$E_t = E_R := \mu[K + (1-\beta)\tau^O + \epsilon]i_R^*,$$

or,

$$E_t = E_R := \mu\left(\frac{K}{\beta} - \frac{1-\beta}{\beta}\hat{\tau}^Y + \epsilon\right) i_R^*,$$

where i_R^* is subject to (3.7).

However, observe that any outcome for E_R consistent with any $(\hat{\tau}^Y, \tau^O)$ satisfying the optimal policy is welfare neutral—it does not affect the maximal welfare attained under the policy satisfying (3.4) and (3.5). Specifically, using (3.2a), (3.2c) and (3.3), the value of the Ramsey optimal plan (i.e. the maximal Ramsey welfare value), given initial state π_0, is

$$V(\pi_0) = \mu\left\{\pi_0\left(1 - \tau_0^{O,*} + 2B\tau_0^{O,*}\right) + \frac{[(1+\beta) - K - i_R^* + 2B(K+\epsilon)]\, i_R^*}{1-\beta}\right\},$$

where $\tau_0^{O,*}$ satisfies (3.4), and i_R^* is subject to (3.7). Since investment is constant under the Ramsey optimal policy (3.7), then the welfare criterion evaluated under the optimal policy, will also be independent of the composition of $(\hat{\tau}^Y, \tau^O)$. Thus, we may consider the highest (welfare-neutral) environmental state attainable under any selection of $(\hat{\tau}_t^Y, \tau_t^O)$ consistent with (3.5), for all $t \geq 1$. This shall be denoted as a particular $E_R =: \bar{E}_R$.

In particular, when the feasible lowest Ramsey $\hat{\tau}^Y$ is zero, the associated constrained efficient allocation has the following interpretation for optimal intergenerational environmental sustainability. The young generation is encouraged to invest in clean technology under this Ramsey plan, while the old generation is induced to take up all the tax burden. In turn, when the young become old, they are also induced to optimally bear the tax burden, so as to encourage the young agents of the next generation to invest in clean technology; so on and so forth.

In this way, the maximal environmental state $\bar{E}_R$ is maintained through a shared responsibility between the young and the old, and this sharing scheme is enforced by this particular Ramsey optimal fiscal plan.

4. Political Conflict of Interest

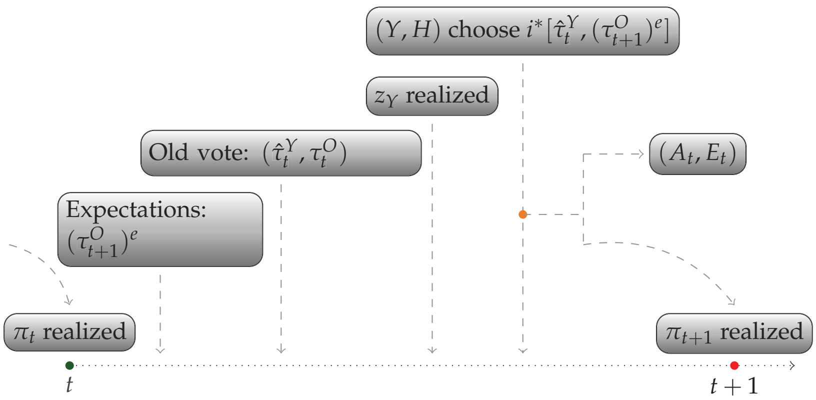
In this section we work towards establishing how political conflicts of interest may distort policy and equilibrium allocations. Ultimately, we would like to perform comparative analyses, in term of living standard, the environmental outcome and social welfare, between our previous Ramsey optimal policy outcome and the politico-economic outcomes.

Consider now, the same economic environment as before, but taxes are determined by majority voting. Following Hassler et al. (2007), we assume that in each period $t \in \mathbb{N}$, only the old agents vote.[12] The median voter in each period will depend on the distribution of old agents, which in turn, depends on the aggregate state variable π_t.

Unlike the Ramsey planner who, by construction, commits to implementing optimal current and future policies, here in each generation, decisive voters (or equivalently opportunistic politicians) care only about maximizing their own finite-life total payoffs over policy. In the model, this naturally gives rise to a problem of lack of commitment, by politicians, to implementing Ramsey optimal policy.

Figure 1 summarizes the timing assumption in the repeated voting game. At the beginning

FIGURE 1. Timing of repeated voting game.



of period $t \in \mathbb{N}$, the aggregate state π_t is publicly observed. Agents form their expectations, $(\tau_{t+1}^O)^e$, about political choices of the tax on the future old.[13] Given expectations, the old vote on their preferred policies $(\hat{\tau}_t^Y, \tau_t^O)$.[14] Young agents realize their idiosyncratic productivity type,

[12] This is observationally equivalent to the following alternative timing assumption: (i) The voting on $(\hat{\tau}_t^Y, \tau_t^O)$ occurs at the end of the previous period $(t-1)$; (ii) the young in $t-1$ vote, and the old do not vote (as it yields no utility gain to them at that point); (iii) observing the natural states π_t and $z_Y \in \{H, L\}$ the new young in period t decide on i_t^*; and (iv) the time-t young then vote on the following period's policy.

[13] As will be apparent in the equilibrium concept later, expectations $(\tau_{t+1}^O)^e$ matter for current voters because it is an argument of the (Y, H)-type agents' investment best response function.

[14] More precisely, voters would also have a decision on the composition of government expenditure—i.e. whether to spend on environmental improvement A_t, or, on general transfers s_t. Since we had assumed $\lambda \geq 1/2$, voters at each instance would unanimously choose $s_t = 0$ and $A_t \geq 0$. Thus we can shut down this additional margin of decision making for voters in the equilibrium descriptions later.

$z_Y \in \{H, L\}$. The (Y, H)-type agents make their investment decisions. Then the next period aggregate state π_{t+1} is realized, along with the co-state of the environment E_t.

We begin by defining voters' preference over policy. The time t and state π_t, (O, z_O)-type agents' preference over policy $(\hat{\tau}_t^Y, \tau_t^O)$, given expectations $(\tau_{t+1}^O)^e$, are

$$w^{O,z_O}(\pi_t; (\hat{\tau}_t^Y, \tau_t^O)) = \mathbb{1}_{\{S\}}(1 - \tau_t^O) + \lambda W(\pi_t, \tau_t^O) + \lambda Z\left(\hat{\tau}_t^Y, (\tau_{t+1}^O)^e\right), \tag{4.1a}$$

for all $z_O \in \{\{S\}, \{L\} \cup \{U\}\}$. The function $\mathbb{1}_{\{S\}}$ equals 1 if $z_O = S$, and, $\mathbb{1}_{\{S\}}$ equals 0 otherwise—i.e. if $z_O \in \{L\} \cup \{U\}$. The function

$$W(\pi_t, \tau_t^O) = \tau_t^O \mu \pi_t, \tag{4.1b}$$

is equivalent to the tax revenue accruing from the rich; and

$$Z\left(\hat{\tau}_t^Y, (\tau_{t+1}^O)^e\right) = [\hat{\tau}_t^Y + \epsilon]\mu i^*(\hat{\tau}_t^Y, (\tau_{t+1}^O)^e), \tag{4.1c}$$

is equivalent to the tax revenue accruing from the current (Y, H)-type agents plus their secondary dividend flow arising from a net positive externality on the environment ($\epsilon > 0$) as a result of their private choices i_t^*. Note that these indirect preferences encode our notion of feasibility as defined in Definition 1.

At each state π_t, and each $t \in \mathbb{N}$, the majority voter may either be from the rich class $\{(O, S)\}$ or the poor $\{(O, L)\} \cup \{(O, U)\}$. If the current state is such that $\mu\pi_t \geq 1/2$, then the majority is rich. Otherwise, it is poor. This makes the notion of a median voter state dependent. A median voter is a voting agent whose indirect utility at state π_t is defined as follows.

Definition 2 (Current median voter)**.** A *current median voter* is one whose indirect utility at state π_t is

$$w^m(\pi_t) = \max_{\tau_t^O, \hat{\tau}_t^Y \in [0,1]} \left\{ (1 - \tau_t^O)\mathbb{1}_{\{\pi_t \geq \frac{1}{2\mu}\}} + \lambda \left[W(\pi_t, \tau_t^O) + Z(\hat{\tau}_t^Y, (\tau_{t+1}^O)^e) \right] \right\}. \tag{4.2}$$

where $\mathbb{1}_{\{X\}} = 1$ if event $\{X\}$ holds, and $\mathbb{1}_{\{X\}} = 0$ otherwise.

We make two observations concerning a current median voter's preference (4.2). First, a median voter's policy preferences over τ_t^O and $\hat{\tau}_t^Y$, respectively, are linearly separable by the functions W and Z. This allows for the two-dimensional policy issues to be treated in a separable manner— i.e. as though they were separate one-dimensional policy issues. Moreover, one can verify that the preference functions W and Z are both single peaked, so that the existence of a median voter in every state π_t is guaranteed (Austen-Smith and Banks, 1999, Theorem 4.4).

Second, there is a clear political conflict of interest. The class $\{(O, S)\}$ would disagree with $\{(O, L)\} \cup \{(O, U)\}$ on τ_t^O. It can be deduced from (4.2), that the former would prefer $\tau_t^O = 0$, and the latter would prefer $\tau_t^O = 1$. However, all classes of voters are unanimous on the issue $\hat{\tau}_t^Y$, as they have identical preference $\lambda Z\left[\hat{\tau}_t^Y, (\tau_{t+1}^O)^e\right]$.

Also observe that, from (4.2), if $\lambda > 1/\mu$, all types of agents are unanimous on policy outcomes. Intuitively, when λ is very large, the (O, S)-type agents do not mind being taxed since the tax revenue contributes to the environmental good which yields them high payoffs at the margin. Thus, they would agree on policy outcomes with the rest of the low-income agents. Therefore, any interesting political conflict of interest would arise only if $\lambda \leq 1/\mu$. In the politico-economic equilibrium characterizations below, we will work with this latter case.

4.1. **Politico-economic equilibrium.** We focus on the class of stationary Markov-perfect equilibria as our equilibrium concept.

Definition 3. Given initial aggregate state π_0, a *stationary Markov-perfect politico-economic equilibrium (SMPE)* is a triple $\langle T^O, P, T^Y \rangle$, where:

(1) $T^O : [0,1] \to [0,1]$ is a tax rule on the current (O,S) agents such that

$$T^O(\pi_t) = \arg\max \left\{ (1-\tau_t^O) \mathbb{1}_{\{\pi_t \geq \frac{1}{2\mu}\}} + \lambda W(\pi_t, \hat{\tau}_t^O) \right\}; \tag{4.3a}$$

(2) $P : [0,1] \to [0,1]$, is the (Y,H)-agents' best response, such that

$$P(\hat{\tau}_t^Y) = i^*[\hat{\tau}_t^Y, (\tau_{t+1}^O)^e]; \tag{4.3b}$$

(3) $T^Y : \emptyset \to [0,1]$ is a net-tax rule on the (Y,H) agents such that

$$T^Y = \arg\max \left\{ \lambda Z \left[\hat{\tau}_t^Y, (\tau_{t+1}^O)^e \right] \right\}; \tag{4.3c}$$

and $(\tau_{t+1}^O)^e = (T^O \circ P)(\hat{\tau}_t^Y)$.

The first component of the equilibrium definition requires that the mapping T^O maximizes the separable policy preference W of the current median voter with respect to the tax on the current old τ_t^O, for every aggregate state π_t.

The second component requires that aggregate state outcomes are consistent with individual level decisions in equilibrium, which produces the equilibrium mapping P. Note that the functional equation (4.3b) defines an operator that maps from the space of (multi)-functions P into itself—i.e. we can write (4.3b) as

$$P(\hat{\tau}_t^Y) = i^*[\hat{\tau}_t^Y, (T^O \circ P)(\hat{\tau}_t^Y)], \tag{4.4}$$

where given functions T^O and i^*, P is a fixed point of the operator defined by the right-hand side of (4.4).

The third component requires that the mapping T^Y maximizes the separable policy preference Z of the current median voter with respect to the policy issue $\hat{\tau}_t^Y$. Implicit in the second and third components comprising a SMPE, is also the requirement that expectations are consistent with equilibrium conditions. That is, the equilibrium composite function $T^O \circ P$ is used to form rational beliefs about future taxes τ_{t+1}^O in order to calculate current SMPE best responses T^Y and P.

4.2. **SMPE characterization.** Readers familiar with Hassler et al. (2007) will recognize that this section is an extended analysis of their politico-economic equilibria. In our model, the private-choice externality on the environmental public good (via $\epsilon > 0$) is a new feature of our SMPE characterization and outcomes. Moreover, there will exist another new class of politico-economic equilibrium. This is discussed later.

Following Hassler et al. (2007), we can exploit the block recursivity of the SMPE definition. We will first characterize the SMPE mapping T^O, then P, and finally T^Y.

4.2.1. *SMPE map T^O.* Describing the SMPE tax function T^O is straightforward. The following observation says that if the current voter population is majority rich, then the SMPE rule prescribes a zero tax on the current (O,S) agents. Otherwise, it says to tax them completely.

Lemma 3. $T^O : [0,1] \to [0,1]$ *is an injective map such that*

$$T^O(\pi_t) = \begin{cases} 0, & \text{if } \pi_t \geq 1/(2\mu) \\ 1, & \text{if } \pi_t < 1/(2\mu) \end{cases}. \tag{4.5}$$

Proof. The solution for the function T^O is obtained from solving (4.3a). □

4.2.2. *SMPE map P.* Next, we describe the equilibrium transition law P.

Lemma 4. $P : [0,1] \to [0,1]$ *is a correspondence such that*

$$P(\hat{\tau}_t^Y) \begin{cases} \in \left\{ \frac{1-\hat{\tau}_t^Y}{2}, \frac{1-\hat{\tau}_t^Y+\beta}{2} \right\}, & \text{if } \hat{\tau}_t^Y \in [0, 1+\beta-1/\mu] \\ = \frac{1-\hat{\tau}_t^Y}{2}, & \text{if } \hat{\tau}_t^Y \in [1+\beta-1/\mu, 1] \end{cases}. \tag{4.6}$$

Proof. See Appendix A.1. □

Correspondence P summarizes the equilibrium best response of (Y, H)-type agents, given $\hat{\tau}_t^Y$, and the agents' beliefs about the tax rate on the old in the next period, $(\tau_{t+1}^O)^e$. By Lemma 3, we have a tractable description of the agents' expectation of future old-age tax—i.e. the agents can expect either a zero, or, a one-hundred percent tax rate on the future old. However, the expectation has to be consistent with the equilibrium strategy as defined in Definition 3. The number $1 + \beta - 1/\mu$ is the largest tax rate consistent with agents' expectation $(\tau_{t+1}^O)^e = 0$. Lemma 4 shows that if $\hat{\tau}_t^Y$ is high enough, then expecting a majority of rich voters in the next period (i.e. a zero future tax on the old) will never be realized. Therefore, the only equilibrium-consistent expectation, in such a case, is to expect majority poor voters in the next period. However, when $\hat{\tau}_t^Y$ is sufficiently small, both beliefs about future tax on the old are consistent with equilibrium strategies.

4.2.3. *SMPE map T^Y and overall characterization.* Now we characterize the last component of a SMPE, along with the rest of the SMPE requirements. Note that T^Y will depend on agents' beliefs about the tax on the future old, $(\tau_{t+1}^O)^e$. This is a consequence of current voters taking into account that the current (Y, H) agents will best respond to $\hat{\tau}_t^Y$ in their investment decisions (i^*). The impact of $\hat{\tau}_t^Y$ is twofold. First, there is a direct effect via their net income flow in the current period. Second, there is an indirect effect via their income flow when old next period, since in equilibrium $(\tau_{t+1}^O)^e = (T^O \circ P)(\hat{\tau}_t^Y)$.

These two effects create a trade-off. On the one hand, increasing $\hat{\tau}_t^Y$ will raise the payoff $\lambda Z(\hat{\tau}_t^Y, (\tau_{t+1}^O)^e)$ directly. On the other, increasing $\hat{\tau}_t^Y$ will lower the current young's investment effort, thereby inducing a possible majority poor next period. The majority poor next period will set $\tau_{t+1}^O = 1$. This will lower the payoff $\lambda Z(\hat{\tau}_t^Y, (\tau_{t+1}^O)^e)$ since Z is a decreasing function of $(\tau_{t+1}^O)^e$.

Following Hassler et al. (2007), it will be useful to distinguish between two possible classes of voting strategies, depending on current voters' beliefs, which ultimately will depend on parameters. We will call these two classes of voting strategies "Machiavellian" and "non-Machiavellian".[15] Loosely, the former is one in which it is possible for current voters to manipulate $(\tau^O_{t+1})^e$ when setting their preferred $\hat{\tau}^Y_t$ in order to generate a larger tax base (of endogenous measure $\mu i^*[\hat{\tau}^Y_t, (\tau^O_{t+1})^e]$) of the (Y, H) agents in the current period. The latter non-Machiavellian voting is the opposite—one in which current voters expect the worst outcome, $(\tau^O_{t+1})^e = 1$. As we shall see later, in some cases, either or both of these voting equilibria may exist.

Non-Machiavellian voting. We begin by first characterizing T^Y in a non-Machiavellian voting SMPE.

Definition 4. A voter is said to be playing a *non-Machiavellian voting strategy* if they set $(\tau^O_{t+1})^e = 1$ and choose

$$T^Y = \arg \max_{\hat{\tau}^Y_t \in [0,1]} \lambda Z(\hat{\tau}^Y_t, 1), \tag{4.7}$$

where the belief $(\tau^O_{t+1})^e = 1$ must be consistent with the requirements of an SMPE, if such a strategy is an equilibrium.

In a Non-Machiavellian equilibrium, we say that there is no room for the voter to manipulate beliefs about the future tax on the old, since the equilibrium-consistent belief is already at the maximal rate, $(\tau^O_{t+1})^e = 1$. Then a non-Machiavellian voting strategy for $\hat{\tau}^Y_t$ is such that $T^Y = (1-\epsilon)/2$. In general, a non-Machiavellian equilibrium is sustained if given $P(\hat{\tau}^Y_t)$,

$$Z\left[(1-\epsilon)/2, 1\right] > Z\left[\hat{\tau}^Y_t, 0\right],$$

for all $\hat{\tau}^Y_t$ that would result in a future majority of rich voters, i.e. $P(\hat{\tau}^Y_t)\mu \geq 1/2$. Next we define the upper bound on such a tax rate.

Definition 5. The largest $\hat{\tau}^Y_t$ consistent with a non-Machiavellian equilibrium is a $\tilde{\theta} := \tilde{\theta}(\beta, \epsilon)$ satisfying

$$Z\left((1-\epsilon)/2, 1\right) = Z\left[\tilde{\theta}, 0\right]; \tag{4.8}$$

and $\tilde{\theta} := \tilde{\theta}(\beta, \epsilon) = [(1 + \beta - \epsilon) - \sqrt{\beta(2 + 2\epsilon + \beta)}]/2$.

Definition 5, together with Lemmata 3 and 4, imply that if $\tilde{\theta} > 1 + \beta - 1/\mu$, the class of non-Machiavellian equilibrium is the only one that emerges. Otherwise, there exists some $\hat{\tau}^Y_t \in [\tilde{\theta}, 1 + \beta - 1/\mu]$ such that $Z\left((1-\epsilon)/2, 1\right) < Z\left[\hat{\tau}^Y_t, 0\right]$, and this is consistent with agents expecting $(\tau^O_{t+1})^e = 0$.

Proposition 2 (Non-Machiavellian equilibria)**.** *There exists a set of non-Machiavellian (NM) SMPE such that,*

$$T^O(\pi_0) = \begin{cases} 0, & \text{if } \pi_0 \geq 1/(2\mu) \\ 1, & \text{if } \pi_0 < 1/(2\mu) \end{cases}; \tag{4.9a}$$
$$T^O(\pi_t) = 1, \qquad (\forall t \geq 1);$$

[15] Note that Hassler et al. (2007) used, respectively, the terminology of "strategic" and "sincere" voting. We use the respective terms, "Machiavellian" and "non-Machiavellian", as strategic and sincere voting already have reserved meanings in the political economy literature.

$$P(\hat{\tau}_t^Y) \begin{cases} \in \left\{ \frac{1-\hat{\tau}_t^Y}{2}, \frac{1-\hat{\tau}_t^Y+\beta}{2} \right\}, & \text{if } \hat{\tau}_t^Y \in [0, \tilde{\theta}(\beta,\epsilon)] \\ = \frac{1-\hat{\tau}_t^Y}{2}, & \text{if } \hat{\tau}_t^Y \in (\tilde{\theta}(\beta,\epsilon), 1] \end{cases}; \tag{4.9b}$$

and,

$$T^Y = \frac{1-\epsilon}{2}. \tag{4.9c}$$

Proof. The existence proof is constructive. Assume at each t, agents know $T^Y = (1-\epsilon)/2$ and expect $(\tau_{t+1}^O)^e = 1$. We want to verify that this assumption is consistent with the requirements of a non-Machiavellian equilibrium. If $T^Y = (1-\epsilon)/2$ and $(\tau_{t+1}^O)^e = 1$, then, $P[(1-\epsilon)/2] = i^*[(1-\epsilon)/2, 1] = (1+\epsilon)/4 < 1/2 \leq 1/(2\mu)$. Therefore, for all $\mu \in (0,1)$, indeed the time-$(t+1)$ majority is poor, and they will prefer $\tau_{t+1}^O = 1$. Thus, current voters' expectations are self-fulfilling.

Given SMPE $P(\hat{\tau}_t^Y)$ as summarized in Lemma 4—i.e. equation (4.6)—and if, the current median voter finds it optimal to set $T^Y = (1-\epsilon)/2$, then a non-Machiavellian equilibrium exists. Now we prove that setting $T^Y = (1-\epsilon)/2$ is indeed optimal for the current median voter. First, note that for $\hat{\tau}_t^Y \in [0, \tilde{\theta}(\beta,\epsilon)]$,

$$Z(\hat{\tau}_t^Y, 0) = \lambda\mu(\hat{\tau}_t^Y + \epsilon)\left[\frac{1-\hat{\tau}_t^Y+\beta}{2}\right]$$

is strictly increasing in $\hat{\tau}_t^Y$, by the definition of $\tilde{\theta}(\beta,\epsilon)$. Second, observe that for all $\hat{\tau}_t^Y$,

$$Z(\hat{\tau}_t^Y, 0) = \lambda\mu(\hat{\tau}_t^Y + \epsilon)\left[\frac{1-\hat{\tau}_t^Y+\beta}{2}\right] > \lambda\mu(\hat{\tau}_t^Y + \epsilon)\left[\frac{1-\hat{\tau}_t^Y}{2}\right] = Z(\hat{\tau}_t^Y, 1).$$

By Definition 5, $Z[\tilde{\theta}(\beta,\epsilon), 0]$ is the maximal tax revenue that can be extracted from the current (Y, H) agents by the current median voter, if the current median voter were to deviate from a status quo of a non-Machiavellian voting equilibrium, for some $\hat{\tau}_t^Y \in [0, \tilde{\theta}(\beta,\epsilon)]$. Moreover, since $Z[\tilde{\theta}(\beta,\epsilon), 0] = Z[(1-\epsilon)/2, 1]$, then, for all $\hat{\tau}_t^Y \in [0, \tilde{\theta}(\beta,\epsilon)]$,

$$Z[(1-\epsilon)/2, 1] > Z(\hat{\tau}_t^Y, 0),$$

so that setting $T^Y = (1-\epsilon)/2$ and $(T^O \circ P)[(1-\epsilon)/2] = 1$ is optimal for the current median voter. Therefore the assumed non-Machiavellian voting strategy is indeed an equilibrium strategy. □

Proposition 3 (Non-Machiavellian SMPE outcome)**.** *The resulting non-Machiavellian (NM) SMPE outcome is unique for $t \geq 1$, with*

$$\hat{\tau}_t^Y = (1-\epsilon)/2; \tag{4.10a}$$

$$\tau_t^O = 1; \tag{4.10b}$$

$$\pi_t = (1+\epsilon)/4; \tag{4.10c}$$

$$E_t = \mu(1+\epsilon)(3+\epsilon)/8. \tag{4.10d}$$

Proof. The non-Machiavellian voting equilibrium outcomes (4.10a)-(4.10d) can be derived from the characterization just proven. □

Note that the non-Machiavellian SMPE characterization admits multiple equilibria, and this is a consequence of the correspondence in (4.9b), when $\hat{\tau}_t^Y \in [0, \tilde{\theta}(\beta,\epsilon)]$; i.e. is too low. However, the non-Machiavellian SMPE outcome is still unique. The proof of Proposition 2 shows that choosing $\hat{\tau}_t^Y < \tilde{\theta}(\beta,\epsilon)$ is never a best response, and, choosing $\hat{\tau}_t^Y = \tilde{\theta}(\beta,\epsilon)$ is optimal and consistent with a non-Machiavellian equilibrium, in which expecting $(\tau_{t+1}^{O})^e = 1$ is consistent with equilibrium. So then, in a non-Machiavellian SMPE outcome, this results in a unique image of $P[(1-\epsilon)/2] = [1-(1-\epsilon)/2)]/2 = (1+\epsilon)/4$. Thus, the remainder outcomes in such an equilibrium is also unique.

However, a non-Machiavellian SMPE may not exist for some parameters. To show this, we define another parameter that summarizes this possibility. Let $\theta \in [0, 1+\beta-1/\mu]$, where $1+\beta-1/\mu \geq 0$, be the largest $\hat{\tau}_t^Y$ that would still induce a majority of rich voters in the following period. Also, all non-Machiavellian SMPE differ from each other only in terms of out-of-equilibrium beliefs. All out-of-equilibrium beliefs consistent with a non-Machevellian SMPE has the property that $\tilde{\theta}(\beta,\epsilon) \geq \theta$. This is proven in the next result.

Proposition 4. *Let $\theta \in [0, 1+\beta-1/\mu]$ be the largest $\hat{\tau}_t^Y$ inducing a majority of rich voters in the following period, $t+1$. If $\tilde{\theta}(\beta,\epsilon) < \theta$, then a non-Machiavellian SMPE does not exist.*

Proof. Consider $\tilde{\theta}(\beta,\epsilon) < \theta$. $Z(\hat{\tau}_t^Y, 0)$ is continuous and increasing at $\hat{\tau}_t^Y = \tilde{\theta}(\beta,\epsilon) < \theta$. If so, and since θ is the largest $\hat{\tau}_t^Y$ supporting a majority of rich voters next period, then the current median voter would prefer some other tax $\hat{\tau}_t^Y \in [\tilde{\theta}(\beta,\epsilon), \theta]$ to setting the non-Machiavellian prescribed $T^Y = (1-\epsilon)/2 < \tilde{\theta}(\beta,\epsilon)$. Recall that the latter alternative induces a majority of poor next period. Therefore, a non-Machiavellian SMPE vanishes when $\theta > \tilde{\theta}(\beta,\epsilon)$. □

Machiavellian voting. Proposition 4 leads us to consider the other class of voting equilibria which we termed Machiavellian. In Machiavellian equilibria, voters have the desire and ability to manipulate the voting outcomes such that a majority of rich voters will emerge in the next period. The voters manipulate the young agents expectation of future tax rate because they know this can improve their own current payoffs. We will first derive a couple of useful observations in Lemma 5 and Lemma 6, and then, we will describe the Machiavellian SMPE and their outcomes in Proposition 5 and Proposition 6, respectively.

Lemma 5. *Assume $0 \leq 1+\beta-1/\mu < (1+\beta-\epsilon)/2$. Let $\theta \in [0, 1+\beta-1/\mu]$, be a tax rate $\hat{\tau}_t^Y = \theta$ that would induce a majority rich next period. If $(T^O \circ P)(\theta) = 0$, then for all $t \in \mathbb{N}$, a current median voter prefers $\hat{\tau}_t^Y = \theta$ to any other $(\hat{\tau}_t^Y)' \in [0, \theta)$.*

Proof. Because the SMPE mapping $P : [0,1] \to [0,1]$ in (4.6) is a correspondence with its image possibly non-monotonic in $\hat{\tau}_t^Y$, there are two cases to consider.

First, consider the case that $P(\hat{\tau}_t^Y)$ is strictly decreasing in $\hat{\tau}_t^Y$. Then $(T^O \circ P)(\theta) = 0$ implies that for all $(\hat{\tau}_t^Y)' \leq \theta$, $(T^O \circ P)[(\hat{\tau}_t^Y)'] = 0$ (self enforcing) and therefore, by (4.6), $P[(\hat{\tau}_t^Y)'] = (1-(\hat{\tau}_t^Y)'+\beta)/2$. As a result the relevant payoff function is $Z(\hat{\tau}_t^Y, 0)$ on the set $[0,\theta]$. Now, $Z(\hat{\tau}_t^Y, 0) = \lambda\mu(\hat{\tau}_t^Y+\epsilon)(1-\hat{\tau}_t^Y+\beta)/2$ is a quadratic function in $\hat{\tau}_t^Y$ and attains a maximum at $(1+\beta-\epsilon)/2$. Since $\theta \leq 1+\beta-1/\mu < (1+\beta-\epsilon)/2$, then $Z(\hat{\tau}_t^Y, 0)$ is increasing on $[0,\theta]$. This implies that $Z(\theta,0) > Z((\hat{\tau}_t^Y)', 0)$ for all $(\hat{\tau}_t^Y)' \in [0,\theta)$. That is, a current median voter can still raise higher revenue from taxing the current (Y,H) agents as long as $(\hat{\tau}_t^Y)' < \theta$. Therefore, if $P(\hat{\tau}_t^Y)$ is monotone decreasing in $\hat{\tau}_t^Y$, then the statement of the Lemma is proved.

Second, consider the possibility that $P(\hat{\tau}_t^Y)$ is not strictly decreasing in $\hat{\tau}_t^Y$, as described in the upper branch of the SMPE map in (4.6). If $P(\hat{\tau}_t^Y)$ is non-monotonic in $\hat{\tau}_t^Y$, then there exists some subset $S \subset [0,\theta)$ such that for all $\hat{\tau}_t^Y \in S$, $(T^O \circ P)(\hat{\tau}_t^Y) = 1$. However, we have proven (see the proof of Proposition 2) that $Z(\hat{\tau}_t^Y, 0) > Z(\hat{\tau}_t^Y, 1)$ for all $\hat{\tau}_t^Y \in [0,1]$. Since $\theta \leq 1+\beta-1/\mu < (1+\beta-\epsilon)/2$, then we can deduce that $Z(\hat{\tau}_t^Y, 0)$ is strictly increasing and continuous on $[0,\theta]$. Therefore, as in the previous case, setting $\hat{\tau}_t^Y = \theta$ is still optimal—i.e. $Z(\theta,0) > Z(\hat{\tau}_t^Y, 0) > Z[(\hat{\tau}_t^Y)'', 1]$, for all $\hat{\tau}_t^Y \in [0,\theta)$ and all $(\hat{\tau}_t^Y)'' \in S$. □

Lemma 6. *Assume* $0 \leq (1+\beta-\epsilon)/2 \leq 1+\beta-1/\mu$. *If* $(T^O \circ P)[(1+\beta-\epsilon)/2] = 0$, *then for all* $t \in \mathbb{N}$, *a current median voter prefers* $\hat{\tau}_t^Y = (1+\beta-\epsilon)/2$ *to any other* $(\hat{\tau}_t^Y)' \in [0,(1+\beta-\epsilon)/2) \cup ((1+\beta-\epsilon)/2, 1+\beta-1/\mu]$.

Lemma 6 follows directly from Lemma 5, since at $\hat{\tau}_t^Y = (1+\beta-\epsilon)/2$, the function $Z(\hat{\tau}_t^Y, 0)$ attains a maximum, and $Z(\hat{\tau}_t^Y, 0)$ is strictly increasing and continuous on $[0,(1+\beta-\epsilon)/2)$. This result is new in our extension of Hassler et al. (2007). Again, this is a consequence of the net positive spillover effect on the environment from private decisions (i.e. $\epsilon > 0$). In particular, such an equilibrium case could occur if, all else given, ϵ is sufficiently large.

Intuitively, Lemma 5 says that given regularity conditions, if the current majority voter is rich, then the dominant strategy of the decisive voter is to extract from the young the highest possible tax rate, θ; and this is consistent with an equilibrium in which a majority of rich voters emerges again next period. In contrast, because of the existence of externalities (ϵ) and therefore another possible configuration of the regularity conditions, we now also have the additional Lemma 6. Lemma 6 says that the optimal strategy for the decisive voter is to tax the current young at a unique rate $\hat{\tau}_t^Y = (1+\beta-\epsilon)/2$; and this would ensure that a majority of rich voters survives in the next period. As in Hassler et al. (2007), despite the possible non-monotonicity in $P(\hat{\tau}_t^Y)$, either Lemma 5 or Lemma 6—depending on parameters (β, ϵ, μ)—tells us that in terms of Machiavellian SMPE, we need only focus on the equivalent class of Machiavellian SMPE in which $P(\hat{\tau}_t^Y)$ is always monotone.

We are now ready to summarize all the previous results. The following proposition provides the complete characterization of a Machiavellian SMPE.

Proposition 5 (Machiavellian equilibria)**.** *Assume* $\beta \geq [2-\mu(1+\epsilon)]^2/(4\mu)$. *Then there exists a set of Machiavellian SMPE such that, for all* $t \in \mathbb{N}$, *we have:*

$$T^O(\pi_t) = \begin{cases} 0, & \text{if } \pi_t \geq 1/(2\mu) \\ 1, & \text{if } \pi_t < 1/(2\mu) \end{cases}; \tag{4.11a}$$

$$P(\hat{\tau}_t^Y) = \begin{cases} \frac{1-\hat{\tau}_t^Y+\beta}{2}, & \text{if } \hat{\tau}_t^Y \in [0,\theta] \\ = \frac{1-\hat{\tau}_t^Y}{2}, & \text{if } \hat{\tau}_t^Y \in (\theta,1] \end{cases}; \tag{4.11b}$$

and:

(M1). *If* $0 \leq 1+\beta-1/\mu < (1+\beta-\epsilon)/2$, *then,*

$$T^Y = \theta \in [\tilde{\theta}(\beta,\epsilon), 1+\beta-1/\mu]; \tag{4.11c}$$

(M2). *If* $0 \leq (1+\beta-\epsilon)/2 \leq 1+\beta-1/\mu$, *then,*

$$T^Y = \frac{1+\beta-\epsilon}{2} \in [\tilde{\theta}(\beta,\epsilon), 1+\beta-1/\mu]; \tag{4.11d}$$

Proof. The assumption $\beta \geq [2-\mu(1+\epsilon)]^2/(4\mu)$ suffices to ensure that the set $[\tilde{\theta}(\beta,\epsilon), 1+\beta-1/\mu]$ is non-empty, and that, $\tilde{\theta}(\beta,\epsilon) \leq 1+\beta-1/\mu$. Then the existence of, and characterization in (4.11a)-(4.11d) follow from Lemma 5 and Lemma 6, respectively. □

Note that $\theta \in [\tilde{\theta}(\beta,\epsilon), 1+\beta-1/\mu]$ in the $M1$ class of equilibria is indeterminate—i.e. equation (4.11c) in Proposition 5. An interpretation of the $M1$ class of equilibria is similar to that in Hassler et al. (2007). The $M1$ equilibria features a lack of coordination among voters so that there still needs to be some extraneous factor θ that helps to pin down a particular selection or equilibrium outcome. As a result, any value of θ in that interval can constitute an $M1$-equilibrium tax rate on the young. Observe, though, in our additional $M2$ class of Machiavellian equilibrium, there is no indeterminacy. The interpretation in our model is that when the marginal net externality effect ϵ is sufficiently large, one can rule out the the possibility that now, choosing some arbitrary $\hat{\tau}^Y = \theta \in [\tilde{\theta}(\beta,\epsilon), 1+\beta-1/\mu]$, is no longer a best response. This was the essence of Lemma 6. In words, this implies that a large enough externality effect, all else equal under a Machiavellian regime, helps to rule out any possibility of lack of coordination on an equilibrium.

Proposition 6 (Machiavellian SMPE outcomes)**.** *Assume $\beta \geq [2-\mu(1+\epsilon)]^2/(4\mu)$. For every $t \geq 1$ such that $\theta \in [\tilde{\theta}(\beta,\epsilon), 1+\beta-1/\mu]$, a Machiavellian SMPE outcome is given by the following:*

(M1). *If $0 \leq 1+\beta-1/\mu < (1+\beta-\epsilon)/2$, then outcomes are indeterminate and depend on θ:*

$$\hat{\tau}_t^Y = \theta; \tag{4.12a}$$

$$\tau_t^O = 0; \tag{4.12b}$$

$$\pi_t = \frac{(1-\theta+\beta)}{2}; \tag{4.12c}$$

$$E_t = \frac{\mu(\epsilon+\theta)(1-\theta+\beta)}{2}; \tag{4.12d}$$

(M2). *If $0 \leq (1+\beta-\epsilon)/2 \leq 1+\beta-1/\mu$, then outcomes are uniquely:*

$$\hat{\tau}_t^Y = \frac{1+\beta-\epsilon}{2}; \tag{4.13a}$$

$$\tau_t^O = 0; \tag{4.13b}$$

$$\pi_t = \frac{(1+\beta+\epsilon)}{4}; \tag{4.13c}$$

$$E_t = \frac{\mu(1+\beta+\epsilon)^2}{8}. \tag{4.13d}$$

The unique Machiavellian equilibrium (M2) in Proposition 5 is a new equilibrium possibility in our model (c.f. Hassler et al., 2007). The corresponding (M2) equilibrium outcome is in Proposition 6. This (M2) SMPE outcome is unique since by Lemma 6, in such a SMPE, voters find it optimal not to choose any $\hat{\tau}_t^Y \in [\tilde{\theta}(\beta,\epsilon), 1+\beta-1/\mu]$ where $\hat{\tau}_t^Y \neq (1+\beta-\epsilon)/2$, but instead they optimally choose $\hat{\tau}_t^Y = (1+\beta-\epsilon)/2$ consistent with their equilibrium belief $(\tau_{t+1}^O)^e = 0$. That is, by choosing in this equilibrium $\hat{\tau}_t^Y = (1+\beta-\epsilon)/2$, when the economy is such that ϵ is sufficiently large, the multiplicity of investment best responses, induced by multiple equilibria in the SMPE correspondence (4.9b), will never be observed as an (M2)-SMPE outcome. Finally, we collect the two possible SMPE notions in the overall result below.

Theorem 1 (Existence of SMPE)**.** *Suppose $\lambda \leq 1/\mu$.*

(1) *If $\beta < [2-\mu(1+\epsilon)]^2/(4\mu)$, then a SMPE is uniquely a non-Machiavellian equilibrium (NM) as stated in Proposition 2.*

(2) *If $\beta \geq [2-\mu(1+\epsilon)]^2/(4\mu)$, then there exists either:*
   - (a) *an infinite set of Machiavellian equilibria ($M1$ type) as stated in Proposition 5 ($M1$), or,*
   - (b) *a unique Machiavellian equilibria ($M2$ type) as stated in Proposition 5 ($M2$), or,*
   - (c) *a unique non-Machiavellian equilibrium (NM) as stated in Proposition 2.*

Suppose $\lambda > 1/\mu$, then there exists a unique equilibrium such that for all $t \in \mathbb{N}$, $T^O(\pi_t) = 1$, $T^Y = (1-\epsilon)/2$, and $P(\hat{\tau}_t^Y) = (1-\hat{\tau}_t^Y)/2$.

Observe, from Theorem 1, that the magnitude of λ relative to $1/\mu$ helps classify politico-economic equilibria but it does not affect the equilibrium outcomes directly. This is due to the fact that the voting over τ_t^O and $\hat{\tau}_t^Y$ is separable and agents' preferences over $\hat{\tau}_t^Y$ are identical across income types.

Also, note that the net tax rates on the young ($\hat{\tau}_t^Y$) in the politico-economic equilibria are lower than their respective counterparts in the same economies without the externality ($\epsilon = 0$).[16] The latter is equivalent to the economy of Hassler et al. (2007). This is because of the "double dividend" effect arising from private investment. This effect is as follows. Since there is net positive externality from more usage of the clean technology, which is positively associated with successful investments, the preferred tax on the young is lower (than if $\epsilon = 0$) to encourage more investment. The majority old voter benefits from more environmental good (and possibly pay zero tax in the event of M1 and M2 equilibria), and, the current young, in expectation, benefit from lower tax, higher expected income, and also more environmental consumption.

5. Discussion on politico-economic outcomes

In this section, we study the implications of each politico-economic equilibrium (SMPE) outcome. First, as a positive analysis, we study comparative SMPE outcomes in the allocations of average income and the environmental state, in section 5.1. Second, from a normative angle, we study comparative welfare analysis between the SMPE and the Ramsey benchmark, in section 5.2. We also look at the relative over- or under-allocation (relative to Ramsey) of the environmental public good using examples, in section 5.3. In particular, we illustrate how politico-economic distortions affect the public-provision component of this good, and how, externality effects may further twist the outcomes, relative to each SMPE's benchmark Ramsey economy.

To ensure that the results are comparable across these economies, we will require that Assumption 2 holds in all the politico-economic equilibria.[17] We fix some notation for ease of comparison. We label a corresponding Ramsey equilibrium, the Non-Machiavellian SMPE, the first case and second case of the Machiavellian SMPE as R, NM, $M1$ and $M2$, respectively. Thus, these respective equilibria (or equilibrium strategies) are denoted by σ_j; and their corresponding equilibrium outcomes are $(i_j^*, \pi_j, \bar{y}_j, \bar{E}_j)$, where $j \in \{R, NM, M1, M2\}$.

5.1. Income-environment relationship under politico-economic equilibria.

Recall that average income is given by $\bar{y}_t = \mu(i_t^* + \pi_t)/2$, and the environmental outcome is $\bar{E}_t = \mu[i_t^*(\hat{\tau}_t^Y + \epsilon) + \pi_t \tau_t^O]$. Further, in all the economies under a Ramsey equilibrium (see equation (3.7)) and all the SMPE (see equations (4.10c), (4.12c) and (4.13c)), we have outcomes for the state that are respectively, time-invariant, i.e. $\pi_t = \pi_{t+1} = i_t^* = \pi_j$, for all $t \geq 1$, where $j \in$

[16] This also tends to be true in the M1 equilibria in which $\hat{\tau}^Y = \theta \in [\tilde{\theta}(\beta,\epsilon), 1+\beta-1/\mu]$. Our economy with the externality channel admits lower realizations of θ, since $\tilde{\theta}(\beta,\epsilon) < \tilde{\theta}(\beta,0)$.

[17] Recall that Assumption 2 will not put much restriction on parameters in the Ramsey benchmark outcome. Moreover, the parameter B is irrelevant for the politico-economic equilibria.

$\{R, NM, M1, M2\}$. Therefore, the relationship between $\bar{y}_t$ and $\bar{E}_t$ depends on which class of equilibrium the economy is in.

For economies in the Non-Machiavellian SMPE (NM) or the second case of Machiavellian SMPE ($M2$), the magnitude of the externality effect of investment, ϵ, is key in determining the average income and environmental outcome as specified in Proposition 3 and Proposition 6 ($M2$). The relationships between income and environmental outcome for NM and $M2$ are stated in the following proposition.

Proposition 7. *Assume the only source of fundamental cross-country variation is the magnitude of the private externality on the environmental good, ϵ. Then the environmental outcome $\bar{E}$ is positively related with average income $\bar{y}$ in the Non-Machiavellian (NM) regime, and in the Machiavellian ($M2$) politico-economic equilibrium.*

Consider first the NM regime of SMPE. In NM, the equilibrium jumps in one period, from its initial state, to a unique steady-state outcome, $\bar{y}_{NM} = \mu(1+\epsilon)/4$, and, $\bar{E}_{NM} = \mu(1+\epsilon)(3+\epsilon)/8$. It can be shown that $\bar{y}_{NM}$ is strictly increasing in ϵ and $\bar{E}_{NM}$ is also monotonically increasing in ϵ when $\epsilon > -2$, which always holds given that $\epsilon \in (0,1)$. Therefore, the average income and environmental outcome move in the same direction when we consider cross-country variations in terms of one single parameter, ϵ, in a Non-Machiavellian SMPE. Next, consider a unique Machiavellian SMPE ($M2$). For all $t \geq 1$, we have $\bar{y}_{M2} = \mu(1+\epsilon+\beta)/4$ and $\bar{E}_{M2} = \mu(1+\beta+\epsilon)^2/8$. Given $\epsilon > 0 > -(1+\beta)$, it follows that the same qualitative relationship between average income and environment arises in $M2$, as in the Non-Machiavellian SMPE.

Remark 1. Note that NM equilibrium is more likely to occur when β is large or μ is small—i.e. $1+\beta-1/\mu < 0$, or, $\tilde{\theta}(\beta,\epsilon) > 1+\beta-1/\mu$. The $M2$ Machiavellian equilibrium is more likely to emerge when μ and ϵ are both large: $(1+\beta-\epsilon)/2 < 1+\beta-1/\mu$.

The income-environment relationship is more complicated in the first case of Machiavellian SMPE—i.e. $M1$. In this class of equilibrium, the major source of variations in average income is not ϵ, but the equilibrium tax rate on the young, θ, which is determined extraneously and exists in a continuum as specified in Proposition 6 ($M1$). However, the environmental outcome depends on both ϵ and this indeterminate tax rate, $\theta \in [\tilde{\theta}(\beta,\epsilon), 1+\beta-1\mu]$.

Proposition 8. *In the first case of Machiavellian SMPE ($M1$), the income and environmental outcome are either unrelated or negatively related.*

From Proposition 6 ($M1$), we know that: $\bar{y}_{M1} = \mu(1+\beta-\theta)/2$ and $\bar{E}_{M1} = \mu(\epsilon+\theta)(1+\beta-\theta)/2$. Consider first, fixing a tax rate $\hat{\tau}_t^Y = \theta$, and varying ϵ. We can verify that average income does not change, while environmental outcome is increasing in ϵ. In other words, average income and environmental outcome are unrelated. Second, if we fix all the parameters including ϵ, then different economies satisfying $M1$ can have different outcomes on $\hat{\tau}_t^Y = \theta \in [\tilde{\theta}(\beta,\epsilon), 1+\beta-1/\mu]$. In this experiment, average income is monotonically decreasing in θ. As $\theta < (1+\beta-\epsilon)/2$ is a condition for $M1$ to exist, the environmental outcome is increasing in θ. Therefore, when θ changes, income and environmental outcomes move in the opposite direction—i.e. they are negatively related.

Remark 2. Conditional on parameters inducing the emergence of $M1$ equilibria, the variation across economies in $M1$ depends solely on policy indeterminacy (or policy sunspots) as described in Proposition 6 ($M1$). However, if it is observed that the $M1$ regime countries have the same taxes, then intrinsic differences between them (e.g. varying ϵ) give us an unrelated relationship between income and environment.

5.1.1. *Interpretation of empirical cross-country income-environment relationship.* Suppose we were to observe the outcomes of the various equilibrium regimes from the model. The ordering of these regimes, according to their induced average-income outcomes, is ambiguous. In other words, depending on parameters, we may actually observe patterns of a positive relationship (in either the NM or $M2$ regimes) ordered before patterns of a negative relationship (in the multiple equilibria of $M1$ regimes), or even no relationship at all.

There is a parallel ambiguity in the empirical literature that attempts to estimate such a relationship. Subsequent replication of the seminal paper of Grossman and Krueger (1995), for example Harbaugh et al. (2002), found that the empirically estimated relationship between some measure of the environment and average income is sensitive to new data and econometric specification. Note that these reduce-form empirical models are estimated based on cross-country panel data. For our purposes, we can only relate to the cross-country interpretation of these estimated relationships, since our model is silent on the question of growth.[18] In particular, the empirical evidence, as in our model, suggest some positive, some negative, and some nonexistent relationship between average income and the environment across countries. Moreover, the evidence is not entirely clear on the ordering of these relationships across the domain of cross-country average incomes.

5.2. **Welfare comparison with Ramsey benchmark.** Given a strategy σ, its induced social welfare will be denoted generically as $V(\pi_0;\sigma)$. In Proposition 1, we showed that an optimal Ramsey plan σ_R^* satisfying (3.4) and (3.5), attains the maximal constrained-efficient value of the social welfare function in (3.2a) — viz. $V(\pi_0;\sigma_R^*) := \max_\sigma V(\pi_0;\sigma) = V(\pi_0)$. Given the strict concavity of the problem in each period, the optimal Ramsey plan is unique in terms of the present value of taxes. Any present value of tax rates, $\tau_t^Y(\sigma) + \beta\tau_{t+1}^O(\sigma)$, that differs from that under the optimal Ramsey plan in (3.5) is thus Pareto dominated by the Ramsey outcome, in welfare terms.

Therefore, the comparison of welfare outcomes across the Ramsey plan (σ_R) and the politico-economic equilibria (σ_{M1}, σ_{M2}, or σ_{NM}) is equivalent to comparing the present value of tax rates, i.e. $\hat{\tau}_t^Y(\sigma) + \beta\tau_{t+1}^O(\sigma)$, for all $t \geq 0$. Observe that, since $i_t^* = \pi_{t+1} = [1 + \beta - (\hat{\tau}_t^Y + \beta\tau_{t+1}^O)]/2$ is monotonically decreasing in the present value of tax rates, comparing welfare is then equivalent to comparing the optimal investment level (or the equilibrium proportion of old rich agents). If a politico-economic equilibrium can induce the same equilibrium proportion of old rich agents as the Ramsey optimal plan, then that politico-economic equilibrium is just as efficient as the Ramsey equilibrium. Any equilibrium outcome that delivers a present value of taxes differing from that of the Ramsey equilibrium's, will be Pareto dominated by the Ramsey equilibrium.

[18] Such an interpretation is valid given that the estimated relationship is a reduced functional form that does not depend on time or country, even though the estimated functional form is a projection on a family of functions that depends on panel data.

NM and M2 SMPE. It is straightforward to analyze the Non-Machiavellian equilibrium (NM) as in Proposition 2 and the additional Machiavellian equilibrium ($M2$) unique to our model, as in (4.11d) in Proposition 5. The equilibrium π_t for all $t \geq 1$ in these two sets of equilibria are constant, and respectively given by:

$$i^*_{NM} = \pi_{NM} = \frac{1+\epsilon}{4},$$

and,

$$i^*_{M2} = \pi_{M2} = \frac{1+\beta+\epsilon}{4}.$$

Recall that the corresponding proportion of the rich in the Ramsey outcome is also constant, given by:

$$i^*_R = \pi_R = \frac{B(1+\beta+\epsilon)}{4B-1};$$

where R denotes Ramsey equilibrium. It can be verified that $\pi_R > \pi_{NM}$ and $\pi_R > \pi_{M2}$, which implies that the above two politico-economic equilibrium outcomes can never Pareto dominate the Ramsey outcome. However, if $J \to \infty$, which means the government has an extreme bias towards low-productivity agents, then the Ramsey allocation approaches the one corresponding to the second case of the Machiavellian equilibria, $M2$.[19]

M1 SMPE. The welfare induced by the first case, $M1$, in (4.11c) of Proposition 5 is ambiguous as there exists a set of such equilibria. We therefore look at the "best" equilibrium outcome (in a revenue sense), $\theta = 1+\beta-1/\mu$, from the set of equilibria.[20] Then the equilibrium proportion of old rich agents turns out to be:

$$i^*_{M1} = \pi_{M1} = \frac{1}{2\mu}.$$

By setting $J = 1$, the independence of μ in the Ramsey allocation makes all the following cases possible, depending on parameters:

$$\pi_{M1} \begin{cases} < \pi_R, & \text{if } \mu > \frac{4\lambda-1}{2\lambda(1+\beta+\epsilon)}, \\ = \pi_R, & \text{if } \mu = \frac{4\lambda-1}{2\lambda(1+\beta+\epsilon)}, \\ > \pi_R, & \text{if } 1/2 < \mu < \frac{4\lambda-1}{2\lambda(1+\beta+\epsilon)} \text{ and } \beta+\epsilon < 3-1/\lambda \end{cases} . \tag{5.1}$$

Observe that it is possible for the politico-economic equilibrium to be as efficient as the Ramsey equilibrium, when $\pi_{M1} = \pi_R$, as in the second case of (5.1). However, when $\pi_{M1} > \pi_R$, as in the third case of (5.1), politico-economic equilibrium leads to over-investment relative to the Ramsey allocation. The converse is true when $\pi_{M1} < \pi_R$. That is, there is Pareto-inferior under-investment in clean technology.

Consider the case when $\pi_{M1} > \pi_R$. This can be explained either by a small μ or by a small ϵ. If μ is small, then to induce a majority of high-income agents, the $M1$ Machiavellian equilibrium will require a high level of investment. In contrast, a Ramsey planner only cares about the total discounted social welfare and disregards who is the majority in the economy.

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[19] $J \to \infty$ implies $B \to \infty$.

[20] For the old voters, $Z(\hat{\tau}^Y_t, 1)$ is increasing in $\hat{\tau}^Y_t$ under the first case in Proposition 5. Therefore, $1+\beta-1/\mu$ is the largest θ in the feasible set and gives the largest value for $Z(\hat{\tau}^Y_t, 1)$.

A small ϵ means little positive externality effect on the environment from private investment. A Ramsey planner in this case will lean towards more public provision of the environmental good, by raising the tax rates. However, in the Machiavellian equilibrium the desire to manipulate the political outcome distorts the mix between private externality on, and, public provision of, the environment outcome. As a result, political manipulation may force too much, or too little, inefficient allocation of the environmental good. We will return to this again in the following section.

5.3. **Comparisons of average income and environment.** The previous analysis has implicitly discussed the comparison of average incomes between the SMPE and each of their respective Ramsey equilibrium. This is because average income $\bar{y}$ is given by $\bar{y} = \mu\pi$ in all stationary (Ramsey or politico-economic) equilibria (see section 3.2.2). From the analyses in section 5.2, we can therefore deduce that, in most cases, each relevant Ramsey allocation generates higher average income than its relevant politico-economic equilibrium counterpart.[21]

The normative comparison of the environmental co-state, however, is complicated by the indeterminacy of individual tax rates in the Ramsey equilibrium. It is also more complicated when it comes to the first case of the Machiavellian equilibria ($M1$) where $\hat{\tau}_t^Y$ is also indeterminate. Therefore, it is analytically ambiguous whether each politico-economic or Ramsey equilibrium will deliver a higher environmental-good outcome. We thus illustrate, with three pairs of numerical examples, that either one can dominate the other.[22] In these examples, we perform comparative-statics on the key parameter that governs the private externality channel, ϵ, conditional on other parameters delivering each of the SMPE classes (i.e. the NM, $M1$ and $M2$ equilibria).[23]

Examples. Figures 2 to 4 plot public expenditure (A_t) and environmental outcome (E_t) as functions of the net tax rate on the young ($\hat{\tau}_t^Y$) given the tax rate on the old (τ_t^O), for each possible SMPE class and each corresponding Ramsey equilibrium. The vertical difference between the graphs of these two functions is the externality component of the environmental outcome, i.e. $\mu\epsilon i^*$. The circles in the figures correspond to $\bar{E}_t$ and A_t outcomes under either a Ramsey (R) or a political-economic (NM, $M1$ or $M2$) equilibrium.[24] The left panel of each figure considers a SMPE class against its corresponding Ramsey equilibrium where we have small externalities (low ϵ). The right considers the same regime and comparison, but with a higher ϵ.

First, observe that the public expenditure curve of a particular class of SMPE does not change with varying ϵ for each class of SMPE, as the tax rate on the old is fixed at either zero ($M1$ or $M2$ equilibria) or one (NM equilibrium). However, the position of the equilibrium outcome moves along the curve as the equilibrium tax rate $\hat{\tau}^Y$ varies with ϵ. Also, note that $\pi_j < \pi_R$ always

[21] However, under the third case of the Machiavellian equilibria satisfying (5.1), democratic voting can generate higher average income than a Ramsey social planner.

[22] Since this is an exercise to show that the ordering between a Ramsey allocation, and a SMPE allocation of the environmental public good can go either way, the subjectivity on what a Ramsey planner's welfare function in the OLG model is not an issue. In these illustrations, we stick to the assumptions that a Ramsey planner is one who attaches the weight β^t on each date-t generation's lifetime total payoffs, and, that the planner has no bias toward the low productivity agents, $J = 1$.

[23] Supplementary Appendix D provides the actual calculations presented in the figures below.

[24] Under Ramsey, we will consider a selection from a Ramsey optimal policy such that $E_t = \bar{E}_R$, i.e. the environmental outcome is maximal. Recall, from section 3.2.3, that this selection has no effect on optimal Ramsey welfare.

holds, for $j \in \{NM, M1, M2\}$ in these examples.[25] This implies that each Ramsey allocation generates a larger private externality effect, $\mu\epsilon i^*$, on the environmental outcome than all the corresponding SMPE do in our examples.

Second, conditional on other parameters inducing each type of politico-economic equilibrium, we have that for sufficiently high (low) degree of net private externality, ϵ, there can be an over-allocation (under-allocation) of the environment relative to a corresponding Ramsey allocation.

Explanation of examples. We first state an overall intuition for the results and then discuss the specific examples.

Under a SMPE, a myopic median voter only considers the trade off between the current old and the current young; where the welfare of the current young matters insofar as it affects the voter's own payoffs. This is in contrast to an infinite-horizon Ramsey planner who, in the model, maximizes a total present value of each generation's welfare. Therefore, as in Hassler et al. (2007), a politico-economic equilibrium distorts the tax burden borne between the young and the old. In particular, the conflict of interest between rich and poor old voters restricts the equilibrium tax rate on the old, τ^O, to be either zero or one hundred percent under majority voting. This equilibrium policy restriction on τ^O rules out a more efficient allocation that is available to a Ramsey planner, as characterised by (3.7), and thus distorting the environmental outcome as a result. Such distortions then help to shape particular resolutions of the "intensive-versus-extensive" marginal trade-offs faced by the policy maker, as outlined in section 2.6.

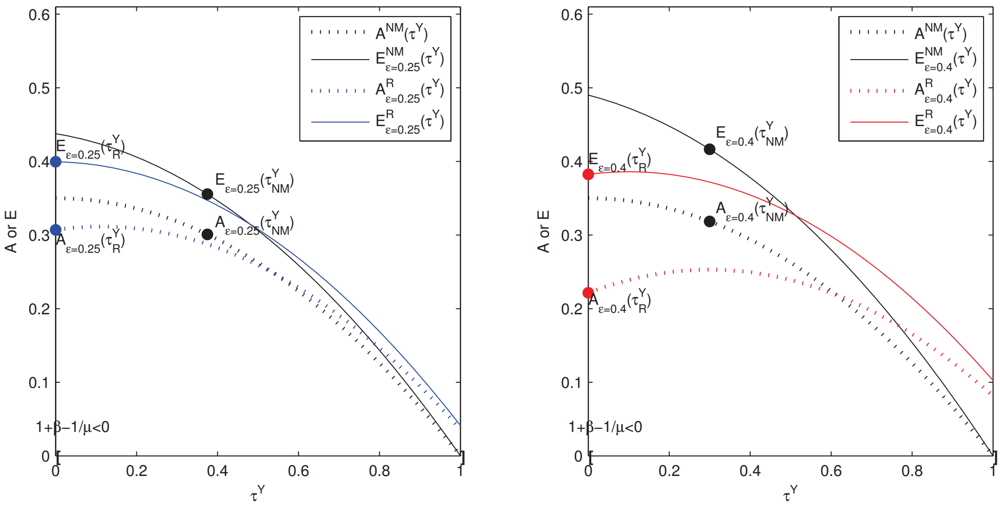
However, now the externality channel (via ϵ) also matters for the resolution of the trade-offs. Varying ϵ affects both the Laffer curve effect and the double-dividend effect. First, consider the effect of ϵ on the Laffer curve. A larger ϵ leads to a smaller present value of tax rates, thus a larger extensive margin (i.e. the tax base or the proportion of agents being taxed). However, as in the Ramsey policy (see Proposition 1) in each class of the SMPE, it is clear that the tax rates are weakly decreasing in ϵ (see Proposition 2 and Proposition 5). This implies that the intensive margin in the Laffer curve channel in this instance is weakly decreasing in ϵ, regardless of the policy regime; and the effect of a larger ϵ on the intensive margin of taxes is in the opposite direction to the effect on the extensive margin. The resolution of these extensive-versus-intensive margins will be different under each SMPE and their corresponding Ramsey equilibrium, given all the other parameters.

Second, ϵ also affects the intensive and extensive margin through the externality component of the environmental outcome (the double-dividend channel). A larger ϵ directly increases the intensive margin in this channel, or, the per unit externality effect. As the extensive margin is the same as the one in the Laffer curve channel ($i^* = \pi$), a larger ϵ also increases the extensive margin in the double-dividend channel. In summary, the effect of ϵ on the environmental co-state depends on the magnitudes of the opposing changes in both channels.

Specifically, consider first the unique Non-Machiavellian equilibrium (NM), in Examples 1 and 2, as summarized in Figure 2. When ϵ is small, the negative impact of an extremely large $\tau^O = 1$ and $\hat{\tau}^Y = (1-\epsilon)/2$ on investment, and therefore π_{NM} (i.e. the tax base or extensive margin) may be dominated by the direct gain from the intensive margin of the tax

[25] This is also true for most cases except for some parametrization in $M1$ shown in the third case of (5.1). But such a case admits a narrow range of parameters.

FIGURE 2. Examples 1 and 2 ($\mu = 0.7, \beta = \frac{1}{3}, \lambda = 1$): NM outcomes under different ϵ with corresponding Ramsey outcomes.

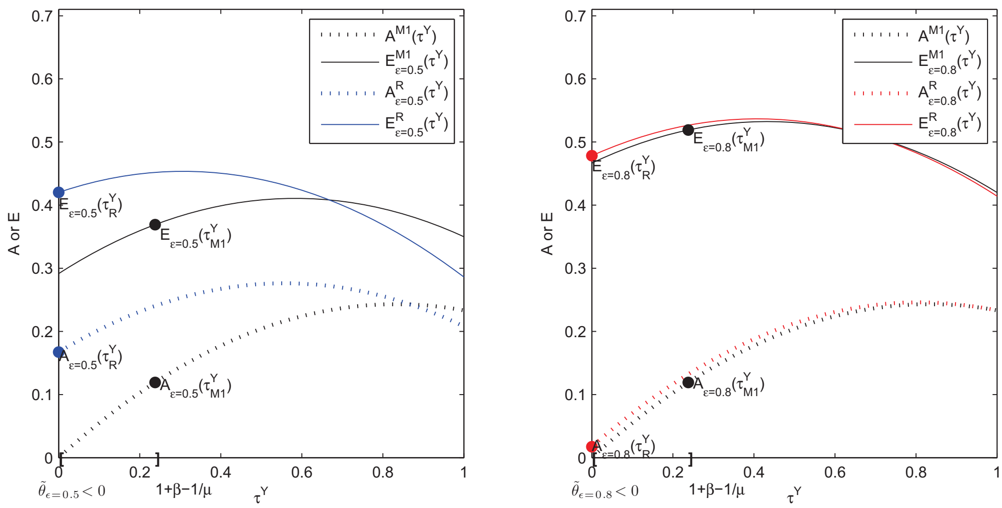


rates. Furthermore, the double dividend channel may be dominated as well. In contrast, the corresponding Ramsey planner would exploit the externality channel much more, and thus induces a higher outcome for E compared to the NM politician. However, with a large enough ϵ, all else unchanged, the direction of this dominance can be reversed, as $\tau^O = 1$ and $\hat{\tau}^Y = (1 - \epsilon)/2$—the policies' intensive margin—is falling with ϵ. Moreover, the second extensive margin through the externality effect on E become more important as ϵ becomes bigger. This further compounds the over-provision in terms of increasing the public component A_t. However, in the case of the corresponding Ramsey planners, as ϵ becomes large enough, a corresponding planner induces less public contribution in terms of A_t, so that A_t falls with ϵ. This tends to lower E under the corresponding Ramsey planner, even though the planner exploits the externality channel much more than the NM politicians.

However, in the Machiavellian equilibria ($M1$ and $M2$) as in Examples 3 - 6, the excessive tax-policies' intensive margin is from a misplacement of tax burden towards the current young agents, while τ^O is restricted to be zero. Consider Examples 3 and 4 yielding the $M1$ class of SMPE (see Figure 3). We select a particular $M1$ outcome in these examples, where the tax rate $\hat{\tau}^Y = \theta = 1 + \beta - \mu^{-1}$, so that it is independent of ϵ. By doing so, the Laffer-curve channel is explicitly shut down with respect to ϵ, as we vary ϵ. In these examples, we see that as ϵ increases, the E outcome increases purely from the intensive margin (due to a higher ϵ) in the double-dividend or the direct externality channel. There is no effect via the public contribution to E since there is no Laffer curve effect. In contrast, the corresponding Ramsey planners would induce less public contribution and rely more on the externality channel. Thus when ϵ is large enough, the $M1$ outcome for E can be over-allocated relative to Ramsey.

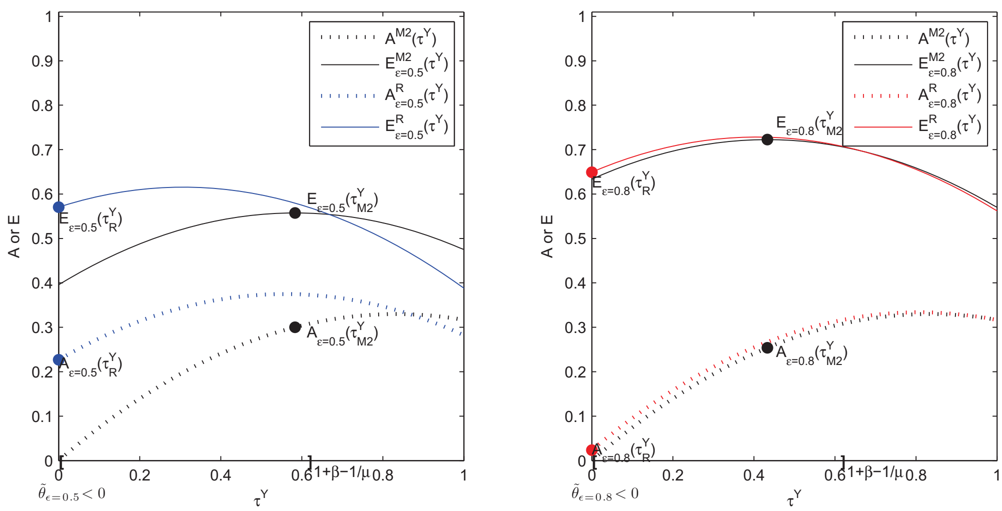
Finally, consider Examples 5 and 6, in which we have the new $M2$ class of SMPE with a unique equilibrium (see Figure 4). Here, as in the $M1$ class, equilibrium tax on the old is $\tau^O = 0$. However, the $M2$ equilibrium tax on the young, $\hat{\tau}^Y = (1 + \beta - \epsilon)/2$, is decreasing in ϵ. When ϵ is small enough, the intensive margin from taxation dominates in the $M2$ equilibrium, resulting in an over-supply of public contribution to E, relative to Ramsey. At the same time, as established earlier, the politicians exploit the externality channel less than the corresponding

FIGURE 3. Example 3 and 4 ($\mu = 0.7, \beta = \frac{2}{3}, \lambda = 1$): $M1$ outcomes under different ϵ with corresponding Ramsey outcomes.



Ramsey planner. However, for large enough ϵ, the orderings may switch. In particular, in this example, with a larger ϵ, the $M2$ politicians turn out to induce less public spending A_t on the environmental good, although relative to its corresponding Ramsey planner, this is still over supplied. The $M2$ politicians whilst still over-supplying in terms of the public contribution to E, now exploit more from the externality channel with a larger ϵ. In contrast, the corresponding Ramsey planner now induces less public spending on the environment contribution and uses the externality channel even more.

FIGURE 4. Example 5 and 6 ($\mu = 0.95, \beta = \frac{2}{3}, \lambda = 1$): $M2$ outcomes under different ϵ with corresponding Ramsey outcomes.



In other words, while the redistributive or Laffer-curve distortion from political myopia is present (as in Hassler et al., 2007), and it may result in one being on the left-side of what would be a Ramsey optimal point on the Laffer curve (with respect to the tax rate on the young), the presence of externalities (ϵ) can change that balance. In particular, our examples show that when ϵ is large enough, the myopic policy maker prefers to trade off the tax-rate intensive margin, in return for a larger policy extensive margin; and as a double dividend, a larger extensive margin of the externality channel together with a larger intensive spillover margin of ϵ. However, we also learn from these examples, that the (net) positive private externality on the environment is under-exploited in all classes of the political equilibria, relative to their Ramsey outcomes.

6. Conclusion

In this paper, we built upon the tractable dynamic politico-economic model of Hassler et al. (2007), and consider a public good that can be affected by externalities from private decisions. We showed that a class of equilibria induced by politically manipulative voters arises, when the externality channel is small, given all else equal. Also, another class in which there is no room for political manipulation and possesses a unique equilibrium can arise, either when agents are more patient, or, when young agents face a low probability of drawing high individual productivities. We showed how this externality features in our politico-economic equilibria. These equilibria nest the equilibria of Hassler et al. (2007) as special cases. In addition to these equilibria with Hassler et al. (2007) features, another new unique equilibrium with manipulative voters, can emerge when the externality effect and proportion of high-productivity young agents are sufficiently large.

The democratic voting equilibria are all weakly Pareto-dominated by the Ramsey equilibrium. This is because the political equilibria generate either too much or too little successful investment associated with the net externality effect, relative to their hypothetical planner's allocation in each respective economy. The possibility that the environmental state can be under-allocated under democratic voting relative to Ramsey allocation provides a useful interpretation of the findings in some empirical studies (see e.g. Kinda, 2010)—that democratic institutions may hurt environmental quality. Interestingly, in the model, the environmental state can also be higher than that in the Ramsey equilibrium; but this outcome can be Pareto inferior. In other words, the environment is over-allocated under such a case of democratic voting. Normatively, these results suggest that careful measurement of such externality effects should be accounted for in informing environmental policy debates.

Finally, our results can stylistically account for cross-country variations and lack of consensus in empirically estimated income-environment relationships directly. Furthermore, it is also possible that even if countries are intrinsically identical, variations can arise out of indeterminate politico-economic equilibrium outcomes. The model equilibria provide a possible interpretation for the ambiguity in the empirical relationship between average income and the environment, at least from a cross-country perspective. In particular, it emphasizes the need to account for political and/or democratic processes underlying economic and environmental policy, in reduced-form empirical regressions.

Appendix A. Omitted Proofs

A.1. **Proof of Lemma 4.**

Proof. $P : [0,1] \to [0,1]$ is given by

$$P(\hat{\tau}_t^Y) := i^*[\hat{\tau}_t^Y, (\tau_{t+1}^O)^e] = \frac{(1-\hat{\tau}_t^Y) + \beta[1-(\tau_{t+1}^O)^e]}{2}. \tag{A.1}$$

Any rational beliefs system on $(\tau_{t+1}^O)^e$ must be such that $(\tau_{t+1}^O)^e \in [0,1]$. Therefore, any rational beliefs system would imply that the fixed point $P : [0,1] \to [0,1]$ to the equilibrium functional equation (4.4) has to be such that $P(\hat{\tau}_t^Y) \in \left[(1-\hat{\tau}_t^Y)/2, (1-\hat{\tau}_t^Y+\beta)/2\right]$. However, in an SMPE, rational beliefs $(\tau_{t+1}^O)^e$ must also be consistent with equilibrium strategies, so then, by Lemma 3, $(\tau_{t+1}^O)^e = (T^O \circ P)(\hat{\tau}_t^Y) \in \{0,1\} \subset [0,1]$. Therefore, we have the following possibilities. If the current net tax on the young is too low, i.e. if $(1-\hat{\tau}_t^Y+\beta)/2 \geq 1/(2\mu)$, then $P(\hat{\tau}_t^Y) \in \left\{(1-\hat{\tau}_t^Y)/2, (1-\hat{\tau}_t^Y+\beta)/2\right\}$ since the majority rich next period will set $\tau_{t+1}^O = 0$ consistent with current voters' SMPE belief that $(T^O \circ P)(\hat{\tau}_t^Y) = 0$; and if the strict equality holds, then there are equal proportions of rich and poor voters next period, so then $P(\hat{\tau}_t^Y)$ can be either of the two points in $\left\{(1-\hat{\tau}_t^Y)/2, (1-\hat{\tau}_t^Y+\beta)/2\right\}$. Otherwise, if the current net tax on the young is too high, i.e. if $(1-\hat{\tau}_t^Y+\beta)/2 < 1/(2\mu)$, then $P(\hat{\tau}_t^Y) = (1-\hat{\tau}_t^Y)/2$, since the majority is poor next period, and they will set $\tau_{t+1}^O = 1$. This is consistent with voters' SMPE beliefs that $(T^O \circ P)(\hat{\tau}_t^Y) = 1$. $\square$

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— Supplementary Appendix —

APPENDIX B. ALTERNATIVE RAMSEY PROBLEM

In the main paper, we assume that $0 \leq b_t \leq \tau_t^Y$. We may allow, in general, for only the requirement that $b_t \geq 0$, so that the net tax on the (Y, H) agents, $\hat{\tau}_t^Y := \tau_t^Y - b_t$, may be unbounded below. The (Y, H)-agent's best response function is now:

$$i_t^* := i^*(\hat{\tau}_t^Y, \tau_{t+1}^O) = \min\left\{\frac{(1-\hat{\tau}_t^Y) + \beta\left(1-\tau_{t+1}^O\right)}{2}, 1\right\}. \tag{B.1}$$

The next section provides a model-consistent lower bound on $\hat{\tau}_t^Y$ in such a setting.

B.1. Natural lower bound on $\hat{\tau}_t^Y$.

The following result provides a natural lower bound which is state-dependent.

Lemma 7. *The natural lower bound on $\hat{\tau}_t^Y$ is given by $\underline{\hat{\tau}}_t^Y$ satisfying*

$$\underline{\hat{\tau}}_t^Y = -\tau_t^O \pi_t, \tag{B.2}$$

and $\underline{\hat{\tau}}_t^Y$ is bounded in $[-1, 0]$ for all $t \in \mathbb{N}$.

Lemma 7 is necessary, along with other feasibility conditions defined next, for ensuring that a government's planning problem is well defined.

Proof. From the government budget constraint (2.4), where $s_t = 0$, we have

$$A_t = \tau_t^O \mu \pi_t + \hat{\tau}_t^Y \mu i_t^*.$$

Let $W(\pi_t, \tau_t^O) := \tau_t^O \mu \pi_t$ —i.e the tax revenue from the current population of (O, S)-type agents. Then we can write

$$\mu i_t^* \hat{\tau}_t^Y = A_t - W(\pi_t, \tau_t^O) \geq -W(\pi_t, \tau_t^O),$$

where the last weak inequality obtains from the fact that $A_t \geq 0$. This implies that $\hat{\tau}_t^Y \geq -\tau_t^O \pi_t / i_t^*$. Since $i_t^* \equiv \min\left\{i^*(\underline{\hat{\tau}}_t^Y + \beta\tau_{t+1}^O), 1\right\} \in [0,1]$, then in any equilibrium where the net tax on the (Y, H)-type agents is minimal, i.e. when $\hat{\tau}_t^Y = \inf\{\hat{\tau}_t^Y | \hat{\tau}_t^Y \in (-\infty, 1]\} =: \underline{\hat{\tau}}_t^Y$ (or when the rebate b_t is maximal) for any given τ_{t+1}^O, the induced effort level must be at its upper bound, $i^*(\underline{\hat{\tau}}_t^Y, \tau_{t+1}^O) = 1$. Therefore, $\underline{\hat{\tau}}_t^Y = -\tau_t^O \pi_t \geq -1$, where the last weak inequality is a result of the fact that $\tau_t^O \in [0,1]$ and $\pi_t \in [0,1]$, for all $t \in \mathbb{N}$. □

Definition 6. A sequence of outcomes $\{A_t, \hat{\tau}_t^Y, \tau_{t+1}^O\}_{t=0}^\infty$ is a *feasible allocation* if it satisfies, for all $t \in \mathbb{N}$:

(1) young agents best response (B.1);
(2) the evolution of the aggregate state (2.3);
(3) government fiscal solvency (2.4); and
(4) the environmental feedback law (2.5);
(5) feasibility:

$$A_t \geq 0, \tau_{t+1}^O \in [0,1], \text{ and, } \hat{\tau}_t^Y \in [\underline{\hat{\tau}}_t^Y, 1], \tag{B.3}$$

where $\underline{\hat{\tau}}_t^Y$ satisfies (B.2).

The time-t, state-π_t-contingent feasible set of government policies is denoted as $\mathcal{F}(\pi_t) := \left\{(\hat{\tau}_t^Y, \tau_{t+1}^O) | (B.1), (2.3)\text{-}(2.5) \text{ and } (B.3) \text{ hold}\right\}$.

B.2. **Benchmark Ramsey planning problem.** The Ramsey planner maximizes a weighted sum of agents' payoffs subject to the requirement that the planner's allocation is feasible. More generally, suppose the planner may also attach a different (relative) weight $J \geq 1$ to the payoffs of agents of types (Y, L) and (O, L). That is, a larger J measures a larger bias of the planner towards low-productivity agents; and $J = 1$ implies that the planner assigns the same weight to the payoffs of all types of agents. The value to the planner beginning from a given initial state π_0 is

$$\begin{aligned} V(\pi_0) = \max_{\{\hat{\tau}_t^Y, \tau_t^O\}_{t=0}^{\infty}} \Bigg\{ \mu \left[\pi_0 V_0^{O,S} + (1-\pi_0) V_0^{O,U} \right] + (1-\mu) J V_0^{O,L} \\ + \sum_{t=0}^{\infty} \beta^t \left[\mu V_t^{Y,H} + (1-\mu) J V_t^{Y,L} \right] \Bigg| \tau_0^O \in [0,1] \text{ and} \\ \left(\hat{\tau}_t^Y, \tau_{t+1}^O \right) \in \mathcal{F}(\pi_t) \Bigg\}, \end{aligned}$$

subject to $(V_0^{O,S}, V_0^{O,U}, V_0^{O,L}, V_t^{Y,H}, V_t^{Y,L})$ satisfying (2.1a)-(2.1e). That is the planner maximizes the weighted $(J \geq 1)$ sum of all agents' lifetime payoffs, subject to the requirement of feasibility given in Definition 1.

Let $B := [\mu + (1-\mu)J]\lambda$. After some algebra, it can be shown that the Ramsey (dual) problem is equivalently given by:

$$V(\pi_0) = \mu \left\{ \max_{\tau_0^O} \left[\pi_0 \left(1 - \tau_0^O + 2B\tau_0^O \right) + \tilde{V}(\pi_0; \tau_0^O) \right] \right\}, \tag{B.4a}$$

where

$$\tilde{V}(\pi_0; \tau_0^O) = \max_{\{\hat{\tau}_t^Y, \tau_{t+1}^O\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta^t U(\hat{\tau}_t^Y, \tau_{t+1}^O) \middle| \left(\hat{\tau}_t^Y, \tau_{t+1}^O \right) \in \mathcal{F}(\pi_t) \right\}, \tag{B.4b}$$

and,

$$\begin{aligned} U(\hat{\tau}_t^Y, \tau_{t+1}^O) = \Bigg[\left(1 - \hat{\tau}_t^Y \right) + \beta \left(1 - \hat{\tau}_{t+1}^O \right) - i_t^* \\ + 2B \left(\hat{\tau}_t^Y + \epsilon + \beta \tau_{t+1}^O \right) \Bigg] i_t^*, \end{aligned} \tag{B.4c}$$

where i_t^* is defined in (2.2). Note that the only place where the state π_t may matter—i.e. making the problem dynamic—is through the natural lower bound on net tax on the young, as summarized in the set $\mathcal{F}(\pi_t)$. This becomes obvious when the problem (B.4a)-(B.4c) is written out explicitly (see Appendix C).

There are two things to note about the Ramsey planning problem. First, the constrained optimization problem in (B.4a)-(B.4c), has two parts. The first term on the right of (B.4a) is a static problem involving τ_0^O. It is independent of any continuation strategy, $\sigma_R|_{(\pi_0;\tau_0^O)} := \{\hat{\tau}_t^Y(\pi_0; \tau_0^O), \tau_{t+1}^O(\pi_0; \tau_0^O)\}_{t=0}^{\infty}$. Second, we will show that the problem beginning from any given $(\pi_0; \tau_0^O)$ summarized by $\tilde{V}(\pi_0; \tau_0^O)$, i.e. the second term on the right of (B.4a), can be guaranteed to be analytically characterized (and independent of $(\pi_0; \tau_0^O)$) under a mild assumption on primitive parameters.

We first deal with the second observation, as summarized in Lemma 8 below, and then incorporate the first observation (see Lemma 2) to arrive at a complete characterization of the optimal Ramsey policy path, σ_R^*, in Proposition 1. The problem described in (B.4a)-(B.4c) is analogous to that found in Hassler et al. (2007). However, the approach to obtain the second observation is different here, due to the assumption that $b_t \geq 0$.

Given Assumption 2, the next result in Lemma 8 says that the generic dynamic Ramsey planning problem (B.4a)-(B.4c) can be reduced to an infinite series of static and identical optimization problems, as the only state-dependent element of the generic dynamic problem—i.e. the endogenous lower bound $\underline{\hat{\tau}}_t^Y = -(\pi_t \tau_t^O)$ —is never binding.

Note that the minimal value of the lower bound on $\hat{\tau}_t^Y$ is $\inf \left(\{ \underline{\hat{\tau}}_t^Y \}_{t \in \mathbb{N}} \right) = -1$. Therefore, given (2.2)-(2.4) and the following conditions:

$$A_t \geq 0, \tau_{t+1}^O \in [0,1], \text{ and}, \hat{\tau}_t^Y \in [-1,1], \tag{B.5}$$

we can alternatively define

$$\mathcal{F}^\dagger := \left(\bigcup_{t \in \mathbb{N}} \left\{ (\hat{\tau}_t^Y, \tau_{t+1}^O) | (B.1), (2.5) - (2.4), \text{ and } (B.5) \text{ hold } \forall t \in \mathbb{N} \right\} \right),$$

which is a non-empty, *non-state-contingent* set of feasible allocations. In other words, the feasible set $\mathcal{F}^\dagger$ does not depend on the state π_t for any $t \in \mathbb{N}$.

Lemma 8. *The natural lower bound (B.2) is never binding, and, the optimal value to the Ramsey planner beginning from* any *given* $(\pi_0; \tau_0^O)$ *can be written as*

$$\tilde{V}(\pi_0; \tau_0^O) = \max_{\tau_t^Y, \tau_{t+1}^O} \left\{ \frac{U(\hat{\tau}_t^Y, \tau_{t+1}^O)}{(1-\beta)} : (\hat{\tau}_t^Y, \tau_{t+1}^O) \in \mathcal{F}^\dagger \right\}. \tag{B.6}$$

Proof. Suppose there exists some $t \in \mathbb{N}$ in which the lower bound $\underline{\hat{\tau}}_t^Y$ is binding, and therefore, the planner's problem is dynamically linked by the state π_t via $\mathcal{F}(\pi_t)$. We want to show that this results in a set of contradictions to all possible cases (describing an optimal planning outcome) induced by this assumption.

If the lower bound $\underline{\hat{\tau}}_t^Y$ is binding for some, or all $t \in \mathbb{N}$, then the problem can be written as shown in Appendix C. Denote $(\overline{\eta}_t^Y, \underline{\eta}_t^Y, \overline{\eta}_t^O, \underline{\eta}_t^O)$ as the time-t Lagrange multipliers on the respective constraints: $\hat{\tau}_t^Y \leq 1$, $\hat{\tau}_t^Y \geq \underline{\hat{\tau}}_t^Y = -\tau_t^O \pi_t$, $\tau_{t+1}^O \leq 1$ and $\tau_{t+1}^O \geq 0$. The Karush-Kuhn-Tucker (KKT) optimality conditions with respect to $\hat{\tau}_t^Y$ and τ_{t+1}^O for all $t \in \mathbb{N}$ are, respectively,

$$(1-4B)[\hat{\tau}_t^Y + \beta\tau_{t+1}^O] = (1-2B)(1+\beta) + 2B\epsilon + 2(\overline{\eta}_t^Y - \underline{\eta}_t^Y) + \beta\underline{\eta}_{t+1}^Y \tau_{t+1}^O, \tag{B.7a}$$

and,

$$\begin{aligned}(1-4B)[\hat{\tau}_t^Y + \beta\tau_{t+1}^O] = (1-2B)(1+\beta) + 2B\epsilon + 2\beta^{-1}(\overline{\eta}_t^O - \underline{\eta}_t^O) \\ - [(1+\beta) - (\hat{\tau}_t^Y + 2\beta\tau_{t+1}^O)]\underline{\eta}_{t+1}^Y.\end{aligned} \tag{B.7b}$$

Under the assumption that $\underline{\hat{\tau}}_t^Y$ is binding for some $t \in \mathbb{N}$, then it must be that $\underline{\eta}_t^Y > 0$. Given this, there are six cases to consider:

C(1). $\underline{\eta}_t^Y > 0$, with $\underline{\eta}_{t+1}^Y = 0$, and $\tau_{t+1}^O = 1$;
C(2). $\underline{\eta}_t^Y > 0$, with $\underline{\eta}_{t+1}^Y = 0$, and $\tau_{t+1}^O = 0$;
C(3). $\underline{\eta}_t^Y > 0$, with $\underline{\eta}_{t+1}^Y = 0$, and $\tau_{t+1}^O \in (0,1)$;
C(4). $\underline{\eta}_t^Y > 0$, with $\underline{\eta}_{t+1}^Y > 0$, and $\tau_{t+1}^O = 1$;
C(5). $\underline{\eta}_t^Y > 0$, with $\underline{\eta}_{t+1}^Y > 0$, and $\tau_{t+1}^O = 0$;
C(6). $\underline{\eta}_t^Y > 0$, with $\underline{\eta}_{t+1}^Y > 0$, and $\tau_{t+1}^O \in (0,1)$;

Let $i \in \{1, 2, ..., 6\}$. Define an indicator function

$$\mathbb{1}_{\{i\}} = \begin{cases} 1 & \text{if C}(i) \text{ is true} \\ 0 & \text{otherwise} \end{cases}.$$

Note that the optimality conditions $(B.7a) - (B.7b)$, under the assumptions of Cases C(1) to C(6), can then be respectively written as

$$\hat{\tau}_t^Y + \beta\tau_{t+1}^O = \frac{(1+\beta)(2B-1) - 2B\epsilon}{4B-1} + \sum_i \mathbb{1}_{\{i\}} \left(\zeta_{i,t} + \zeta_{i,t+1}\right), \tag{B.8a}$$

and

$$\hat{\tau}_t^Y + \beta\tau_{t+1}^O = \frac{(1+\beta)(2B-1) - 2B\epsilon}{4B-1} + \sum_i \mathbb{1}_{\{i\}} \left(\psi_{i,t} + \psi_{i,t+1}\right), \tag{B.8b}$$

The composite functions $(\zeta_{i,t}, \psi_{i,t})$ are such that, for example, if $i = 4$, then we have

$$\begin{aligned}
\zeta_{4,t} &= 2(4B-1)^{-1}\underline{\eta}_t^Y, \\
\zeta_{4,t+1} &= -(4B-1)^{-1}\beta\underline{\eta}_{t+1}^Y, \\
\psi_{4,t} &= -2\beta^{-1}(4B-1)^{-1}\overline{\eta}_t^O, \\
\psi_{4,t+1} &= (4B-1)^{-1}\left[(1-\beta) - \underline{\hat{\tau}}_t^Y\right]\underline{\eta}_{t+1}^Y.
\end{aligned}$$

Denote any Ramsey planner's continuation tax plan (not necessarily optimal) starting from a fixed $(\pi_0; \tau_0^O)$ as $\sigma_R(\pi_0; \tau_0^O) := \{\hat{\tau}_t^Y(\sigma_R), \tau_{t+1}^O(\sigma_R)\}_{t=0}^{\infty}$. We shall abbreviate this as σ_R. Denote an optimal strategy as σ_R^*. The induced total payoff under any σ_R is defined as

$$\begin{aligned}
v(\pi_0; \tau_0^O | \sigma_R) &= \frac{1}{4}\sum_{t=0}^{\infty} \beta^t \left\{ U(\hat{\tau}_t^Y, \tau_{t+1}^O) \right\} \\
&= \frac{1}{4}\sum_{t=0}^{\infty} \beta^t \left\{ \left[(1+\beta) - (\hat{\tau}_t^Y + \beta\tau_{t+1}^O)\right] \right. \\
&\quad \left. \times \left[\left((1+\beta) - (\hat{\tau}_t^Y + \beta\tau_{t+1}^O)\right) + 4B(\hat{\tau}_t^Y + \beta\tau_{t+1}^O + \epsilon)\right] \right\},
\end{aligned}$$

where it is understood that $\hat{\tau}_t^Y \equiv \hat{\tau}_t^Y(\sigma_R)$ and $\tau_{t+1}^O \equiv \tau_{t+1}^O(\sigma_R)$. Note that the per-period indirect utility U is quadratic in the present value of taxes, $\hat{\tau}_t^Y + \beta\tau_{t+1}^O$, and therefore, the value function $v(\cdot|\sigma_R)$ is also quadratic. Thus, $v(\pi_0; \tau_0^O|\sigma_R)$ attains a global maximum when there is a strategy $\tilde{\sigma}_R$ such that

$$\hat{\tau}_t^Y(\tilde{\sigma}_R) + \beta\tau_{t+1}^O(\tilde{\sigma}_R) = \frac{(1+\beta)(2B-1) - 2B\epsilon}{4B-1} \equiv K, \qquad \forall t \in \mathbb{N}. \tag{B.9}$$

However, we need to next check if a strategy such as $\tilde{\sigma}_R$ is feasible. Note that, since the upper bounds on $\hat{\tau}_t^Y$ and τ_{t+1}^O are both 1, then the upper bound for the present value of taxes $\hat{\tau}_t^Y + \beta\tau_{t+1}^O$ each period is $1 + \beta$. Moreover, the lower bound on the present value of taxes per period is $\underline{\hat{\tau}}_t^Y \in [-1, 0]$, by Lemma 7. Since $B > 1/2$ and $\beta \in (0, 1)$ and $\epsilon \in (0, 1)$, then we have $K < 1 + \beta$. Moreover, since $B > 1/2$, then Assumption 2 is sufficient and necessary for $K > 0$. Therefore, $K \in (0, 1 + \beta)$, implying that $\tilde{\sigma}_R$ is a feasible *and* strictly interior strategy such that it is also optimal: $v(\pi_0; \tau_0^O|\tilde{\sigma}_R) = v(\pi_0; \tau_0^O|\sigma_R^*)$.

By assumption, a strategy $\sigma_R^{C(i)}$ satisfying any of Cases C(1)-C(6), and therefore (B.8a)-(B.8b), must yield the planner an indirect utility of $v(\pi_0, \tau_0^O|\sigma_R^{C(i)}) = v(\pi_0, \tau_0^O|\sigma_R^*)$. However, note that (B.8a)-(B.8b) is always equivalent to (B.9) plus strictly non-zero terms:

$$\sum_i \mathbb{1}_{\{i\}} \left(\zeta_{i,t} + \zeta_{i,t+1}\right) \text{ or } \textstyle\sum_i \mathbb{1}_{\{i\}} \left(\psi_{i,t} + \psi_{i,t+1}\right).$$

Since the indirect utility function $v(\cdot|\sigma_R)$ is quadratic, any strategy $\sigma_R^{C(i)}$ generated by (B.8a)-(B.8b) for some $t \in \mathbb{N}$, must yield at least one per-period payoff that is strictly dominated:

$$U[\hat{\tau}_t^Y(\sigma_R^{C(i)}), \tau_{t+1}^O(\sigma_R^{C(i)})] < U[\hat{\tau}_t^Y(\tilde{\sigma}_R), \tau_{t+1}^O(\tilde{\sigma}_R)],$$

and all other per-period payoffs being at most, weakly dominated. This implies that

$$v(\pi_0, \tau_0^O|\sigma_R^{C(i)}) < v(\pi_0, \tau_0^O|\sigma_R^*) = \tilde{V}(\pi_0, \tau_0^O).$$

We have a contradiction.

□

The characterization of the solution to this more general Ramsey problem is exactly the same as in the main text, which was stated in Proposition 1. However, the optimal Ramsey tax plan has a domain which admits a larger set of tax rates, $\mathcal{F}(\pi_t) \ni (\hat{\tau}_t^Y, \tau_{t+1}^O)$, for each π_t. This is because $\hat{\tau}_t^Y$ can take on negative values admissible with respect to its natural lower bound.

Appendix C. Dynamic Ramsey Problem

The dynamic Ramsey planning problem (B.4a)-(B.4c) following $(\pi_0; \tau_0^O)$, written explicitly, is:

$$\begin{aligned}
\tilde{V}(\pi_0; \tau_0^O) = \max_{\{\hat{\tau}_t^Y, \tau_{t+1}^O\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \Big\{ & \left[(1+\beta) - (\hat{\tau}_t^Y + \beta\tau_{t+1}^O)\right] \\
& \times \left[\left((1+\beta) - (\hat{\tau}_t^Y + \beta\tau_{t+1}^O)\right) + 4B(\hat{\tau}_t^Y + \beta\tau_{t+1}^O + \epsilon)\right] \\
& + \overline{\eta}_t^Y(1 - \hat{\tau}_t^Y) + \underline{\eta}_t^Y\left[\hat{\tau}_t^Y + \tau_t^O\pi_t\right] \\
& + \overline{\eta}_t^O(1 - \tau_{t+1}^O) + \underline{\eta}_t^O\tau_{t+1}^O\Big\},
\end{aligned} \tag{C.1}$$

where

$$\pi_t = \min\left\{\frac{(1+\beta) - (\hat{\tau}_{t-1}^Y + \beta\tau_t^O)}{2}, 1\right\}.$$

Appendix D. Examples: Comparative Equilibrium Analyses

This appendix provides the actual calculations for the examples illustrated in section 5.3. The examples appear in pairs, with each pair showing that the environmental state in a Ramsey equilibrium can be either higher or lower that its corresponding class of politico-economic equilibria. We set $J = 1$ in all these examples.

Example 1. Let $\mu = 0.7, \beta = 1/3, \lambda = 1, \epsilon = 1/4$.
The Ramsey optimal policy path is: $\hat{\tau}^Y + \beta\tau^O = K = 0.28$. Let $\hat{\tau}^Y$ attain the lower bound. Then $\hat{\tau}^Y = 0$ and $\tau^O = 0.83$. The highest feasible environmental public good outcome under this Ramsey equilibrium is $\bar{E}_R \approx 0.40$. For the voting equilibrium, we need to first determine which set of equilibrium it belongs. It can be verified $1 + \beta - 1/\mu < 0$, and then it follows that the non-Machiavellian equilibrium (i.e. NM) is the emergent equilibrium. Therefore, $\hat{\tau}^Y = (1-\epsilon)/2 = 0.37$, and $\bar{E}_{NM} = \mu(1+\epsilon)(3+\epsilon)/8 \approx 0.35$. So in this example, $\bar{E}_R > \bar{E}_{NM}$.

Example 2. Let $\mu = 0.7, \beta = 1/3, \lambda = 1, \epsilon = 0.4$.
Note that we only change the size of the externality effect ϵ and keep other parameters unchanged as in Example 1. The politico-economic equilibrium still falls in non-Machiavellian

equilibrium class, but $\hat{\tau}^Y = (1-\epsilon)/2 = 0.30$. And $\bar{E}_{NM} = \mu(1+\epsilon)(3+\epsilon)/8 \approx 0.41$. The Ramsey optimal policy path changes: $\hat{\tau}^Y + \beta\tau^O = K = 0.18$. This results in setting $\hat{\tau}^Y = 0, \tau^O = 0.55$ to attain the highest feasible environmental state under Ramsey equilibrium. Then $\bar{E}_R \approx 0.38$. So in this example, $\bar{E}_{NM} > \bar{E}_R$.

In example 3 and 4, we increase β to $2/3$. This gives rise to the emergence of the first case of Machiavellian equilibria. Then we vary ϵ given all the other parameters unchanged.

Example 3. Let $\mu = 0.7, \beta = 2/3, \lambda = 1, \epsilon = 0.5$.
The Ramsey optimal policy path is: $\hat{\tau}^Y + \beta\tau^O = K = 0.22$. Let $\hat{\tau}^Y$ attain the lower bound. Then $\hat{\tau}^Y = 0$ and $\tau^O = 0.33$. The highest feasible environmental public good outcome under this Ramsey equilibrium is $\bar{E}_R \approx 0.42$. For the voting equilibrium, we need to first determine which set of equilibrium it belongs. It can be verified $\tilde{\theta} = -0.20 < 0.24 = 1+\beta-1/\mu$, and $(1+\beta-\epsilon)/2 = 0.58 > 0.24 = 1+\beta-1/\mu$, then it follows that the first case of Machiavellian equilibria (i.e. $M1$) arises. Pick the "best" equilibrium where $\hat{\tau}^Y = \theta = 1+\beta-1/\mu = 0.24$.[26] Therefore, $\bar{E}_{M1} = (1+\beta+\epsilon-1/\mu)/2 \approx 0.37$. So in this example, $\bar{E}_R > \bar{E}_{M1}$.

Example 4. Let $\mu = 0.7, \beta = 2/3, \lambda = 1, \epsilon = 0.8$.
Note that we only change the value of ϵ compared to Example 3. The politico-economic equilibria regime does not change. Since β and μ are unchanged, the tax rate on the young does not change and the environmental outcome of the "best" equilibrium from $M1$ becomes: $\bar{E}_{M1} = (1+\beta+\epsilon-1/\mu)/2 \approx 0.52$. However, the Ramsey optimal policy path changes: $\hat{\tau}^Y + \beta\tau^O = K = 0.02$. This results in setting $\hat{\tau}^Y = 0, \tau^O = 0.03$ to attain the highest feasible environmental state under Ramsey equilibrium. Then $\bar{E}_R \approx 0.48$. So in this example, $\bar{E}_{M1} > \bar{E}_R$.

In example 5 and 6, we increase μ to 0.95 while keep β and λ to be the same as example 3 and 4. This gives rise to the emergence of the second case of Machiavellian equilibria. We vary ϵ given all the other parameters unchanged.

Example 5. Let $\mu = 0.95, \beta = 2/3, \lambda = 1, \epsilon = 0.5$.
The Ramsey optimal policy path is: $\hat{\tau}^Y + \beta\tau^O = K = 0.22$. Let $\hat{\tau}^Y$ attain the lower bound. Then $\hat{\tau}^Y = 0$ and $\tau^O = 0.33$. The highest feasible environmental public good outcome under this Ramsey equilibrium is $\bar{E}_R \approx 0.57$. For the voting equilibrium, we need to first determine which set of equilibrium it belongs. It can be verified that $\tilde{\theta} < 0$, and, $1+\beta-1/\mu = 0.61 > 0$; hence $\tilde{\theta} < 1+\beta-1/\mu$. And $(1+\beta-\epsilon)/2 = 0.58 < 1+\beta-1/\mu$ then it follows that the second case of Machiavellian equilibria (i.e. $M2$) emerges. Therefore, $\hat{\tau}^Y = (1+\beta-\epsilon)/2 = 0.58$, and $\bar{E}_{M2} = \mu(1+\epsilon+\beta)^2/8 \approx 0.56$. So in this example, $\bar{E}_R > \bar{E}_{M2}$.

Example 6. Let $\mu = 0.95, \beta = 2/3, \lambda = 1, \epsilon = 0.8$.
Note that we only change the value of ϵ and keep other parameters unchanged as in Example 5. The politico-economic equilibrium regime does not change, but $\hat{\tau}^Y = (1+\beta-\epsilon)/2 = 0.43$, and $\bar{E}_{M2} = \mu(1+\epsilon+\beta)^2/8 \approx 0.72$. But the Ramsey optimal policy path changes: $\hat{\tau}^Y + \beta\tau^O = K = 0.02$. This results in setting $\hat{\tau}^Y = 0, \tau^O = 0.03$ to attain the highest feasible environmental state under Ramsey equilibrium. Then $\bar{E}_R \approx 0.65$. So in this example, $\bar{E}_{M2} > \bar{E}_R$.

[26] The "best" equilibrium outcome means the highest payoff that the median voter can attain in the continuum of $M1$ equilibria.