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# Dress to Impress: Brands as Status Symbols 

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# Dress to Impress: Brands as Status Symbols* 

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#### Abstract

We analyzed the market for indivisible, pure status goods. Firms produce and sell different brands of pure status goods to a population that is willing to signal individual abilities to potential matches in another population. Individual status is determined by the most expensive status good one has. There is a stratified equilibrium with a finite number of brands. Under constant tax rates, a monopoly sells different brands to social classes of equal measure, while in contestable markets, social classes have decreasing measures. Under optimal taxation, contestable markets have progressive tax rates, while a monopoly faces an adequate flat tax rate to all brands. In contrast with the literature, subsidies may be socially optimal, depending on the parameters, in both market structures.


Keywords: brand, competition, free entry, matching, monopoly, signaling, status, tax, welfare.

JEL classification: C78, H23, L12, L15.

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## 1 Introduction

Consumers strategically purchase particular kinds of goods to signal their social status. The literature has extensively covered the demand for these status goods, their impact on economic growth, and the policies used to correct for externalities. More recently, researchers have incorporated more details of the supply of status goods. ${ }^{1}$ These authors have created models where fully separating equilibrium is attained with perfectly customized status goods in which those signaling their status reveal all information. This full customization can either be in the quality space, as in Board (2008), or in the quantity space, as in Cole et al. (1992) and Hopkins and Kornienko (2005).

However, full customization is usually not found in reality. Typically, markets offer to consumers indivisible status goods with pre-specified qualities. For instance, a Rolex Daytona Cosmograph $40-\mathrm{mm}$ steel watch costs $\$ 13,450.00$. The next model, the Rolex Daytona Cosmograph 40-mm yellow gold and steel watch, is $11.5 \%$ more expensive. ${ }^{2}$ Some automobile models having the same basic design and features are offered on different brands by the same firm. ${ }^{3}$

Brand names have always been attached to status. Advertisement campaigns of brands such as Audi, BMW, Rolex, Cartier and Giorgio Armani frequently refer to how society perceives an individual who owns their products. The literature on marketing has recognized social status as a key feature in the market for luxury goods. ${ }^{4}$

The first step in understanding the relationship between brand creation and status is the analysis of the case of pure status goods. Pure status goods generate no direct utility to their owners. Jewelry and fashion accessories are examples of products that resemble pure status goods. These industries' businesses trade substantial amounts of money. ${ }^{5}$ A person looking for ways to impress others may find it valuable to wear diamond rings. Impressing others can help to establish contacts for future partnerships in many areas, including business, marriage, and politics.

[^1]This paper models the demand and supply of pure status goods to a population, the Greens. Each Green wishes to signal his ability to the population of Reds. We find the number of brands available to Greens and their respective prices under monopoly and in a contestable market. Regardless of the market structure, prices, quantity of different brands of status goods available and social strata locations are determined endogenously.

There is a fixed cost for developing each new brand. Full customization is not attained in equilibrium. Instead, firms offer a finite number of brands. A social norm (convention) ranks brands according to their status levels. The status of an individual is determined by the brand of highest status level that he owns. Hence, no individual purchases more than one status good. All Green individuals who buy a particular brand of status good are pooled together, forming the corresponding social stratum. Hence, private information is never completely revealed. If there is just one brand, Greens are divided in two classes: haves and have nots. With two brands, there are three classes, and so on, as in Bagwell and Bernheim (1996), and Burdett and Coles (1997).

Reds and Greens have complementary abilities. So, positive assortative matching is the most efficient outcome. Status goods can purvey useful information by improving the accuracy of the signaling and matching processes (the accuracy of signals increases with the number of brands). Thus, conspicuous consumption is not a pure waste as in Frank (1985), Ireland (1994, 2001), and Rege (2008).

To the best of our knowledge, our paper is the first to point out the role of fixed costs in brand creation and status goods market equilibrium. Here, fixed costs prevent complete separation/customization, and there is a stratified equilibrium. Fixed costs are deeply related to social stratification: for any exogenous tax schedule, when the development cost of an extra brand increases, the matching process is less efficient because the number of brands offered decreases. As a consequence, larger brand development costs lead to equilibria that are "closer" to the completely random matching between the two populations.

Regardless of the market structure, an increase in tax rates tends to decrease the number of brands available, and we approach the completely random matching between Greens and Reds. In general, the model explains how the market structure and the tax policy affect prices, the number of status goods brands offered, the efficiency of the matching process, and, consequently, the social welfare.

When facing no taxes or flat tax rates, a monopoly offers a finite spectrum of brands,
each brand serves a market niche, and all niches have equal measure. Serving social strata of equal measure is a socially desirable result. ${ }^{6}$ However, there may exist too many or too few social strata. The profit-maximizing monopoly may choose to provide a number of brands that is larger or smaller than the socially optimal since it takes into account only the behavior of consumers, the Greens, disregarding the externalities to Reds. If the externality on the Reds is sufficiently low (high), the equilibrium number of brands is larger (smaller) than what would maximize the social welfare. Under a monopoly, the socially optimal policy is an adequate flat tax rate that leads the firm to provide the socially optimal number of brands.

In contestable markets, firms compete for market niches. The assumptions of free entry and free exit drive industry profits to zero because firms fear the entry of new competitors. This possibility is guaranteed by the assumption that incumbents and potential entrants have access to identical technologies. There is no closed-form solution to the number of brands and measures of strata. However, there is a recursive algorithm to find these values for any set of parameters. Under any flat tax schedule, higher strata have smaller measures than lower strata. In the absence of taxes, there might be overconsumption or underconsumption of status goods; that is, the number of brands may not be socially optimal. In contestable markets, welfare maximization implies progressive tax rates.

In the absence of taxes or subsidies for status goods, a monopoly might yield an outcome that is socially preferable to a contestable market or vice-versa, depending on the parameters. We evaluated the intervention of a benevolent government that maximizes the utilitarian welfare by applying a Pigouvian consumption tax to status goods. ${ }^{7}$ The optimal tax policy depends on the industry configuration, and it equalizes the marginal social losses (from fixed costs of brand creation) and gains (from increased matching efficiency due to complementarity in matching payoffs). It may be the case that some (or even all) brands are subsidized. Indeed, in both market structures, optimal taxes on status goods can be positive, zero or negative, depending on the externalities that status goods generate to the Red population. These externalities depend on how the aggregate matching output is shared within the matched pair. If the share of the Reds is increased, then the externalities imposed

[^2]on Reds also increase. If externalities are relatively high, optimal taxes can be zero or even negative, while relatively low externalities imply in high taxes. This is in contrast to a long literature on conspicuous consumption as a negative externality.

Since the publication of Veblen's work in 1899, scholars have tried to explain the role of status in human relations. Duesenberry (1949) and Pollack (1976) created the notion of positional goods, in which the utility of an individual depends of its consumption relative to the consumption of the same good enjoyed by others. ${ }^{8}$ Cole et al. (1992, 1995), Burdett and Coles (1997), Postlewaite (1998), and Hopkins and Kornienko (2004, 2005, 2009) pointed out that conspicuous consumption can be used as signals in matching markets, and that the market for such goods will resemble those of positional goods. The signals ensure the implementation of the most efficient matching, which is positive assortative. However, if matching concerns were not present, consumers would not engage in conspicuous consumption at all; thus, in these authors' view, conspicuous consumption constitutes a wasteful activity. Becker et al. (2005) showed that status concerns affect consumers' risk-taking behavior, generating an optimal income distribution. These authors focused on the demand side only.

Pesendorfer (1995) and Bagwell and Bernheim (1996) pioneered the study of the supply of status goods. Pesendorfer showed how fashion cycles can be related to social stratification. Bagwell and Bernheim show how social stratification is related to conspicuous consumption. They considered a competitive market where firms offer different quality goods that have both signaling and intrinsic values, finding conditions under which status goods would be sold above marginal cost (Veblen effects). In their model, quality is given exogenously. Diaz et al. (2008) and Rayo (forthcoming) created models of status goods with endogenous quality. Diaz et al. show that a monopolist typically oversupplies quality because of its signal value. This overprovision of quality is higher for the most expensive varieties of the status good, making conspicuous consumption higher for wealthier individuals. Rayo (forthcoming) shows that a monopolistic status good producer might decide not to offer a full spectrum of quality choices if price discrimination in the vicinity of a particular point is not profitable, leading to local pooling of consumers at a particular quality level. He argues that firms can strategically pool consumers together by creating "gaps" in the spectrum of varieties offered to extract higher information rents. We propose an alternative explanation for the absence

[^3]of full separation in equilibrium: the presence of fixed costs.
We contribute to this literature by showing that: (1) The number of options available to consumers can be endogenous; (2) Full customization is not typically obtained when fixed costs are present, even at a local level; and (3) Technology parameters and market structure might influence not only the number of brands available, but also their prices, targeted consumer population and, ultimately, social welfare. While both our paper and Rayo (forthcoming) are concerned with the effects of status in the variety of goods provided, we differ in a fundamental way: the technological hypotheses are quite different. In our model, fixed costs rather than price discrimination drives the stratification result. Unlike in Rayo, there are no intervals where consumers are completely separated. We also evaluate the impact of tax policy on social welfare. Due to differences in our technology hypotheses, our conclusions regarding public policies and social welfare differ significantly.

Our contribution is also related to Damiano and Li (2007, 2008). They show that monopolistic status good firms can provide full separation if a modified match value function exhibits complementarity in types. There is no simultaneous game equilibrium in a duopolistic network market, but there are sequential game equilibria with market specialization. Our approach allows us to obtain an equilibrium with multiple providers of status good, as long as markets are contestable. Our work also relates to Board (2009), in the sense that local overstratification might occur in our model when multiple firms supply the status good market.

Section 2 describes the model. Section 3 discusses the demand for status goods. Section 4 computes socially optimal allocations. Section 5 studies the monopolistic market for status goods. Section 6 analyzes contestable markets. Section 7 investigates the effects of taxation on these markets, and the last section concludes. Appendix $A$ contains all proofs. Appendix $B$ presents the analogous results when the matching output displays Leontief technology.

## 2 Model Setup

There are two populations of equal unitary measure: Greens and Reds. There is a unique consumption good traded at unit price (numéraire). Every Green has an initial endowment, denoted $y$, while every Red has an initial endowment of zero. Greens are indexed by $i \in$ $G=[0,1]$ and Reds by $j \in R=[0,1]$. These indices reflect individual abilities, and, in each
population, the abilities are uniformly distributed. Reds' abilities are perfectly observable. The ability of each Green is his private information, but the distribution of Greens' abilities is common knowledge to all players. ${ }^{9}$

In addition to the consumption good, there are also pure status goods; that is, goods that generate no direct benefit. By hypothesis, each status good is indivisible. Each Green may buy status goods from different brands to signal his individual ability to prospective partners. A social norm characterizes the status level of each brand; that is, there is a linear (i.e., complete and strict) order for the different brands of status goods. When there are $N$ brands available, each Green with ability $i \in G$ has a status level denoted by $s(i) \in\{0,1, \cdots, N\}$. The status goods that each Green owns are perfectly observable. A social norm establishes that the social status of each Green is equal to the maximum status level of the brands of status goods that he owns. Hence, each Green gains no additional status from buying multiple units of the same brand or by purchasing status goods from brands with lower status levels than the one that he already owns.

If a Green does not buy any status good, then his status level is denoted $s(i)=0$. We describe this situation as if he had bought the brand $n=0$ at price $p_{0}=0$. Let $p_{n}$ denote the price of the status good providing status level $n$. Given the prices $p_{n}$, for $n \in\{0,1, \cdots, N\}$, each Green decides which brand $n$ to purchase, if any. This leaves him with $x=y+T-\left(1+\tau_{n}\right) p_{n}$ units of the consumption good, where $T \in \mathbb{R}$ is a lump-sum transfer he receives, $\tau_{n} \in[\underline{\tau}, \bar{\tau}]$ is the tax rate, charged by the government, of brand $n$, and $\underline{\tau}$ and $\bar{\tau}$ are exogenous constants such that $\underline{\tau}>-1$ and $\bar{\tau}$ is larger than the socially optimal tax rates $\widehat{\tau}$ and $\widehat{\hat{\tau}}_{N^{*} .}{ }^{10}$ The government redistributes the tax revenue to Greens via the lump-sum transfers $T$ to achieve a balanced budget. The government does not tax the consumption good.

Every agent of each population matches exactly one individual from the other population to jointly produce more of the consumption good. The joint production function, denoted

[^4]$z(i, j)$, is Cobb-Douglas (multiplicative); namely $z(i, j)=i j$. Multiplicative technologies are analytically convenient because we can prove (see Proposition 1 and Appendix) that the demand for brand $n_{1}$ does not depend on the price of any brand $n_{2}$ such that $n_{2}<n_{1} .{ }^{11}$

Let $\phi>0$ be an exogenous constant measuring the benefits to Reds from the joint production. When matched, a Green of ability $i$ and a Red of ability $j$ jointly produce $(1+\phi) z(i, j)$ units of the consumption good and share them in the fixed proportion $1: \phi$. Because the abilities of Greens and Reds are gross complements in the joint production function, the socially optimal matching is positively assortative, as in Becker (1973, 1974).

All agents maximize their individual utilities. Every Green with ability $i \in G$ has a quasilinear utility function given by $U(x, i, j)=x+z(i, j)$, where $x$ represents the quantity of consumption good that he consumes outside of matching, $j \in R$ represents the ability of his match, and $z(i, j)$ represents the amount of consumption good that he takes from the joint production with his partner. Because of the quasilinearity in Greens' utilities, consumption of status goods is not a function of the initial endowment. Thus, the utilitarian welfare function grows linearly in $y .{ }^{12}$ When matched with a Green of ability $i$, the utility of a Red individual of ability $j$ is $\phi z(i, j)$.

New brands of status goods can be created at cost $c>0$, exogenously given, called the development cost, and interpreted as a sunk cost that is necessary to install a production plant or register a new brand or patent. Once a firm creates a brand, this firm can produce as many units as it pleases without any additional cost. The zero marginal cost hypothesis is fitting because we are modeling pure status goods. We can think of a pure status good as a tag with the brand's logotype that is produced at an arbitrarily low marginal cost. The largest cost component comes from developing the brand's name and logotype and carving a market niche.

A matching between Greens and Reds is a bijective, measure preserving function $m$ : $G \rightarrow R$. A matching is weakly stable if no agent has a profitable deviation, given his/her information. We focus only on stratified equilibria. In this kind of equilibrium, there are endogenous variables $i_{n}$, with $0=i_{0}<\cdots<i_{N+1}=1$, named strata limit abilities. Every Green with an ability in the interval $\left[i_{n}, i_{n+1}\right) \subset G$ buys exactly one unit of a status good

[^5]of brand $n$, and matches a Red having an ability randomly drawn from the corresponding interval of the Red population; i.e., $\left[i_{n}, i_{n+1}\right) \subset R$. In other words, matches are random inside each corresponding stratum of the two populations.

Why are the matchings in stratified equilibria always weakly stable? Consider a Red in the top stratum, $N$. Her expected utility in the stratified equilibrium is larger than the expected utility from matching a Green of any other status level $n \in\{0,1, \cdots, N-1\}$. Switching to another Green in stratum $N$ does not increase her expected utility. Greens in the top stratum cannot do better than signaling as they do in equilibrium. Given the random matching in stratum $N$, we can continue this recursive argument in stratum $N-1$, then in stratum $N-2$, and so on, until we reach stratum $n=0$.

## 3 Demand for Status Goods

Assume that every Green has a large initial endowment. More precisely, assume from now on that:

$$
\begin{equation*}
y>1+\frac{\bar{\tau}}{1+\underline{\tau}} . \tag{1}
\end{equation*}
$$

Then, every Green can afford to buy at least one unit of any brand of status good.
Lemma 1 Suppose that $\tau_{n} \in[\underline{\tau}, \bar{\tau}]$ for every $n \in\{1,2, \cdots, N\}$ and that the initial endowment of each Green agent satisfies inequality (1). Then, every Green enjoys a positive amount of the consumption good even if she buys one unit of the most expensive brand of status good.

Let $E_{i}[\cdot]$ denote the expectation operator of a Green with ability $i$ conditional on the information available to him. Assume that Greens behave competitively (i.e., no coalitions are formed), and take as given the transfers $T$, the number $N$ of status brands, all prices $p_{n}$ and tax rates $\tau_{n}$, for $n \in\{0,1, \cdots, N\}$. Define $\tau_{0}=0, i_{0}=0, p_{0}=0$, and $i_{N+1}=1$. Each Green with ability $i \in G$ solves the problem:

$$
\max _{n \in\{0,1, \cdots, N\}} x+E_{i}[\operatorname{im}(i) \mid s(i)=n] \quad \text { such that } \quad x \leq y+T-p_{n}\left(1+\tau_{n}\right)
$$

The sequence of $i_{n}$ 's define the social strata. By construction, Green individuals with abilities equal to $i=i_{n}$, for some $n \in\{1, \cdots, N\}$, are indifferent between brands $n$ and $n-1$. For the sake of simplicity, in this case (tie), assume that the agent chooses the higher status brand.

Lemma 2 A Green individual with ability $i$ in the interior of any interval $\left(i_{n}, i_{n+1}\right), n \in$ $\{0,1, \cdots, N\}$ strictly prefers brand $n$ to all others.

An argument similar to the one in the proof of Lemma 2 can establish that any Green individual with ability $i=i_{n}$, for some $n \in\{1, \cdots, N\}$, prefers brand $n$ and brand $n-1$ to any other brand $n^{\prime} \notin\{n, n-1\}$.

Definition 1 For fixed prices $p_{n}, n \in\{1, \cdots, N\}$, we say that the gap conditions hold if and only if

$$
0<i_{1}<i_{2}<\cdots<i_{N}<1
$$

Instead of using the prices $p_{n}$ directly, it is convenient to work with adjusted prices $\alpha_{n}$, defined as

$$
\begin{equation*}
\alpha_{n}=2\left(1+\tau_{n}\right) p_{n}, \quad \forall n \in\{1, \cdots, N\} . \tag{2}
\end{equation*}
$$

The gap conditions must hold in any stratified equilibrium; otherwise, at least one brand of status goods would have no demand, and then the value of $N$ adjusts itself to conform to the number of brands that are demanded by a positive measure of agents. As a consequence, adjusted prices of consecutive brands $\alpha_{n}$ and $\alpha_{n+1}$, must not be "too close" to each other, as illustrated by Example 1. In particular, it is possible to find $p_{n}>p_{n+1}$ as long as $\alpha_{n+1}-\alpha_{n}$ is positive and sufficiently large, so that $i_{n}<i_{n+1}$. Proposition 1 establishes a bijective relation between prices (or adjusted prices) and strata limit abilities.

Proposition 1 (Inverse Demand for Status Goods) For every $n \in\{0,1,2, \cdots, N\}$, the strata limit abilities and inverse demand for status goods of brand $n$ are given by: $i_{N}=\alpha_{N}$, $i_{N-1}=\alpha_{N-1} / \alpha_{N}$,

$$
i_{n}=\left\{\begin{array}{cc}
\left(\alpha_{N} \alpha_{N-2} \alpha_{N-4} \cdots \alpha_{n}\right) /\left(\alpha_{N-1} \alpha_{N-3} \cdots \alpha_{n+1}\right), & \text { if } N-n \text { is even } ; \\
\left(\alpha_{N-1} \alpha_{N-3} \alpha_{N-5} \cdots \alpha_{n}\right) /\left(\alpha_{N} \alpha_{N-2} \cdots \alpha_{n+1}\right), & \text { if } N-n \text { is odd, }  \tag{4}\\
\alpha_{n}=i_{n} i_{n+1},
\end{array}\right.
$$

and

$$
\begin{equation*}
p_{n}=\frac{i_{n} i_{n+1}}{2\left(1+\tau_{n}\right)} . \tag{5}
\end{equation*}
$$

In particular, the demand for brand $n_{1}$, namely $i_{n_{1}+1}-i_{n_{1}}$, does not depend on adjusted prices $\alpha_{n_{2}}$ when $n_{2}<n_{1} .{ }^{13}$

[^6]Example 1 Suppose that $N=2$ and $\tau_{n} \equiv 0$, for every $n \in\{1,2\}$. By Proposition 1, the strata limit abilities $i_{1}$ and $i_{2}$ satisfy the following:

$$
\left\{\begin{array}{c}
i_{1} i_{2}=2 p_{1}, \\
i_{2}=2 p_{2} .
\end{array}\right.
$$

Assume that $p_{2}$ is chosen. Price $p_{1}$ cannot be set arbitrarily close to $p_{2}$; otherwise, the demand for $n=1$ vanishes because there is a discrete increment in $z(i, j)$ for Green agents that switch their status level from $s(i)=1$ to $s(i)=2$, but only a small sacrifice of resources ( $p_{2}-p_{1}$ ). For instance, if $p_{2}=0.4$, then $i_{2}=0.8$. Because we must have $i_{1}<i_{2}$, then $2 p_{1}=i_{1} i_{2}<\left(i_{2}\right)^{2}=0.64$, which implies that $p_{1}<0.32$, and $p_{2}-p_{1}>0.08$. If $p_{2}=0.4$ and $p_{1} \geq 0.32$, then no Green individual buys brand $n=1$. For both brands to exist there must be a positively measured price gap. In general, the exact gap conditions are in Remark 2 of Appendix A.

Price $p_{1}$ does not affect the demand for the brand $n=2$, which is equal to $1-i_{2}$, because $1-i_{2}=1-2 p_{2}$. If $p_{2}=0.4$ and $p_{1}=0.1$, then $i_{1}=0.25, i_{2}=0.8$, and the average ability of consumers of brand $n=1$ is $(0.25+0.8) / 2=0.525$. The Green agent with ability $i=0.8$ continues to be indifferent between the two available brands if $p_{1}$ has an infinitesimal increment while $p_{2}$ remains fixed. Why? On one hand, the Green with ability $i=0.8$ is tempted to buy the cheapest brand because now the average ability of its consumers is slightly above 0.525 . On the other hand, the price of brand $n=1$ is higher after the increment in $p_{1}$. These two opposite effects exactly balance each other for the Green with ability $i=0.8$.

## 4 Welfare

This section considers a benevolent social planner who decides each individual's allocation of goods, which brands of status goods and how many units to produce to maximize the utilitarian welfare, denoted by $W$. The social planner weights equally the utility of each individual in the economy, and, aside from assigning status goods, she does not interfere on the matching process.

The utilitarian welfare is the integral of the utilities of all Greens and all Reds plus the aggregate profit. Hence:

$$
\begin{equation*}
W=\sum_{n=0}^{N}\left\{\int_{i_{n}}^{i_{n+1}} y+E_{i}\left[i m^{*}(i) \mid s(i)\right] d i\right\}+\sum_{n=0}^{N}\left\{\int_{i_{n}}^{i_{n+1}} E_{i}\left[\phi i m^{*}(i) \mid s(i)\right] d i\right\}-c N . \tag{6}
\end{equation*}
$$

The first term on the welfare function indicates the utility of Greens after the expenditure with status goods is returned to them. The second term represents the utility of Reds. Although Reds do not make any strategic decision, they benefit, on average, from finer social strata because $z(i, j)=i j$ exhibits complementarity ( $z$ is a supermodular function), and, therefore, high-ability Reds benefit the most from having high-ability partners. Positive assortative matching is the most efficient one. With finer partitions of ability spaces, the implemented matching "gets closer" to the positive assortative matching, with randomization occurring only within intervals of smaller measure. Thus, there is a positive externality from Green's choice of status goods on Reds' utility.

Let $W^{*}$ denote the maximized value of $W$. Socially optimal allocations solve the following maximization problem:

$$
\begin{equation*}
W^{*}=\max _{N \in \mathbb{R}_{+}}\left\{\max _{i_{1}<\cdots<i_{N}} \sum_{n=0}^{N} \int_{i_{n}}^{i_{n+1}} y+(1+\phi) E_{i}\left[i m^{*}(i) \mid s(i)\right] d i-c N\right\} . \tag{7}
\end{equation*}
$$

Maximization problem (7) is solved in two steps. First, we fix the value $N$ and solve the problem inside the brackets, choosing the strata limit abilities $i_{n}^{*}$. Then, we choose the socially optimal number of brands, denoted $N^{*}$. Let $i_{n}^{*}$ represent the strata limit abilities that maximizes the welfare function. Proposition 2 calculates the socially optimal strata limit abilities $i_{n}^{*}$ and the socially optimal number of brands $N^{*}$. The value $N^{*}$ equalizes the marginal social gain from having an extra brand with its marginal social cost. The marginal social gain arises from improving the efficiency of the matching process by improving the informativeness of the signals. The marginal social cost of each extra brand is its development cost $c$.

To obtain $N^{*}>0$, from now on, assume that $c \leq 1 / 6$. Hence, for every $\phi>0$, it is always true that $[(1+\phi) /(6 c)]^{1 / 3}-1>0$. Thus, the function $W(N)$, defined in equation (6), has a unique maximum such that $N^{*}>0$ and $W^{\prime}\left(N^{*}\right)=0$. In fact, function $W(N)$ is smooth, concave, $W^{\prime}(0)>0$, and $W^{\prime}(N)<0$ for sufficiently large values of $N$.

Proposition 2 (Socially Optimal Number of Brands) Suppose that $c \leq 1 / 6$.
(a) The socially optimal strata are of equal length. For every $n \in\{1,2, \cdots, N+1\}$ :

$$
i_{n}^{*}-i_{n-1}^{*}=\frac{1}{N+1} .
$$

(b) The socially optimal number of brands is given by: ${ }^{14}$

$$
\begin{equation*}
N^{*}=\left(\frac{1+\phi}{6 c}\right)^{1 / 3}-1 \tag{8}
\end{equation*}
$$

(c) The maximized welfare is larger than welfare in random matching (i.e., when no status goods is available).

Under the socially optimal allocation, the measure of Greens that do not buy any status good is equal to:

$$
i_{1}^{*}=\frac{1}{N^{*}+1}=\left(\frac{6 c}{1+\phi}\right)^{1 / 3}
$$

This quantity increases with the cost $c$ and decreases with $\phi$.
By how much the stratified equilibrium improves upon a market with no status goods? Let $W_{0}$ denote the welfare if there are no status goods. Then:

$$
W^{*}-W_{0}=\frac{(1+\phi) N^{*}\left(N^{*}+2\right)}{12\left(N^{*}+1\right)^{2}}-c N^{*}=c+\frac{1+\phi}{12}-\frac{3}{2}\left(\frac{(1+\phi) c^{2}}{6}\right)^{1 / 3} .
$$

Since $\phi>0$, the usual hypothesis $c \leq 1 / 6$ implies that $c \leq(1+\phi) / 6$. Thus, a straightforward calculation proves that the difference $W^{*}-W_{0}$ is decreasing with $c$. When $c=0$, then $W^{*}-W_{0}=(1+\phi) / 12$. When $c=(1+\phi) / 6$, then $W^{*}-W_{0}=0$.

## 5 Monopoly

In this section, assume that there is a unique profit-maximizing firm producing all brands of status goods. Let $\pi^{M}$ denote the maximized profit. Also, assume that the tax rate $\tau \in[\underline{\tau}, \bar{\tau}]$ is exogenously given and is the same for all brands. Section 7 proves that this is an optimal tax schedule for a benevolent government maximizing the utilitarian welfare.

The monopolist chooses the number of brands available, $N^{M} \in\{0,1,2, \cdots\}$, and their prices. Let $p^{M}=\left(p_{1}^{M}, \cdots, p_{N}^{M}\right)$ and $N^{M} \geq 0$ denote the optimal price vector and the optimal number of brands for the monopoly, respectively. After the monopoly moves, each Green decides which brand $n \in\left\{0,1, \cdots, N^{M}\right\}$ to buy. Then, each Red observes Greens' status levels $s(i) \in\left\{0,1, \cdots, N^{M}\right\}$ and matches a Green randomly chosen in her social stratum. Following Cole et al. $(1992,1995)$ and Okuno-Fujiwara and Postlewaite (1995), we define a stratified equilibrium in a monopolistic market.

[^7]Definition 2 A stratified equilibrium in a monopolistic status good market is given by a number of brands $N^{M}$, a set of strata limit abilities $\left\{i_{n}^{M}\right\}_{n=0}^{N^{M}}$, a set of status good prices $\left\{p_{n}^{M}\right\}_{n=0}^{N^{M}}$, a social norm ranking the different brands $n \in\left\{1,2, \cdots, N^{M}\right\}$ of status goods, and a matching $m: G \rightarrow R$ between Greens and Reds such that:

1. The monopolist maximizes its profit, given the equilibrium demand of Greens;
2. Green individuals maximize their expected utility, given the social norm and the equilibrium values of $N^{M}$ and $\left\{p_{n}^{M}\right\}_{n=0}^{N^{M}}$;
3. The market for each brand of status good $n \in\left\{0,1, \cdots, N^{M}\right\}$ clears; that is, equation (5) holds for the equilibrium sequences $\left\{i_{n}^{M}\right\}_{n=0}^{N^{M}}$ and $\left\{p_{n}^{M}\right\}_{n=0}^{N^{M}}$;
4. The matching between Greens and Reds is weakly stable and randomly assigns, for each Green $i \in\left[i_{n}^{M}, i_{n+1}^{M}\right) \subseteq G$, a match $j=m(i) \in\left[i_{n}^{M}, i_{n+1}^{M}\right) \subseteq R$ in the corresponding stratum of the Reds population.

The stratified equilibrium is not unique. There is a pooling equilibrium where Reds do not take status goods as signals of Greens' abilities, no one purchases any status good, and all matches are random. Given the way that Reds' expectations are formed, it is optimal for every Green to keep all of his initial endowment of consumption goods. The firm does not offer status goods because there is no demand.

The monopolist chooses $N^{M}$ and $p^{M}$ by solving the problem:

$$
\begin{gather*}
\pi^{M}=\max _{N \in \mathbb{R}_{+}}\left\{\max _{p \in \mathbb{R}_{+}^{N}}\left\{\sum_{n=1}^{N} p_{n}\left(i_{n+1}(p)-i_{n}(p)\right)-c N\right\}\right\}, \quad \text { such that: }  \tag{9}\\
0<i_{n}(p)<i_{n+1}(p) \leq 1, \quad \forall n \in\left\{0,1,2, \cdots, N^{M}\right\}
\end{gather*}
$$

where each $i_{n}(p), n \in\left\{1,2, \cdots, N^{M}\right\}$, is calculated from vector $p$ by formulas (2) and (3). Proposition 3 establishes the existence and characterizes the stratified equilibrium in a monopolistic market. We will always assume that $c$ is sufficiently small so that the optimal decision is the production of more than zero brands of status goods.

Proposition 3 (Existence and Characterization of Equilibrium in a Monopolistic Market) Assume that $c<1 /[3(1+\tau)]$. Then, there is a stratified equilibrium in the monopolistic
status good market in which strata limit abilities, prices, profit, and the number of brands available satisfy:
(a) The strata limit abilities $i_{n}^{M}$ are given, for every $n \in\left\{1, \cdots, N^{M}\right\}$, by:

$$
\begin{equation*}
i_{n}^{M}=\frac{n}{N^{M}+1} . \tag{10}
\end{equation*}
$$

(b) The demand for each brand is constant. For every $n \in\left\{1, \cdots, N^{M}\right\}$ :

$$
\begin{equation*}
i_{n}^{M}-i_{n-1}^{M}=\frac{1}{N^{M}+1} \tag{11}
\end{equation*}
$$

(c) The optimal number of brands for the monopoly, denoted $N^{M}$, is:

$$
\begin{equation*}
N^{M}=\left(\frac{1}{3 c(1+\tau)}\right)^{1 / 3}-1 \tag{12}
\end{equation*}
$$

(d) The optimal prices for the monopoly are, for every $n \in\left\{0,1, \cdots, N^{M}\right\}$ :

$$
\begin{equation*}
p_{n}^{M}=\frac{9^{1 / 3} c^{2 / 3} n(n+1)}{2(1+\tau)^{1 / 3}} \tag{13}
\end{equation*}
$$

(e) The monopoly's profit is:

$$
\begin{equation*}
\pi^{M}=\frac{1}{6(1+\tau)}-\frac{(3 c)^{2 / 3}}{2(1+\tau)^{1 / 3}}+c \tag{14}
\end{equation*}
$$

The number of brands $N^{M}$ offered by the monopoly in equilibrium is non-negative and decreases with the development cost $c$ and the tax rate $\tau$. The number of brands offered by a monopoly does not depend on the benefits of the joint production to Reds, $\phi$, if the tax $\tau$ is exogenous.

Profit $\pi^{M}$ decreases with the development cost $c$ and the tax rate $\tau$; that is, $d \pi^{M} / d c<0$ and $d \pi^{M} / d \tau<0 .{ }^{15}$ The demand for status goods does not depend on $\phi$. Hence, monopoly prices and profit do not depend on the parameter $\phi$. Under socially optimal taxation (see Section 7), the tax rate and number of brands will be functions of $\phi$. The proportion of Greens that do not buy any status good is $i_{1}^{M}=[3 c(1+\tau)]^{1 / 3}$, a measure that increases with $c$ and $\tau$.

[^8]
## 6 Contestable Markets

In contestable markets, defined by Baumol et al. (1977, 1982), there is free entry and free exit. If all firms have access to the same technology, incumbents have incentives to deter entry by charging prices that do not allow any potential entrant to obtain positive profit. The threat of entry from a contestant firm employing a "hit-and-run" strategy drives all profits to zero, even in industries where the number of firms is small.

### 6.1 Basics

Whenever we refer to a contestable market, we do not make a distinction between a brand and a firm; each firm holds only one brand and maximizes its profit. There are operating firms and potential entrants. Each operating firm pays the development cost $c>0$ once and produces a unique brand of status good. Let $N^{C}$ stand for the number of brands available. Firm $n \in\left\{1, \cdots, N^{C}\right\}$ offers brand $n$ at a price denoted $p_{n}^{C}$, and obtains profit $\pi_{n}^{C}$. Strata limit abilities are denoted $i_{n}^{C}$, with $n \in\left\{0,1, \cdots, N^{C}+1\right\}, i_{0}^{C}=0$ and $i_{N^{C}+1}^{C}=1$. The total number of brands available, strata limit abilities and prices are determined endogenously in equilibrium.

There are a large number of potential entrants, and each is considering whether to enter the market by offering a new brand $E$, at price $p^{E}$, and obtaining profit $\pi^{E}$. Potentially, a new brand could share the market with brand $n$ if $p^{E}=p_{n}^{C}$ or simply occupy a new market niche if $p^{E}<p_{1}^{C}, p^{E}>p_{N^{C}}^{C}$ or $p_{n}^{C}<p^{E}<p_{n+1}^{C}$, for some $n \in\left\{1, \cdots, N^{C}-1\right\}$.

Proposition 4 proves that there is a stratified equilibrium in which all firms, both operational and potential entrants, earn zero profit. Thus, the revenue of each operational firm is equal to $c$, and it is not possible for an entrant to share part or all of the demand with an incumbent while still making a positive profit. Staying out of the market is optimal.

If the entrant's price is such that $p_{n}^{C}<p^{E}<p_{n+1}^{C}$, there will be new strata limit abilities $\tilde{\imath}_{1}, \cdots, \tilde{\imath}_{n}, \tilde{\imath}^{E}, \tilde{\imath}_{n+1}, \cdots, \tilde{\imath}_{N}$ such that $\tilde{\imath}_{n+k}=i_{n+k}^{C}$, for every $k>0$. This is because, by Proposition 1, the demand for brands $n+k$, with $k \geq 1$, do not change after the entry.

Definition 3 An Industry Configuration (IC) is constituted by the number of incumbent brands $N^{C}$ and a price vector $p^{C}=\left(p_{1}^{C}, \cdots, p_{N^{C}}^{C}\right)$ that is charged by the incumbent firms for each brand $n \in\left\{1, \cdots, N^{C}\right\}$. An Industry Configuration is said to be feasible if: (i) the market clears; that is, equation (5) holds for every $n \in\left\{1, \cdots, N^{C}\right\}$ when $i_{n}=i_{n}^{C}$; (ii)
incumbent firms make non-negative profits; that is, for every $n \in\left\{1, \cdots, N^{C}\right\}, \pi_{n}^{C} \geq 0$, or, equivalently:

$$
\begin{equation*}
p_{n}^{C}\left(i_{n+1}^{C}-i_{n}^{C}\right)-c \geq 0 \tag{15}
\end{equation*}
$$

An Industry Configuration is said to be sustainable if no entrant can obtain positive profit by taking the incumbents' prices as given; that is:

$$
\begin{equation*}
p^{E}\left(\tilde{\imath}_{n+1}-\tilde{\imath}_{n}^{E}\right)-c \leq 0 . \tag{16}
\end{equation*}
$$

Now, we describe the stratified equilibrium. There are $N^{C}$ firms operating and a large number of potential entrants outside this market. The firm producing and selling brand $n \in\left\{1, \cdots, N^{C}\right\}$ pays the development cost $c$, obtains revenue $p_{n}^{C}\left(i_{n+1}^{C}-i_{n}^{C}\right)$, and earns profit $\pi_{n}^{C}=0$, where $\pi_{n}^{C}=p_{n}^{C}\left(i_{n+1}^{C}-i_{n}^{C}\right)-c$. Following Baumol (1982), Cole et al. (1992, 1995) and Okuno-Fujiwara and Postlewaite (1995), we define stratified equilibrium in a contestable market as follows.

Definition 4 A stratified equilibrium in a contestable status good market is given by a social norm ranking the brands of status goods, an industry configuration $\left\{N^{C}, p^{C}\right\}$, a set of strata limit abilities $\left\{i_{n}^{C}\right\}_{n=1}^{N^{C}}$ representing the demand for the different brands of status goods, and a matching $m(\cdot)$ that randomly assigns, for each Green $i \in\left[i_{n}^{C}, i_{n+1}^{C}\right) \subseteq G$, a Red $j=m(i) \in$ $\left[i_{n}^{C}, i_{n+1}^{C}\right) \subseteq R$, such that:

1. Given the social norm, the strata limit abilities and the matching, then the industry configuration $\left\{N^{C}, p^{C}\right\}$ is feasible and sustainable;
2. Given the social norm, the strata limit abilities and the matching $m(\cdot)$, then each agent maximizes his expected utility; that is, equation (5) holds for $\left\{i_{n}^{C}\right\}_{n=1}^{N^{C}}$ and $\left\{p_{n}^{C}\right\}_{n=1}^{N^{C}}$;
3. Given the social norm, the industry configuration and the strata limit abilities, then the matching is weakly stable.

Strata limit abilities are determined iteratively, from top to bottom. Let $i_{N^{C}+1}^{C}=1$. Suppose that all $\tau_{m}$, for all $m \in\left\{0,1, \cdots, N^{C}\right\}$, are exogenously given and assume that $i_{n+1}^{C}$ is already fixed (thus, the choice of price for firm $n$ does not affect any strata limit ability $i_{n+k}^{C}$, for any $k \geq 1$ ). The profit of firm $n$ is a quadratic and concave function of $i_{n}^{C}$, denoted $\pi_{n}^{C}\left(i_{n}^{C}\right)$.

Substitute the inverse demand equation (5) with $i_{n}=i_{n}^{C}$, into $0=\pi_{n}^{C}=p_{n}^{C}\left(i_{n+1}^{C}-i_{n}^{C}\right)-c$. After some algebra, equation $\pi_{n}^{C}\left(i_{n}^{C}\right)=0$ becomes:

$$
\begin{equation*}
-i_{n+1}^{C}\left(i_{n}^{C}\right)^{2}+\left(i_{n+1}^{C}\right)^{2} i_{n}^{C}-2\left(1+\tau_{n}\right) c=0 \tag{17}
\end{equation*}
$$

Firm $n$ chooses the price $p_{n}^{C}$ that makes $i_{n}^{C}$ equal to the largest real root of the equation $\pi_{n}^{C}\left(i_{n}^{C}\right)=0$. Why the largest root? If firm $n$ targets the smallest root, then it becomes vulnerable to entry by a firm outside the market charging a price higher than $p_{n}^{C}$ but arbitrarily close to it. This entrant firm would capture most of firm n's demand while making a positive profit. ${ }^{16}$ Because firm $n$ would still pay the development cost, it would end up with negative profit.

When firm $n$ chooses the largest root, the entrant charging a slightly larger price would make negative profit because, at the largest root, $d \pi_{n}^{C}\left(i_{n}^{C}\right) / i_{n}<0$. Profits decline with $i_{n}$ at the largest root because the largest root is at the right-hand side of the vertex of the concave parabola. Intuitively, the demand becomes too small to sustain non-negative profit.

Now, suppose that a (potential) entrant charges a price $p^{E}$ such that $p_{n-1}^{C}<p^{E}<p_{n}^{C}$. Then, the entrant's corresponding strata limit ability $\tilde{\imath}^{E}$ would be larger than $i_{n-1}^{C}$. This entrant would earn a negative profit because its demand would be a proper subset of firm $n-1$ equilibrium demand, and, by definition, $p_{n-1}^{C}$ is the largest price that can make the profit of firm $n-1$ non-negative. Mathematically, $d \pi_{n-1}^{C}\left(i_{n-1}^{C}\right) / i_{n-1}<0$ and $\pi_{n-1}^{C}\left(i_{n-1}^{C}\right)=0$ imply that $\pi_{n-1}^{C}\left(\tilde{\imath}^{E}\right)<0$, if $\tilde{\imath}^{E}>i_{n-1}^{C}$. Anticipating this, the potential entrant would not enter the market.

### 6.2 Stratified Equilibrium

Starting from $i_{N+1}^{C}=1$, we recursively calculate the equilibrium strata limit abilities by solving the equation $\pi_{n}^{C}\left(i_{n}^{C}\right)=0$ for $i_{n}^{C}$ as a function of $i_{n+1}^{C}$ :

$$
i_{n}^{C}=\left\{\begin{array}{cl}
\frac{i_{n+1}^{C}}{2}\left(1+\sqrt{1-\frac{8 c\left(1+\tau_{n}\right)}{\left(i_{n+1}^{C}\right)^{3}}}\right), & \text { if } \quad i_{n+1}^{C} \geq 2 c^{1 / 3}\left(1+\tau_{n}\right)^{1 / 3}  \tag{18}\\
0, & \text { otherwise. }
\end{array}\right.
$$

${ }^{16}$ The entrant's profit is positive because the derivative of function $\pi_{n}^{C}\left(i_{n}\right)$, calculated at the smallest root, is positive. This is true because the smallest root is at the left-hand side of the vertex of the concave parabola that is the graph of the function $\pi_{n}^{C}\left(i_{n}\right)$. Function $\pi_{n}^{C}\left(i_{n}\right)$ is increasing at the smallest root and decreasing at the largest root of equation (17).

If equation (17) has no real root, then the entire graph of function $\pi_{n}^{C}\left(i_{n}^{C}\right)$ is below the horizontal axis. If firm $n$ enters the market, it earns a negative profit. In this case, this firm stays out of the market. The quadratic equation (17) has no real root if and only if:

$$
i_{n+1}^{C}<2\left(c\left(1+\tau_{n}\right)\right)^{1 / 3}
$$

When this inequality holds, firm $n$ does not offer a new brand. For instance, if $1<$ $2\left(c\left(1+\tau_{1}\right)\right)^{1 / 3}$, then no firm produces status goods. Algorithm 1 calculates the number $N^{C}$ of brands. It starts with the wrong guess of $N=-1$ to find how many brands the contestable market can accommodate. This wrong guess generates the wrong indices $n$. Once $N^{C}$ is computed, all indices are corrected by adding $N^{C}+1$ to the wrong indices. For instance, variable $i_{0}^{C}$ is relabeled and becomes $i_{N_{+1}}^{C}$, variable $i_{-1}^{C}$ becomes $i_{N^{C}}^{C}, i_{-2}^{C}$ becomes $i_{N^{C}-1}^{C}$, and so on, until $i_{n_{0}}^{C}$ becomes $i_{1}^{C}$. Indices of tax rates $\tau_{n}$ must also be corrected.

Algorithm 1 Because we do not know the value of $N^{C}$ initially, we start the counting process by guessing that $N=-1$. If $1<2\left(c\left(1+\tau_{-1}\right)\right)^{1 / 3}$, then no firm produces status goods. Otherwise, Lemma 3 proves that, starting from $n=N=-1$ and $i_{N+1}^{C}=i_{n+1}^{C}=$ $i_{0}^{C}=1$, as $n$ decreases in each iteration by one unit, we recursively pick the largest root of equation (18) and, eventually, in a finite number of steps, there is some integer $n_{0}$ such that $i_{n_{0}+1}^{C} \geq 2\left(c\left(1+\tau_{n_{0}}\right)\right)^{1 / 3}$ and $i_{n_{0}}^{C}<2\left(c\left(1+\tau_{n_{0}-1}\right)\right)^{1 / 3}$. Then, $N^{C}$ is equal to the absolute value of this particular index, $N^{C}=\left|n_{0}\right|=-n_{0}$. By adding $N^{C}+1$ to all indices of the variables, we re-index them all correctly. Finally, we set $i_{0}^{C}=0$.

Example 2 Suppose that $1>2\left(c\left(1+\tau_{-1}\right)\right)^{1 / 3}$. We start with $n=N=-1$ and $i_{N+1}^{C}=$ $i_{0}^{C}=1$. Because $i_{n+1}^{C}=i_{-1+1}^{C}=1>2\left(c\left(1+\tau_{-1}\right)\right)^{1 / 3}$, the equation $\pi_{-1}^{C}\left(i_{-1}^{C}\right)=0$ has two distinct real roots. Let $i_{-1}^{C}$ denote its largest root. For the next step, assume that $n=-2$. Since $i_{n+1}^{C}=i_{-2+1}^{C}=i_{-1}^{C}>2\left(c\left(1+\tau_{-2}\right)\right)^{1 / 3}$, there are more brands, and the Algorithm can continue. Because $i_{-1}^{C}>2\left(c\left(1+\tau_{-2}\right)\right)^{1 / 3}$, equation $\pi_{-2}^{C}\left(i_{-2}^{C}\right)=0$ has two real roots. Let $i_{-2}^{C}$ be the largest root of $\pi_{-2}^{C}\left(i_{-2}^{C}\right)=0$. Now, $i_{n}^{C}=i_{-2}^{C}<2\left(c\left(1+\tau_{-3}\right)\right)^{1 / 3}$. Then, there is no space for more brands in this market. Algorithm 1 establishes that $N^{C}=|-2|=2$; that is, there are exactly 2 brands. By adding $N^{C}+1=3$ to the indices of all variables, we find the strata limit abilities $i_{N^{C}+1}^{C}=i_{3}^{C}=1$ (previously $i_{0}^{C}$ ), $i_{2}^{C}$ (previously $i_{-1}^{C}$ ), and $i_{1}^{C}$ (previously $\left.i_{-2}^{C}\right)$. Setting $i_{0}=0$ completes the set of strata limit abilities. After adding $N^{C}+1=3$ to the indices, tax rates that were previously indexed $\tau_{-1}$ and $\tau_{-2}$, become, respectively, $\tau_{2}$ and $\tau_{1}$.

Remark 1 Firms in contestable markets offer a higher number of brands than monopolies do for many combinations of the parameters.

Lemma 3 (Finiteness of the Algorithm) The number of firms $N^{C}$ determined by Algorithm 1 is always finite; that is, given the initial guess $N=-1$, there is a number $n_{0} \in \mathbb{Z}$ such that $i_{n_{0}+1} \geq 0$ and $i_{n_{0}}<0$.

Algorithm 1 characterizes an Industry Configuration. By applying equation (5) to the sequence of values $i_{n}^{C}$, we find the prices $p_{n}^{C}$. By design, all firms obtain zero profit. In equilibrium, for each $n \in\left\{1,2, \cdots, N^{C}\right\}$, firm $n$ chooses the price:

$$
\begin{equation*}
p_{n}^{C}=c\left[\frac{i_{n}^{C}}{2}\left(1+\sqrt{1+\frac{8 c\left(1+\tau_{n}\right)}{\left(i_{n}^{C}\right)^{3}}}\right)\right]^{-1} \tag{19}
\end{equation*}
$$

Lemma 4 The Industry Configuration characterized by Algorithm 1 is feasible and sustainable.

Proposition 4 (Equilibrium Characterization in Contestable Markets) If the gap conditions hold, and $1<2\left(c\left(1+\tau_{1}\right)\right)^{1 / 3}$, then there is a stratified equilibrium in a contestable status good market in which the equilibrium industry configuration and strata limit abilities are given by Algorithm 1.

Table 1 reports the equilibrium results for particular values of the parameters for both monopolies and contestable markets. For many combinations of parameters, there are more brands in contestable markets than in a monopolistic market.

Corollary 1 proves that social strata measures are decreasing with $n$, under constant tax rates. In particular, under zero taxes, distinct social strata have different measures.

Corollary 1 In a contestable market, under a flat tax schedule (in particular, this is true if all taxes are zero), higher strata have lower measures than lower strata. Formally, the following function is decreasing: $n \mapsto i_{n+1}^{C}-i_{n}^{C}$, for $n \in\left\{1,2, \cdots, N^{C}\right\}$.

Under a flat tax schedule, cum-tax prices and pre-tax prices increase with $n$. Thus, firms in the high end of the market do not need to sell many units to break even. Many firms operate in these market niches, each one selling to a relatively small measure of Greens. On the other hand, firms selling brands of relatively low status charge relatively low prices and need to sell them to a relatively large measure of Greens to break even.

## 7 Pigouvian Taxation

### 7.1 Why Should We Tax or Subsidize Status Goods?

Signaling generates a positive externality for the Red population because Reds receive, on average, higher payoff from the matching and joint production process when strata are finer. If taxes are nonexistent, there may exist overprovision or underprovision of status good brands compared to the socially optimal level. Naturally, this opens the possibility of welfare improvement through public policy. This is in contrast with previous results. Frank (1985), Rege (2008) and Hoppe et al. (2009) all argue that status goods should be taxed to the limit or banned altogether, since conspicuous consumption is completely wasteful. Ireland $(1994,2001)$ also argues in favor of the control of status good consumption through taxation. In our model, because the number of brands available affects matching efficiency, status good consumption is not a complete waste. If the improvement in matching efficiency generated by one extra brand is larger than the waste generated by its development cost, there might even be space for subsides.

The supply and demand for status goods do not depend on the benefit of the joint production to Reds, $\phi$. Yet Reds' utilities affect the social welfare. In a monopolistic market, this point is made clear by comparing equations (8) and (12). In the former, the number of brands, $N^{*}$, is an increasing function of $\phi$, but in the latter, the number of brands, $N^{M}$, does not depend on $\phi$.

Firms in contestable markets may offer an inefficient number of brands because the demand does not depend on $\phi$. Furthermore, Corollary 1 proves that if the tax rates for all brands are zero, then the measures of social strata decrease with $n$. Since the social optimum occurs when social strata have identical measure, strata obtained in equilibrium in contestable markets are socially undesirable. Table 1 exemplifies this point. If $\phi=1$, the socially optimal solution coincides with the monopoly's solution. When $\phi=1$ and $c=0.02$, a monopoly provides its first brand to Greens in the interval $[1 / 3,2 / 3)$, but a contestable market allocates status goods only to Greens with abilities at or above $i_{1}^{C}=0.54$.

### 7.2 Optimal Taxation for a Monopoly

All socially optimal strata have the same measure. Under a constant tax rate, a monopoly sells the different brands to sets of consumers of equal measure. However, a monopoly will
typically provide a non-optimal number of brands.
Suppose that the government does not interfere in the matching process and chooses the lump-sum transfer $T$ and tax rates $\tau_{n} \in[\underline{\tau}, \bar{\tau}]$ on status goods to maximize welfare subject to the budget balance constraint.

By equating conditions (8) and (12), we find that $N^{*}=N^{M}$ if and only if $\tau=\widehat{\tau}$, where $(1+\phi)(1+\widehat{\tau})=2$. By setting the tax rate to $\tau=\widehat{\tau}$, the government induces the monopoly to provide exactly $N^{*}$ brands. The optimal tax rate $\widehat{\tau}$ does not depend on $n$ or $c$, only on $\phi$.

Proposition 5 (Optimal Taxation in a Monopolistic Industry) Suppose that $c<(1+\phi) / 6$. In a monopolistic status goods industry under socially optimal taxation:
(a) The socially optimal tax rate is constant for all brands, and is given by the following decreasing function of $\phi$ :

$$
\begin{equation*}
\widehat{\tau}=\frac{1-\phi}{1+\phi} . \tag{20}
\end{equation*}
$$

(b) The monopoly chooses to produce the socially optimal number of brands $\left(N^{M}=N^{*}\right)$ and sells every unit of each brand $n \in\left\{1, \cdots, N^{*}\right\}$ at price $\widehat{p}_{n}$, where:

$$
\begin{equation*}
\widehat{p}_{n}=\left(\frac{9(1+\phi) c^{2}}{16}\right)^{1 / 3} n(n+1) \tag{21}
\end{equation*}
$$

(c) For each brand $n \in\left\{1, \cdots, N^{*}\right\}$, its cum-tax price is:

$$
\begin{equation*}
(1+\widehat{\tau}) \widehat{p}_{n}=\frac{3^{2 / 3} c^{2 / 3} n(n+1)}{2^{1 / 3}(1+\phi)^{2 / 3}} \tag{22}
\end{equation*}
$$

Part (b) of Proposition 5 proves that under socially optimal taxation for a monopoly, pre-tax prices increase with $\phi, c$ and $n$. Part (c) of Proposition 5 establishes that cum-tax prices $(1+\widehat{\tau}) \widehat{p}_{n}$ are increasing with $c$ and $n$ and decreasing with $\phi$.

Because the monopoly profit decreases with the tax rate and does not depend directly on $\phi$ and because the optimal tax rate $\widehat{\tau}$ decreases with $\phi$, the monopoly profit increases with $\phi$ under optimal taxation. In addition, optimal taxes do not depend on the parameter $c$.

If $\phi=0$, then $\widehat{\tau}=1$. As $\phi$ grows large, the socially optimal tax rate decreases, approaching $\widehat{\tau}=-1$. Hence, regardless of $\phi$, the socially optimal tax always lies in $-1<\widehat{\tau}<1$. In particular, when Reds consume half of the jointly produced output ( $\phi=1$ ), no corrective taxes are needed; that is, $\widehat{\tau}=0 .{ }^{17}$

[^9]
### 7.3 Optimal Taxation in a Contestable Market

The government can induce the socially optimal allocation by choosing a progressive tax schedule with rates that increase with status levels at the "right speed." Proposition 6 makes this statement precise.

Proposition 6 (Optimal Taxation in Contestable Markets) Suppose that the market for the status good is contestable. For each $n \in\left\{1, \cdots, N^{*}\right\}$, let $\widehat{\widehat{\tau}}_{n}$ and $\widehat{\hat{p}}_{n}$ denote the socially optimal tax rates and the prices when these taxes are applied, respectively. Suppose that $c<(1+\phi) / 16$. Then:
(a) The tax schedule that implements allocations maximizing welfare is, for every $n \in$ $\left\{1, \cdots, N^{*}\right\}$ :

$$
\begin{equation*}
\widehat{\widehat{\tau}}_{n}=\frac{3 n(n+1)}{1+\phi}-1 \tag{23}
\end{equation*}
$$

(b) Under this tax schedule, pre-tax prices are constant for every brand; that is, for every $n \in\left\{1, \cdots, N^{*}\right\}:$

$$
\begin{equation*}
\widehat{\hat{p}}_{n}=c\left(\frac{1+\phi}{6 c}\right)^{1 / 3} \tag{24}
\end{equation*}
$$

(c) Under socially optimal taxation, cum-tax prices $\left(1+\widehat{\widehat{\tau}}_{n}\right) \widehat{\hat{p}}_{n}$ are increasing in the status level $n$. Moreover, they are equal to the cum-tax prices obtained in the case of a monopoly with the optimal constant tax rate $\widehat{\tau}$; that is, for every $n \in\left\{1, \cdots, N^{*}\right\}$ :

$$
\left(1+\widehat{\widehat{\tau}}_{n}\right) \widehat{\hat{p}}_{n}=(1+\widehat{\tau}) \widehat{p}_{n}=\frac{3^{2 / 3} c^{2 / 3} n(n+1)}{2^{1 / 3}(1+\phi)^{2 / 3}}
$$

Part (a) indicates that welfare maximization in a contestable market implies that the government implements a progressive tax schedule. As in the monopoly case, optimal tax rates $\widehat{\hat{\tau}}_{n}$ depend only on $\phi$, the relative benefit of production to the Reds. Furthermore, when the Reds' relative benefit from joint production is larger, socially optimal tax rates are lower ( $\widehat{\widehat{\tau}}_{n}$ decreases with $\phi$ ). Taxes decrease with $\phi$ because the magnitude of the positive externality (due to more efficient signaling, resulting in more efficient matching) is higher when $\phi$ is large. Remark 3 shows the necessary conditions for the government to subsidize at least one brand. It also shows the necessary condition for the government to subsidize all brands, which essentially is that $\phi$ is large enough.

Part (b) of Proposition 6 establishes a rule of thumb for taxing status goods in contestable markets: the government should adjust taxes until all pre-tax prices are equal. This result
is not surprising because the revenue of a firm divided by its demand is equal to the pre-tax price. Thus, all firms' revenues are equal to each other because all producers have zero profit and constant development cost, and the demands of all firms have the same measure, $\left(1+N^{*}\right)^{-1}$. Hence, all pre-tax prices must coincide. In other words, the government should keep all brand premia to itself.

Part (c) of Proposition 6 proves that the cum-tax prices under optimal taxation are the same regardless of the market structure. This conclusion is not surprising, either, because all of the externalities have been internalized. To generate identical allocations in the two market structures, incentives for Greens, given here by the cum-tax price of each brand $n$, must be equal.

## 8 Conclusion

This paper presents a stylized model of the supply and demand of pure status goods in which there are fixed costs, matching concerns, endogenous stratification, and optimal taxation. Full separation of agents in the signaling population cannot be achieved in a stratified equilibrium, regardless of the market structure. Instead, the highest degree of separation occurs in a stratified equilibrium, in which individuals of different but similar abilities purchase status goods of the same particular brand. A government maximizing welfare charges a flat schedule to a monopoly and progressive tax rates in contestable markets.

This paper focuses only on utilitarian welfare. Obviously, different welfare functions might produce different results. The reader interested in policy issues might be willing to understand how the optimal policy changes with the functional form and parameters of the social welfare function.

We have made simplifying assumptions on how the matching outcome generates output by choosing a Cobb-Douglas production function. We have also analyzed the case of Leontief technology. Analyzing other joint production functions could be a topic of future research.

In practice, many goods that are used to signal abilities are not pure status goods. Luxury cars, Persian rugs, and haute couture clothing are examples of status goods that have intrinsic value. Future studies could try to extend the current framework, analyzing how status concerns interact with quality, quantity and brand creation. Extensions could also explore the case in which abilities in both populations are private information. Reds
and Greens would buy status goods to signal their individual abilities. The status goods could be sold in a single market or in two separate markets; that is, status goods used by Greens could be equal or completely different from the status goods purchased by Reds.

Finally, there are political economy aspects of taxation deserving mention. In the present model, different tax schedules maximize the utilities of Reds, Greens and the monopoly firm. Hence, these three groups have incentives to invest in lobbying the government for their favorite tax policies.

## A Appendix: Proofs

## A. 1 Demand for Status Goods

Proof of Lemma 1: every Green buys at most one unit of status good. Because for every $i \in G$ and every $j \in R$, it is always the case that $z(i, j) \leq 1$, then no Green can increase his utility by more than 1 unit via the matching and joint production processes. Hence, the cumtax price for any status goods is no larger than 1 . No profit-maximizing firm offers brands generating cum-tax prices above this upper bound because there would be no demand. Thus, $\left(1+\tau_{n}\right) p_{n} \leq 1$ and $-\left(1+\tau_{n}\right) p_{n} \geq-1$, for every brand $n \in\{1, \cdots, N\}$. Because $\tau \leq \tau_{n}$, for every $n \in\{1, \cdots, N\}$, then $(1+\underline{\tau}) p_{n} \leq\left(1+\tau_{n}\right) p_{n} \leq 1$. Hence, $p_{n} \leq(1+\underline{\tau})^{-1}$. Then:

$$
\begin{gathered}
|T| \leq \max \left\{\left|\tau_{n} p_{n}\right| ; n \in\{1, \cdots, N\}\right\}=\max \left\{\left|\tau_{n}\right| p_{n} ; n \in\{1, \cdots, N\}\right\} \leq \\
\leq\left(\max \left\{\left|\tau_{n}\right| ; n \in\{1, \cdots, N\}\right\}\right) \frac{1}{1+\underline{\tau}} \leq \frac{\bar{\tau}}{1+\underline{\tau}} .
\end{gathered}
$$

Because there is a balanced budget, and every Green receives (or pays, if $T<0$ ) the same transfer, namely $T$, then $T>-\bar{\tau} /(1+\underline{\tau})$. By hypothesis, $y>1+\bar{\tau} /(1+\underline{\tau})$. Hence:

$$
x=y+T-\left(1+\tau_{n}\right) p_{n}>1+\frac{\bar{\tau}}{1+\underline{\tau}}-\frac{\bar{\tau}}{1+\underline{\tau}}-1=0 .
$$

This proves that $x>0$.
Proof of Lemma 2: by Lemma 1, every Green keeps some positive amount of the consumption good and can afford to buy at least one unit of any brand of status good. Fix the numbers $n \in\{1,2, \cdots, N\}$ and $k \in\{1,2, \cdots, N-n\}$. For each $i \in[0,1]$, define $\Delta_{n, k}(i)$ by the following expression:

$$
\Delta_{n, k}(i)=2 U\left(y+T-\frac{\alpha_{n+k}}{2}, i, \frac{i_{n+k}+i_{n+k+1}}{2}\right)-2 U\left(y+T-\frac{\alpha_{n}}{2}, i, \frac{i_{n}+i_{n+1}}{2}\right) .
$$

Using $U(x, i, j)=x+z(i, j)$, then $\Delta_{n, k}(i)$ becomes:

$$
\begin{gathered}
\Delta_{n, k}(i)=\left[2 y+2 T-\alpha_{n+k}+i\left(i_{n+k}+i_{n+k+1}\right)\right]-\left[2 y+2 T-\alpha_{n}+i\left(i_{n}+i_{n+1}\right)\right], \\
\Delta_{n, k}(i)=i\left(i_{n+k+1}+i_{n+k}-i_{n+1}-i_{n}\right)-\alpha_{n+k}+\alpha_{n} .
\end{gathered}
$$

Because $i_{n+k+1}>i_{n+1}$ and $i_{n+k}>i_{n}$, then $\Delta_{n, k}(i)$ is an increasing function of $i$. By the definition of $i_{n+k}$, a Green agent with ability $i_{n+k}$ is indifferent between brands $n+k$ and $n+k-1$. Thus, $\Delta_{n+k-1,1}\left(i_{n+k}\right)=0$. Because $i<i_{n+1} \leq i_{n+k}$, then $\Delta_{n+k-1,1}(i)<$ $\Delta_{n+k-1,1}\left(i_{n+k}\right)=0$. Hence, $\Delta_{n+k-1,1}(i)<0$ holds for every $k \in\{1,2, \cdots, N+1-n\}$. Since $\Delta_{n, 1}(i)<0$ and $\Delta_{n+1,1}(i)<0$, then $\Delta_{n, 2}(i)=\Delta_{n, 1}(i)+\Delta_{n+1,1}(i)<0$. Similarly, since $\Delta_{n, 2}(i)<0$ and $\Delta_{n+2,1}(i)<0$, then $\Delta_{n, 3}(i)<0$. Recursively, we prove that $\Delta_{n, k}(i)<0$, for every $k$. This means that a Green of ability $i$ belonging to the open interval $\left(i_{n}, i_{n+1}\right)$ prefers to buy a status good of brand $n$ than to purchase from brand $n+k$, for any $k>0$. An analogous argument proves that this Green also prefers brand $n$ to any other brand $n-k$, with $k>0$.

Proof of Proposition 1 (Inverse Demand for Status Goods): because Reds' abilities are uniformly distributed in the interval $[0,1]$, then:

$$
\begin{equation*}
E_{i_{n}}\left[i_{n} m^{*}\left(i_{n}\right) \mid s\left(i_{n}\right)=n\right]=i_{n}\left(\frac{i_{n+1}+i_{n}}{2}\right), \tag{25}
\end{equation*}
$$

and

$$
\begin{equation*}
E_{i_{n}}\left[i_{n} m^{*}\left(i_{n}\right) \mid s\left(i_{n}\right)=n-1\right]=i_{n}\left(\frac{i_{n}+i_{n-1}}{2}\right) . \tag{26}
\end{equation*}
$$

For every $n \in\{1, \cdots, N\}$, the indifference of a Green agent of ability $i_{n}$ means that:

$$
\begin{gather*}
y+T-\left(1+\tau_{n}\right) p_{n}+E_{i_{n}}\left[i_{n} m\left(i_{n}\right) \mid s\left(i_{n}\right)=n\right]= \\
=y+T-\left(1+\tau_{n-1}\right) p_{n-1}+E_{i_{n}}\left[i_{n} m\left(i_{n}\right) \mid s\left(i_{n}\right)=n-1\right] . \tag{27}
\end{gather*}
$$

Substituting equations (25) and (26) into equation (27) and using (2), we find:

$$
\begin{equation*}
\alpha_{n}=\alpha_{n-1}+i_{n} i_{n+1}-i_{n} i_{n-1}, \quad \forall n \in\{1, \cdots, N\} . \tag{28}
\end{equation*}
$$

Because $n=0$ means buying no status goods, $p_{0}=0$. By (2), $\alpha_{0}=0$. Substituting $n=1, i_{0}=0$ (which holds by assumption), and $\alpha_{0}=0$ in equation (28), we obtain $\alpha_{1}=i_{1} i_{2}-i_{1} i_{0}=i_{1} i_{2}$. When $n=2$, equation (28) states that $\alpha_{2}=\alpha_{1}+i_{2} i_{3}-i_{2} i_{1}=i_{2} i_{3}$.

Continuing to iterate, we find that $\alpha_{n}=i_{n} i_{n+1}$ holds for every $n$. Using (2), we obtain equation (5). From equation (4), we can prove that for every $n \in\{2,3, \cdots, N+1\}$ :

$$
i_{n-2}=\left(\frac{\alpha_{n-2}}{\alpha_{n-1}}\right) i_{n}
$$

From this and the fact that $i_{N+1}=1$, the strata limit abilities are computed iteratively, starting with $i_{N}=\alpha_{N}$ and $i_{N-1} i_{N}=\alpha_{N-1}$, to find equation (3).

Remark 2 Given adjusted prices $\alpha_{n}, n \in\{1,2, \cdots, N\}$, gap conditions hold if and only if:

$$
\left\{\begin{array}{c}
\alpha_{N}<1, \\
\alpha_{N-1}<\left(\alpha_{N}\right)^{2}, \\
\alpha_{N-2}<\left(\alpha_{N-1} / \alpha_{N}\right)^{2}, \\
\alpha_{N-3}<\left(\left[\alpha_{N} \alpha_{N-2}\right] / \alpha_{N-1}\right)^{2}, \\
\alpha_{N-4}<\left(\left[\alpha_{N-1} \alpha_{N-3}\right] /\left[\alpha_{N} \alpha_{N-2}\right]\right)^{2}, \\
\cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \omega^{2}, \text { if } N \text { is even, or } \\
\alpha_{1}<\left(\left[\alpha_{N} \alpha_{N-2} \cdots \alpha_{2}\right] /\left[\alpha_{N-1} \alpha_{N-3} \cdots \alpha_{3}\right]\right)^{2}, \\
\alpha_{1}<\left(\left[\alpha_{N-1} \alpha_{N-3} \cdots \alpha_{2}\right] /\left[\alpha_{N} \alpha_{N-2} \cdots \alpha_{3}\right]\right)^{2}, \text { if } N \text { is odd. }
\end{array}\right.
$$

## A. 2 Welfare

Proof of Proposition 2 (Socially Optimal Number of Brands):
Part (a). The expected utility of every Green or Red individual of ability $i \in\left[i_{n}, i_{n+1}\right]$ is given by:

$$
\begin{equation*}
E_{i}\left[i m^{*}(i) \mid s(i)\right]=i\left(\frac{i_{n+1}+i_{n}}{2}\right) . \tag{29}
\end{equation*}
$$

Using (29) to calculate the expected values and evaluating the integral inside equation (7), we find (30). Thus, the maximized welfare is:

$$
\begin{equation*}
W^{*}=y+\max _{N \in \mathbb{R}_{+}}\left\{\max _{i_{1}<\cdots<i_{N}} \sum_{n=0}^{N}(1+\phi)\left(i_{n+1}-i_{n}\right)\left(\frac{i_{n+1}+i_{n}}{2}\right)^{2}-c N\right\} . \tag{30}
\end{equation*}
$$

Let $N$ be a fixed positive integer. Consider the sub-problem:

$$
\begin{equation*}
\max _{i_{1}<\cdots<i_{N}}\left\{\sum_{n=0}^{N}(1+\phi)\left(i_{n+1}-i_{n}\right)\left(\frac{i_{n+1}+i_{n}}{2}\right)^{2}-c N\right\} \tag{31}
\end{equation*}
$$

with boundary conditions $i_{0}=0$ and $i_{N+1}=1$. Taking the derivatives with respect to each $i_{n}$ of the objective function in the sub-problem (31), we find that, $\forall n \in\{1, \cdots, N\}$ :

$$
\left[\left(i_{n+1}+i_{n-1}\right)-2 i_{n}\right]\left(i_{n+1}-i_{n-1}\right)=0
$$

Because, by construction, $i_{n+1}>i_{n-1}$, we must have $\left[\left(i_{n+1}+i_{n-1}\right)-2 i_{n}\right]=0$. The secondorder condition, $-2\left(i_{n+1}-i_{n-1}\right)<0$, is trivially satisfied. Hence, the first-order conditions of the sub-problem (31) imply that $\forall n \in\{1, \cdots, N\}$ :

$$
\begin{equation*}
i_{n+1}-i_{n}=i_{n}-i_{n-1} . \tag{32}
\end{equation*}
$$

The unique solution of this system of first-order difference equations, subject to the boundary conditions $i_{0}=0$ and $i_{N+1}=1$, is given by:

$$
\begin{equation*}
i_{n}^{*}=\frac{n}{N+1}, \quad \forall n \in\{1, \cdots, N\} \tag{33}
\end{equation*}
$$

The result follows immediately by substituting equation (33) into the expression $i_{n}^{*}-i_{n-1}^{*}$.
Part (b). By substituting the socially optimal strata limit abilities $i_{n}^{*}=n /(N+1)$ into the welfare given by (30) and algebraically manipulating the outcome, we obtain the following:

$$
W^{*}=y+\max _{N \in \mathbb{N}}\left\{\frac{1+\phi}{4(N+1)^{3}}\left[4 \sum_{n=0}^{N^{*}} n^{2}+4 \sum_{n=0}^{N^{*}} n+\sum_{n=0}^{N^{*}} 1\right]-c N\right\} .
$$

After more manipulation, this expression reduces to:

$$
\begin{equation*}
W^{*}=y+\max _{N \in \mathbb{R}_{+}}\left\{\frac{4(1+\phi) N(N+2)+3(1+\phi)-12 c N(N+1)^{2}}{12(N+1)^{2}}\right\} \tag{34}
\end{equation*}
$$

The first- and second-order derivatives of the objective function in (34) with respect to the number of brands are, respectively:

$$
\frac{(1+\phi)}{6(N+1)^{3}}-c \quad \text { and } \quad \frac{-(1+\phi)}{2(N+1)^{4}}
$$

Because $-(1+\phi) /\left[2(N+1)^{4}\right]<0$, the objective function in (34) is concave, and the firstorder condition generates its unique global maximum at $N=N^{*}$, which is given by:

$$
N^{*}=\left(\frac{1+\phi}{6 c}\right)^{1 / 3}-1
$$

Part (c). Because $W(N)$ is concave and smooth, there is a unique vale $N=N^{*}$ such that $W^{\prime}\left(N^{*}\right)=0$. Under the parameters' restriction, $N^{*}>0$. Then, $W\left(N^{*}\right)>W(N)$, for every $N \neq N^{*}$. In particular, $W\left(N^{*}\right)>W(0)$. However, $W(0)$ is the welfare if there are no status goods and all matches are random.

## A. 3 Monopoly

Proof of Proposition 3 (Existence and Characterization of Equilibrium in a Monopolistic Market):

Part (a). The monopoly's problem in (9) can be rewritten as:

$$
\begin{equation*}
\pi^{M}=\max _{N \in \mathbb{Z}_{+}}\left\{\max _{i_{1}<\cdots<i_{N}}\left\{\sum_{n=1}^{N} \frac{i_{n} i_{n+1}}{2(1+\tau)}\left(i_{n+1}-i_{n}\right)-c N\right\}\right\} \tag{35}
\end{equation*}
$$

by substituting the inverse demand equation (4) into (9) and realizing that there is a bijection between the price and stratum limit spaces, as established in Proposition 1. Therefore, for a fixed $N$, maximizing in the price space is equivalent to choosing optimal strata limit abilities $i_{1}^{M}<\cdots<i_{N}^{M}$. Let $N$ be a fixed positive integer. Consider the following sub-problem:

$$
\begin{equation*}
\max _{i_{1}<\cdots<i_{N}}\left\{\sum_{n=1}^{N} \frac{i_{n} i_{n+1}}{2(1+\tau)}\left(i_{n+1}-i_{n}\right)-c N\right\} \tag{36}
\end{equation*}
$$

with boundary conditions $i_{0}=0$ and $i_{N+1}=1$. Take the derivative with respect to each $i_{n}$ of the monopoly's objective function in the sub-problem (36) to obtain $i_{n+1}^{2}-2 i_{n} i_{n+1}+$ $2 i_{n} i_{n-1}-i_{n-1}^{2}=0$, or, equivalently:

$$
\left[\left(i_{n+1}+i_{n-1}\right)-2 i_{n}\right]\left(i_{n+1}-i_{n-1}\right)=0
$$

The second-order condition, $-2\left(i_{n+1}-i_{n-1}\right)<0$, is trivially satisfied. Because, by construction, $i_{n+1}>i_{n-1}$, we have that:

$$
\begin{equation*}
\left(i_{n+1}+i_{n-1}\right)-2 i_{n}=0 \tag{37}
\end{equation*}
$$

Consequently, $i_{n+1}-i_{n}=i_{n}-i_{n-1}$; that is, demands for all status goods have the same measure. Solving the system of difference equations (37) with boundary conditions $i_{0}=0$ and $i_{N+1}=1$, we obtain equation (10).

Part (b). By equation (10), $i_{n}^{M}=n /(N+1)$ and $i_{n+1}^{M}=(n+1) /(N+1)$. Therefore, $i_{n+1}^{M}-i_{n}^{M}=[(n+1)-n] /(N+1)=1 /(N+1)$.

Part (c). Substituting (10) into (35), simplifying, and taking $N$ as a real number, we obtain the following:

$$
\begin{equation*}
\pi^{M}=\max _{N \in \mathbb{R}_{+}}\left\{\frac{N(N+2)}{6(1+\tau)(N+1)^{2}}-c N\right\} \tag{38}
\end{equation*}
$$

The first-order condition determines the unique global maximum because the objective function of the maximization problem (38) is concave. In fact, its second-order derivative with
respect to the number of brands is $-1 /\left[(1+\tau)(N+1)^{4}\right]<0$. Taking the first-order condition of problem (38) and simplifying gives us equation (12).

Part (d). Substituting equation (10) into (5), results in:

$$
\begin{equation*}
p_{n}^{M}=\frac{n(n+1)}{2(1+\tau)(N+1)^{2}} . \tag{39}
\end{equation*}
$$

Substituting equation (12) into (39), we obtain equation (13).
Part (e). Substituting equation (12) back into sub-problem (38) and simplifying gives us the result.

Therefore, every player is playing a best response given the equilibrium action of other players. The stratified matching is weakly stable. Equation (3) holds with $N=N^{M}$ and $i_{n}=i_{n}^{M}$ for all $n \in\left\{0,1, \cdots, N^{M}\right\}$; the market for each brand $n$ of status good clears.

## A. 4 Contestable Markets

Proof of Lemma 3 (Finiteness of the Algorithm): because $\tau<-1+1 / 8 c$, then $1-$ $8 c(1+\underline{\tau})>0$. Because $c>0$ and $\tau_{n} \geq \underline{\tau}>-1$, then, for every positive integer $n$, $i_{n} \leq \frac{i_{n+1}}{2}(1+\sqrt{1-8 c(1+\underline{\tau})})$, and $\frac{1}{2}<\frac{1}{2}(1+\sqrt{1-8 c(1+\underline{\tau})})<1$. Hence, the sequence $\left\{i_{n}^{C} \mid n \in\left\{1, \cdots, N^{C}\right\}\right\}$ decays to zero at a rate which is at least as fast as the decay rate of a geometric sequence with factor $(1+\sqrt{1-8 c(1+\underline{\tau})}) / 2$. For any positive constant, no matter how small, there is a sufficiently large (yet finite) value $n$ such that $i_{n+1}$ is smaller than this constant. There is some finite positive integer number $n_{0}$ such that $i_{n_{0}+1}<$ $2(c(1+\underline{\tau}))^{1 / 3} \leq 2\left[c\left(1+\tau_{n_{0}}\right)\right]^{1 / 3}$.

Proof of Lemma 4: consider a deviation by a firm outside the market. If it enters the market charging the same price $p_{n}^{C}$ as a firm that owns brand $n \in\left\{1, \cdots, N^{C}\right\}$, it has to pay the entire cost $c$ and share the demand with the incumbent firm selling brand $n \in\left\{1, \cdots, N^{C}\right\}$. Since the incumbent is already earning zero profit, the entrant cannot make positive profit. If it enters charging a price $p^{E}>p_{N^{C}}^{C}$, it steals part of the demand from firm $N^{C}$, but it obtains a negative profit (because profit is decreasing in $i_{N}$ in the interval $i \in\left(i_{N^{C}}^{C},+\infty\right)$ ). If it enters charging a price $p^{E}$ such that $p_{n}^{C}<p^{E}<p_{n+1}^{C}$ for some brand $n \in\{0,1, \cdots, N-1\}$, then it takes a fraction of the demand of firm $n$. However, this fraction is smaller than 1 because Proposition 1 establishes that $i_{n+1}^{C}$ does not change as a result of the entry, but the lower end of the demand becomes larger than $i_{n}^{C}$. Using the obvious notation, $i_{n}^{C}<\tilde{\imath}^{E}<i_{n+1}^{C}$. Hence, $i_{n+1}^{C}-\tilde{\imath}^{E}<i_{n+1}^{C}-i_{n}^{C}$. Therefore, the entrant would
obtain a negative profit because profits decrease with the lower boundary of the demand if $\tilde{\imath}^{E}>i_{n}^{C}$.

Proof of Proposition 4 (Equilibrium Characterization in Contestable Markets): because $m^{*}(\cdot)$ is stable, no agent has incentives to look for another partner. Given $m^{*}(\cdot)$, Green agents maximize their utility, so they have no incentive to change their decisions. Each firm maximizes its profit, given Greens' demands and other firms' strategies, and thus has no incentives to deviate. Equation (3) holds with $N=N^{C}$ and $i_{n}=i_{n}^{C}$ for all $n$; thus, the market for the status good clears. To compute the values $i_{n+1}^{C}$ as functions of $i_{n}^{C}$, we use equation (17). The result is the following:

$$
i_{n+1}^{C}=\frac{i_{n}^{C}}{2}\left(1+\sqrt{1+\frac{8 c\left(1+\tau_{n}\right)}{\left(i_{n}^{C}\right)^{3}}}\right) .
$$

Substituting the equation above back on the zero profit condition $p_{n}^{C}\left(i_{n+1}^{C}-i_{n}^{C}\right)=c$ gives us equation (19). Hence, the stratified equilibrium is fully characterized.

Proof of Corollary 1: let $\tau_{n}=\tau$ for all $n$. Because profits of operational firms equal zero in contestable markets, then $i_{n+1}^{C}-i_{n}^{C}=c / p_{n}^{C}$, for every $n \in\left\{1,2, \cdots, N^{C}\right\}$. Using $\pi_{n}^{C}=0$, the definition of adjusted prices in equation (2), $\tau_{n}=\tau$, and equation (4), we find the following:

$$
i_{n+1}^{C}-i_{n}^{C}=\frac{c}{p_{n}^{C}}=\frac{2 c(1+\tau)}{\alpha_{n}}=\frac{2 c(1+\tau)}{i_{n}^{C} i_{n+1}^{C}}
$$

Because the function $n \mapsto i_{n}^{C} i_{n+1}^{C}$ is increasing, the function $n \mapsto i_{n+1}^{C}-i_{n}^{C}$ is decreasing.

## A. 5 Pigouvian Taxation

Proof of Proposition 5 (Optimal Taxation in a Monopolistic Industry):
Part (a). To prove (20), substitute the monopoly's best reply (12) into the formula for the socially optimal number of brands (8).

Part (b). Just substitute equations (8) and (20) into equation (39).
Part (c). Equation (22) comes from combining formulas (20) and (21).
Proof of Proposition 6 (Optimal Taxation in Contestable Markets): hypothesis $c<$ $(1+\phi) / 16$ is necessary and sufficient to make Algorithm 1 generate at least one status good; that is, $N^{C} \geq 1$. Without this hypothesis, no status good is produced in a contestable market.

Part (a). Equations (33) and (8) bring allocations that maximize welfare. Using the inverse demand equation (5), we find that $\pi_{n}=0$ if and only if:

$$
\begin{equation*}
1+\tau_{n}=\frac{i_{n+1} i_{n}\left(i_{n+1}-i_{n}\right)}{2 c} \tag{40}
\end{equation*}
$$

Given that the value $i_{n+1}$ is established, the tax rate must be chosen for brand $n$ to obtain $i_{n}=n /\left(N^{*}+1\right)$. Substituting this expression into equation (40), we find:

$$
1+\widehat{\widehat{\tau}}_{n}=\frac{n(n+1)}{2 c\left(N^{*}+1\right)^{3}}
$$

By equation (8), $c\left(N^{*}+1\right)^{3}=(1+\phi) / 6$. Hence, $1+\widehat{\widehat{\tau}}_{n}=3 n(n+1) /(1+\phi)$.
Part (b). We know that $i_{n+1}-i_{n}=\left(N^{*}+1\right)^{-1}$, for every $n \in\left\{0, \cdots, N^{*}\right\}$. Substituting this expression into equation (5), we obtain a constant net price under optimal tax; i.e., $\widehat{\hat{p}}_{n}=c N^{*}$, for every $n \in\left\{1, \cdots, N^{*}\right\}$. Using formula (8) again, equation (24) holds for $n \in\left\{1, \cdots, N^{*}\right\}$.

Part (c). Use the previous formula for $\widehat{\hat{p}}_{n}$ and the result of part (a) to conclude that the cum-tax prices are increasing and convex in the status levels $n$; that is, for any $n \in$ $\left\{1, \cdots, N^{*}\right\}:$

$$
\left(1+\widehat{\widehat{\tau}}_{n}\right) \widehat{\widehat{p}}_{n}=n(n+1)\left(\frac{9 c^{2}}{2(1+\phi)^{2}}\right)^{1 / 3}
$$

The cum-tax prices are equal to those in the case of a monopoly with the optimal constant tax rate $\widehat{\tau}$; that is, $\left(1+\widehat{\widehat{\tau}}_{n}\right) \widehat{\widehat{p}}_{n}=(1+\widehat{\tau}) \widehat{p}_{n}$. To prove this, it is sufficient to multiply equations (23) and (24), and then compare the result with equation (22), which has the formula for cum-tax prices in a monopolistic market.

Remark 3 Formula (23) implies that $\widehat{\widehat{\tau}}_{n}<0$ is equivalent to $\phi>3 n(n+1)-1$. When $n=1$, this inequality becomes $\phi>5$. If $\phi \leq 5$, then there is no subsidy at all, not even for the least expensive brand. If $\phi>5$, then the least expensive brands are subsidized while the more valued brands may have positive tax rates. If $\phi>5$, it may also occur that all brands are subsidized. This would be the case if even the most expensive brand is subsidized; that is, if $1+\phi>3 N^{*}\left(N^{*}+1\right)$. This is equivalent to

$$
(6 c)^{2 / 3}(1+\phi)^{2 / 3}-3(1+\phi)^{1 / 3}+3(6 c)^{1 / 3}>0
$$

which always holds for sufficiently large values of $\phi$.

## B Appendix: Leontief Technology

By following similar steps as in the main text, this section studies the stratified equilibrium under a Leontief technology for the production of the jointly consumed good. In this case, the abilities of Greens and Reds are perfect complements. Most of the results shown in the main text hold with this alternative technology.

From now on, assume that the joint production function is $z(i, j)=\min \{i, j\}$.

## B. 1 Demand for Status Goods

In the stratified equilibrium, the conditional expected joint productions of a Green with ability $i=i_{n}$ if he buys status good of brands $n$ and $n-1$ are, respectively:

$$
\begin{equation*}
E_{i_{n}}\left[z\left(i_{n}, m^{*}\left(i_{n}\right)\right) \mid s_{i_{n}}=n\right]=i_{n}, \tag{41}
\end{equation*}
$$

and

$$
\begin{equation*}
E_{i_{n}}\left[z\left(i_{n}, m^{*}\left(i_{n}\right)\right) \mid s_{i_{n}}=n-1\right]=\frac{i_{n}+i_{n-1}}{2} \tag{42}
\end{equation*}
$$

Equalizing the expected utilities of a Green agent with ability $i=i_{n}$ if he buys status good of brands $n-1$ and $n$, we find the following:

$$
y+T-\left(1+\tau_{n}\right) p_{n}+i_{n}=y+T-\left(1+\tau_{n-1}\right) p_{n-1}+\frac{i_{n}+i_{n-1}}{2} .
$$

For all $n \in\{0,1, \cdots, N\}$, adjusted prices are $\alpha_{n}=2\left(1+\tau_{n}\right) p_{n}$. Thus, $\alpha_{n}=\alpha_{n-1}+i_{n}-i_{n-1}$. Adding the boundary constraints of $i_{0}=0$ and $p_{0}=0$ and solving recursively, we obtain that, for every $n \in\{0,1, \cdots, N\}$, the inverse demands are $\alpha_{n}=i_{n}$. Equivalently, for every $n \in\{0,1, \cdots, N\}:$

$$
\begin{equation*}
p_{n}=\frac{i_{n}}{2\left(1+\tau_{n}\right)} . \tag{43}
\end{equation*}
$$

With Leontief technology, the inverse demand is linear in $i_{n}$ and depends only on the stratum's lower bound, $i_{n}$.

## B. 2 Welfare

Socially optimal allocations solve the following maximization problem:

$$
\begin{equation*}
W^{*}=\max _{N \in \mathbb{R}_{+}}\left\{\max _{i_{1}<\cdots<i_{N}} \sum_{n=0}^{N} \int_{i_{n}}^{i_{n+1}} y+(1+\phi) E_{i}\left[z\left(i, m^{*}(i)\right) \mid s(i)\right] d i-c N\right\} . \tag{44}
\end{equation*}
$$

Maximization problem (44) can be solved in two steps, as in Section 4. We need to compute $E_{i}\left[z\left(i, m^{*}(i)\right) \mid s(i)\right]$. Fix $i \in\left[i_{n}, i_{n+1}\right]$. Then:

$$
E_{i}\left[z\left(i, m^{*}\right) \mid s(i)\right]=\int_{m^{*}=i_{n}}^{i_{n+1}} \min \left\{i, m^{*}\right\} \frac{1}{i_{n+1}-i_{n}} d m^{*}
$$

If $m^{*} \leq i$, then $\min \left\{i, m^{*}\right\}=m^{*}$. If $m^{*}>i$, then $\min \left\{i, m^{*}\right\}=i$. Thus:

$$
\left(i_{n+1}-i_{n}\right) E_{i}\left[z\left(i, m^{*}\right) \mid s(i)\right]=\int_{m^{*}=i_{n}}^{i} m^{*} d m^{*}+\int_{m^{*}=i}^{i_{n+1}} i d m^{*}=\frac{-i^{2}}{2}+\frac{2\left(i_{n+1}\right) i}{2}-\frac{\left(i_{n}\right)^{2}}{2}
$$

In stratum $n$, the integral of expected values $E_{i}\left[z\left(i, m^{*}(i)\right) \mid s(i)\right]$ when $i$ runs along the interval $\left[i_{n}, i_{n+1}\right]$ is:

$$
\begin{align*}
\int_{i=i_{n}}^{i_{n+1}} E_{i}\left[z\left(i, m^{*}(i)\right) \mid s(i)\right] d i & =\frac{1}{2\left(i_{n+1}-i_{n}\right)}\left[-\int_{i=i_{n}}^{i_{n+1}} i^{2} d i+2 i_{n+1} \int_{i=i_{n}}^{i_{n+1}} i d i-\left(i_{n}\right)^{2} \int_{i=i_{n}}^{i_{n+1}} d i\right] \\
& =\frac{-\left(i_{n+1}^{2}+i_{n+1} i_{n}+i_{n}^{2}\right)}{6}+\frac{i_{n+1}\left(i_{n+1}+i_{n}\right)}{2}-\frac{\left(i_{n}\right)^{2}}{2} \\
& =\frac{2 i_{n+1}^{2}+2 i_{n+1} i_{n}-4 i_{n}^{2}}{6} \tag{45}
\end{align*}
$$

Thus:

$$
\begin{gathered}
6 \sum_{n=0}^{N} \int_{i_{n}}^{i_{n+1}} E_{i}\left[z\left(i, m^{*}(i)\right) \mid s(i)\right] d i= \\
=\cdots+2 i_{n}^{2}+2 i_{n} i_{n-1}-4 i_{n-1}^{2}+2 i_{n+1}^{2}+2 i_{n+1} i_{n}-4 i_{n}^{2}+\cdots \\
=\cdots-2\left(i_{n}\right)^{2}+2 i_{n-1}\left(i_{n}\right)+2 i_{n+1}\left(i_{n}\right)+\cdots .
\end{gathered}
$$

We are writing only terms with factors of $i_{n}$ because other terms have zero derivatives when we differentiate with respect to $i_{n}$. Hence:

$$
\frac{d}{d i_{n}}\left(6 \sum_{n=0}^{N} \int_{i_{n}}^{i_{n+1}} E_{i}\left[z\left(i, m^{*}(i)\right) \mid s(i)\right] d i\right)=-4\left(i_{n}\right)+2 i_{n-1}+2 i_{n+1}
$$

The second derivative with respect to $i_{n}$ is negative; thus, the objective function is concave. The first-order condition provides a maximum. Making the derivative equal to zero, we find $2 i_{n}=i_{n-1}+i_{n+1}$. Hence, $i_{n+1}-i_{n}=i_{n}-i_{n-1}$. The measures of all social
strata are identical in a social optimal allocation. Let $i_{n}^{*}$ represent the strata limit abilities that maximize welfare. Then, for every $n \in\{1, \cdots, N\}, i_{n}^{*}-i_{n-1}^{*}=1 /(N+1)$, and:

$$
\begin{equation*}
i_{n}^{*}=\frac{n}{N+1} . \tag{46}
\end{equation*}
$$

Using equation (45), we find that welfare is:

$$
W^{*}=y+\max _{N \in \mathbb{R}_{+}}\left\{\max _{i_{1}<\cdots<i_{N}}\left(\left[\frac{1+\phi}{6} \sum_{n=0}^{N}\left(2 i_{n+1}^{2}+2 i_{n+1} i_{n}-4 i_{n}^{2}\right)\right]-c N\right)\right\} .
$$

Substituting (46) into the expression above, after some algebra, we find the following:

$$
W^{*}=y+\max _{N \in \mathbb{R}_{+}}\left\{\left(\frac{1+\phi}{6(N+1)^{2}} \sum_{n=0}^{N}\left[2(n+1)^{2}+2 n(n+1)-4 n^{2}\right]\right)-c N\right\},
$$

or:

$$
W^{*}=y+\max _{N \in \mathbb{R}_{+}}\left\{\left(\frac{1+\phi}{6}\right)\left(\frac{3 N+2}{N+1}\right)-c N\right\} .
$$

Define $g(N)$ by:

$$
g(N)=\left(\frac{1+\phi}{6}\right)\left(\frac{3 N+2}{N+1}\right)-c N .
$$

Compute $g^{\prime}(N)$ and $g^{\prime \prime}(N)$ as follows:

$$
g^{\prime}(N)=\frac{1+\phi}{6(N+1)^{2}}-c
$$

and

$$
g^{\prime \prime}(N)=\frac{-(1+\phi)}{3(N+1)^{3}}<0 .
$$

Let $N^{*}$ be the unique solution of $g^{\prime}(N)=0$. Then:

$$
\begin{equation*}
\left(N^{*}+1\right)^{2}=\frac{1+\phi}{6 c} . \tag{47}
\end{equation*}
$$

The socially optimal number of brands is:

$$
\begin{equation*}
N^{*}=\left(\frac{1+\phi}{6 c}\right)^{1 / 2}-1 \tag{48}
\end{equation*}
$$

From now on, assume that $c \leq(1+\phi) / 6$. This implies that $N^{*} \geq 0$.

## B. 3 Monopoly

Suppose there is only one firm supplying all brands of the status good in the market, and $\tau_{n}=\tau$, for all $n$. The monopolist maximizes its profit, given by:

$$
\pi^{M}=\max _{N \in \mathbb{Z}_{+}}\left\{\max _{p \in \mathbb{R}_{+}^{N}}\left\{\sum_{n=1}^{N}\left(i_{n+1}(p)-i_{n}(p)\right) p_{n}-c N\right\}\right\} \text { such that } i_{n+1}(p)>i_{n}(p)
$$

Let $N$ be a fixed positive integer. Because $p_{n}=i_{n} /[2(1+\tau)]$, consider the following internal maximization sub-problem:

$$
\max _{i_{1}<\cdots<i_{N}}\left\{\sum_{n=1}^{N} \frac{i_{n}\left(i_{n+1}-i_{n}\right)}{2(1+\tau)}-c N\right\} .
$$

Making the derivative with respect to $i_{n}$ equal to zero, $i_{n-1}+i_{n+1}=2 i_{n}$. Because this holds for every $n \in\{1,2, \cdots, N-1\}$ and because the boundary conditions are $i_{0}=0$ and $i_{N}=1$, the solution of the monopoly's problem implies that strata measures are all equal to each other. For every $n \in\{1, \cdots, N\}, i_{n}^{M}=n /(N+1)$. For every $n \in\{1, \cdots, N\}$, then $i_{n}^{M}-$ $i_{n-1}^{M}=1 /(N+1)$. For every $n \in\{1, \cdots, N\}$, monopoly prices are $p_{n}=n /[2(1+\tau)(N+1)]$. The monopoly profit becomes:

$$
\begin{aligned}
\pi^{M} & =\max _{N}\left\{\left(\sum_{n=1}^{N} \frac{n}{2(1+\tau)(N+1)^{2}}\right)-c N\right\} \\
& =\max _{N}\left\{\frac{N}{4(1+\tau)(N+1)}-c N\right\} .
\end{aligned}
$$

Because the objective function is concave, the first-order condition establishes the global maximum, denoted $N^{M}$. Thus:

$$
\begin{equation*}
\left(N^{M}+1\right)^{2}=\frac{1}{4 c(1+\tau)} \tag{49}
\end{equation*}
$$

Therefore:

$$
\begin{equation*}
N^{M}=\sqrt{\frac{1}{4 c(1+\tau)}}-1 \tag{50}
\end{equation*}
$$

The monopoly prices are linear in $n$ :

$$
p_{n}^{M}=\frac{i_{n}^{M}}{2(1+\tau)}=\frac{n}{2(1+\tau)\left(N^{M}+1\right)}=n \sqrt{\frac{c}{1+\tau}} .
$$

The last equality was obtained using equation (50).

## B. 4 Contestable Markets

Once again, assume that each brand belongs to an independent firm. The profit of firm $n$ is:

$$
\pi_{n}=p_{n}\left(i_{n+1}^{C}-i_{n}^{C}\right)-c=\frac{i_{n}^{C}}{2\left(1+\tau_{n}\right)}\left(i_{n+1}^{C}-i_{n}^{C}\right)-c .
$$

In contestable markets, every firm makes zero profit. The equation $\pi_{n}=0$ becomes:

$$
\begin{equation*}
-\left(i_{n}^{C}\right)^{2}+i_{n+1}^{C} i_{n}^{C}-2 c\left(1+\tau_{n}\right)=0 \tag{51}
\end{equation*}
$$

Solving the equation for $i_{n}^{C}$, we find that if $i_{n+1}^{C} \geq \sqrt{8 c\left(1+\tau_{n}\right)}$, then:

$$
\begin{equation*}
i_{n}^{C}=\frac{i_{n+1}^{C}}{2}\left(1+\sqrt{1-\frac{8 c\left(1+\tau_{n}\right)}{\left(i_{n+1}^{C}\right)^{2}}}\right) \tag{52}
\end{equation*}
$$

We find the values $i_{n}^{C}$ using an algorithm analogous to Algorithm 1; the only difference is to replace $i_{n_{0}+1}^{C} \geq 2\left(c\left(1+\tau_{n_{0}}\right)\right)^{1 / 3}$ by $i_{n_{0}+1}^{C} \geq\left(8 c\left(1+\tau_{n_{0}}\right)\right)^{1 / 2}$. The expressions in (51) and (52) are similar, but not identical to those in the Cobb-Douglas case.

## B. 5 Pigouvian Taxation for Monopolies

By making $N^{*}=N^{M}$ and using equations (47) and (49), the socially optimal tax rate is:

$$
\begin{equation*}
\widehat{\tau}=\frac{1-2 \phi}{2+2 \phi} . \tag{53}
\end{equation*}
$$

The socially optimal tax rate is a decreasing function of $\phi$. If $\phi=0$, then $\tau=0.5$. Hence, $50 \%$ is the maximal socially optimal taxation. If $\phi=0.5$, then $\tau=0$. If $\phi=+\infty$, then $\tau=-1$. As the benefits of matching to Reds grow large, the positive externality of better quality matching increases and the optimal policy converges to full subsidy. ${ }^{18}$

The number of brands, $N^{M}$, satisfies:

$$
N^{M}+1=\sqrt{\frac{1}{4 c(1+\widehat{\tau})}}=\sqrt{\frac{(1+\phi)}{6 c}}
$$

Because the demand equation is given by (43), prices charged by a monopoly facing socially optimal taxation, denoted $\widehat{p}_{n}$, are given by:

$$
\begin{equation*}
\widehat{p}_{n}=\frac{i_{n}}{2(1+\widehat{\tau})}=\frac{(1+\phi) n}{3\left(N^{M}+1\right)}=\frac{(1+\phi) n}{3} \frac{6^{1 / 2} c^{1 / 2}}{(1+\phi)^{1 / 2}}=\left(\frac{\sqrt{6}}{3} \sqrt{c(1+\phi)}\right) n \tag{54}
\end{equation*}
$$

[^10]By equations (53) and (54), cum-tax prices become:

$$
\begin{equation*}
(1+\widehat{\tau}) \widehat{p}_{n}=\frac{3}{2(1+\phi)}\left(\frac{\sqrt{6 c(1+\phi)}}{3}\right) n=\left(\frac{\sqrt{6}}{2} \sqrt{\frac{c}{1+\phi}}\right) n . \tag{55}
\end{equation*}
$$

## B. 6 Pigouvian Taxation in Contestable Markets

Rewrite equation (51) as:

$$
1+\tau_{n}=\frac{i_{n}\left(i_{n+1}-i_{n}\right)}{2 c}
$$

Using $i_{n}^{*}=n /\left(N^{*}+1\right)$ and then equation (47), we find the socially optimal tax $\widehat{\widehat{\tau}}_{n}$ for a contestable market under Leontief technology:

$$
1+\widehat{\widehat{\tau}}_{n}=\frac{i_{n}^{*}\left(i_{n+1}^{*}-i_{n}^{*}\right)}{2 c}=\frac{n}{2 c\left(N^{*}+1\right)^{2}}=\frac{6 c n}{2 c(1+\phi)}=\frac{3 n}{1+\phi} .
$$

Using $\tau_{n}=\widehat{\widehat{\tau}}_{n}$ in equation (43), we obtain the pre-tax prices in a contestable market, denoted by $\widehat{\hat{p}}_{n}$ :

$$
\widehat{\widehat{p}}_{n}=\frac{i_{n}^{*}}{2\left(1+\widehat{\tau}_{n}\right)}=\frac{1+\phi}{6 n} \frac{n}{N^{*}+1}=\frac{1+\phi}{6} \sqrt{\frac{6 c}{1+\phi}}=\frac{\sqrt{6}}{6} \sqrt{c(1+\phi)}
$$

All brands have the same pre-tax price. Using this and equation (54) to compare with the monopoly, we conclude that $\widehat{p}_{n}>\widehat{\hat{p}}_{n}$ if and only if $n>1 / 2$. This is true for every $n \geq 1$. When facing socially optimal taxation, the monopoly charges pre-tax prices that are smaller than those obtained in a contestable market. Cum-tax prices in a contestable market are:

$$
\left(1+\widehat{\hat{\tau}}_{n}\right) \widehat{\hat{p}}_{n}=\frac{3 n}{1+\phi} \frac{\sqrt{6 c(1+\phi)}}{6}=\left(\frac{\sqrt{6}}{2} \sqrt{\frac{c}{1+\phi}}\right) n
$$

By equation (55), under the socially optimal taxation, monopoly cum-tax prices are equal to those in a contestable market, $\left(1+\widehat{\hat{\tau}}_{n}\right) \widehat{\hat{p}}_{n}=(1+\widehat{\tau}) \widehat{p}_{n}$.

## B. 7 Comparing the Leontief and Cobb-Douglas Cases

The aspects of the model that are identical in the two cases are:
I.1. Existence of a Stratified Equilibrium.
I.2. It is efficient that all social strata have the same measure.
I.3. Under monopoly with any exogenous flat tax, all social strata have the same measure.
I.4. In a contestable market with any exogenous flat tax, the sequence of measures of the strata is decreasing.
I.5. There is space for a welfare improving Pigouvian taxation, regardless of the market structure.
I.6. The socially optimal tax rate is constant in a monopolistic market, and progressive in a contestable market.
I.7. The welfare is larger in the stratified equilibrium with socially optimal taxation than the welfare in the case of no status goods and random matching.
I.8. There is always a positive measure of Greens that purchase no status goods; furthermore, when tax rates are flat, there is no social stratum with higher measure than the social stratum of individuals purchasing no status goods. In particular, the "no status goods" stratum is the largest if status good markets are contestable, and has the same measure as all others if status good markets are monopolistic.

## The aspects of the model that are different in the two cases are:

D.1. The strata limit abilities, prices and number of brands are numerically different. In particular, for a given flat tax rate, if $c(1+\tau)<9 / 64$ (alternatively, if $c(1+\tau)>$ $9 / 64$ ), then monopolies offer more (less) brands in the Leontief case than under CobbDouglas technology;
D.2. In the stratified equilibrium, if firms face a flat tax rate, then prices increase convexly with the strata limit abilities $i_{n}$ under Cobb-Douglas (CD) technology, and linearly in the Leontief (Lt) case.
D.3. Under a flat tax rate, the measures of the social strata decrease with $n$ at different rates in the two cases (CD and Lt).
D.4. The socially optimal number of brands (approximated as a real number) is higher under Leontief than under Cobb-Douglas technology.
D.5. The socially optimal tax rates are numerically different. In particular, there may exist parameters under which it is desirable to tax a particular brand of status goods in one case (under a particular technology) and to subsidize in the other (under the other technology).
D.6. In contestable markets, the Pigouvian gross tax rate $1+\widehat{\widehat{\tau}}$ needed to correct externalities grows linearly with $n$ under Leontief technology, and convexly in the CD case.

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Table 1: Stratum Limits and Status Good Prices (Cobb-Douglas Technology)

| c | market | $\phi$ | $N$ |  | Status Good Brand |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 0.02 | Contestab. | - | 7 | $i_{n}$ | 0.000 | 0.540 | 0.653 | 0.736 | 0.804 | 0.862 | 0.913 | 0.958 | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | 0.176 | 0.240 | 0.296 | 0.346 | 0.393 | 0.437 | 0.479 | - | - | - | - |
|  | Monopoly | - | 2 | $i_{n}$ | 0.000 | 0.333 | 0.667 | - | - | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | 0.111 | 0.333 | - | - | - | - | - | - | - | - | - |
|  | Social Planner | 0.5 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
|  |  | 1 | 2 | $i_{n}$ | 0.000 | 0.333 | 0.667 | - | - | - | - | - | - | - | - | - |
|  |  | 2 | 2 | $i_{n}$ | 0.000 | 0.333 | 0.667 | - | - | - | - | - | - | - | - | - |
| 0.04 | Contestab. | - | 3 | $i_{n}$ | 0.000 | 0.650 | 0.803 | 0.912 | - | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | 0.261 | 0.366 | 0.456 | - | - | - | - | - | - | - | - |
|  | Monopoly | - | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | 0.250 | - | - | - | - | - | - | - | - | - | - |
|  | Social Planner | 0.5 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
|  |  | 1 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
|  |  | 2 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
| 0.06 | Contestab. | - | 2 | $i_{n}$ | 0.000 | 0.644 | 0.861 | - | - | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | 0.277 | 0.430 | - | - | - | - | - | - | - | - | - |
|  | Monopoly | - | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | 0.250 | - | - | - | - | - | - | - | - | - | - |
|  | Social Planner | 0.5 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
|  |  | 1 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
|  |  | 2 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
| 0.08 | Contestab. | - | 1 | $i_{n}$ | 0.000 | 0.800 | - | - | - | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | 0.400 | - | - | - | - | - | - | - | - | - | - |
|  | Monopoly | - | 1 | $i_{n}$ | 0.000 | $0.500$ | - | - | - | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | $0.250$ | - | - | - | - | - | - | - | - | - | - |
|  | Social Planner | 0.5 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
|  |  | 1 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
|  |  | 2 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
| 0.10 | Contestab. | - | 1 | $i_{n}$ | 0.000 | 0.724 | - | - | - | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | 0.362 | - | - | - | - | - | - | - | - | - | - |
|  | Monopoly | - | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | 0.250 | - | - | - | - | - | - | - | - | - | - |
|  |  | 0.5 | 0 | $i_{n}$ | 0.000 | - | - | - | - | - | - | - | - | - | - | - |
|  | Social Planner | 1 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
|  |  | 2 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
| 0.12 | Contestab. | - | 1 | $i_{n}$ | 0.000 | 0.600 | - | - | - | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | 0.300 | - | - | - | - | - | - | - | - | - | - |
|  | Monopoly | - | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | 0.250 | - | - | - | - | - | - | - | - | - | - |
|  |  | 0.5 | 0 | $i_{n}$ | 0.000 | - | - | - | - | - | - | - | - | - | - | - |
|  | Social Planner | 1 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
|  |  | 2 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
| 0.14 | Contestab. | - | 0 | $i_{n}$ | 0.000 | - | - | - | - | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | - | - | - | - | - | - | - | - | - | - | - |
|  | Monopoly | - | 0 | $i_{n}$ | 0.000 | - | - | - | - | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | - | - | - | - | - | - | - | - | - | - | - |
|  |  | 0.5 | 0 | $i_{n}$ | 0.000 | - | - | - | - | - | - | - | - | - | - | - |
|  | Social Planner | 1 | 0 | $i_{n}$ | 0.000 | - | - | - | - | - | - | - | - | - | - | - |
|  |  | 2 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |

Table 2: Stratum Limits and Status Good Prices (Leontief Technology)

| $c$ | market | $\phi$ | N |  | Status Good Brand |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 0.02 | Contestab. | - | 11 | $i_{n}$ | 0.000 | 0.218 | 0.401 | 0.501 | 0.581 | 0.650 | 0.711 | 0.768 | 0.820 | 0.868 | 0.915 | 0.958 |
|  |  |  |  | $p_{n}$ | 0.000 | 0.109 | 0.201 | 0.251 | 0.290 | 0.325 | 0.356 | 0.384 | 0.410 | 0.434 | 0.457 | 0.479 |
|  | Monopoly | - | 3 | $i_{n}$ | 0.000 | 0.250 | 0.500 | 0.750 | - | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | 0.125 | 0.250 | 0.375 | - | - | - | - | - | - | - | - |
|  | Social Planner | 0.5 | 3 | $i_{n}$ | 0.000 | 0.250 | 0.500 | 0.750 | - | - | - | - | - | - | - | - |
|  |  | 1 | 3 | $i_{n}$ | 0.000 | 0.250 | 0.500 | 0.750 | - | - | - | - | - | - | - | - |
|  |  | 2 | 4 | $i_{n}$ | 0.000 | 0.200 | 0.400 | 0.600 | 0.800 | - | - | - | - | - | - | - |
| 0.04 | Contestab. | - | 4 | $i_{n}$ | 0.000 | 0.556 | 0.700 | 0.814 | 0.912 | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | 0.278 | 0.350 | 0.407 | 0.456 | - | - | - | - | - | - | - |
|  | Monopoly | - | 2 | $i_{n}$ | 0.000 | 0.333 | 0.667 | - | - | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | 0.167 | 0.333 | - | - | - | - | - | - | - | - | - |
|  | Social Planner | 0.5 | 2 | $i_{n}$ | 0.000 | 0.333 | 0.667 | - | - | - | - | - | - | - | - | - |
|  |  | 1 | 2 | $i_{n}$ | 0.000 | 0.333 | 0.667 | - | - | - | - | - | - | - | - | - |
|  |  | 2 | 3 | $i_{n}$ | 0.000 | 0.250 | 0.500 | 0.750 | - | - | - | - | - | - | - | - |
| 0.06 | Contestab. | - | 2 | $i_{n}$ | 0.000 | 0.686 | 0.861 | - | - | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | 0.343 | 0.430 | - | - | - | - | - | - | - | - | - |
|  | Monopoly | - | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | 0.250 | - | - | - | - | - | - | - | - | - | - |
|  | Social Planner | 0.5 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
|  |  | 1 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
|  |  | 2 | 2 | $i_{n}$ | 0.000 | 0.333 | 0.667 | - | - | - | - | - | - | - | - | - |
| 0.08 | Contestab. | - | 2 | $i_{n}$ |  | 0.400 | 0.800 | - | - | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | 0.200 | 0.400 | - | - | - | - | - | - | - | - | - |
|  | Monopoly | - | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | 0.250 | - | - | - | - | - | - | - | - | - | - |
|  | Social Planner | 0.5 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
|  |  | 1 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
|  |  | 2 | 2 | $i_{n}$ | 0.000 | 0.333 | 0.667 | - | - | - | - | - | - | - | - | - |
| 0.10 | Contestab. | - | 1 | $i_{n}$ | 0.000 | 0.724 | - | - | - | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | 0.362 | - | - | - | - | - | - | - | - | - | - |
|  | Monopoly | - | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | 0.250 | - | - | - | - | - | - | - | - | - | - |
|  | Social Planner | 0.5 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
|  |  | 1 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
|  |  | 2 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
| 0.12 | Contestab. | - | 1 | $i_{n}$ | 0.000 | 0.600 | - | - | - | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | 0.300 | - | - | - | - | - | - | - | - | - | - |
|  | Monopoly | - | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | 0.250 | - | - | - | - | - | - | - | - | - | - |
|  |  | 0.5 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
|  | Social Planner | 1 | 1 | $i_{n}$ | 0.000 | 0.500 |  | - | - | - | - | - | - | - | - | - |
|  |  | 2 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
| 0.14 | Contestab. | - | 0 | $i_{n}$ | 0.000 | - | - | - | - | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | - | - | - | - | - | - | - | - | - | - | - |
|  | Monopoly | - | 0 | $i_{n}$ | 0.000 | - | - | - | - | - | - | - | - | - | - | - |
|  |  |  |  | $p_{n}$ | 0.000 | - | - | - | - | - | - | - | - | - | - | - |
|  |  | 0.5 | 0 | $i_{n}$ | 0.000 | - | - | - | - | - | - | - | - | - | - | - |
|  | Social Planner | 1 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | - |
|  |  | 2 | 1 | $i_{n}$ | 0.000 | 0.500 | - | - | - | - | - | - | - | - | - | $-$ |


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[^1]:    ${ }^{1}$ Some examples of status goods demand models are Cole et al. (1993, 1995), Hopkins and Kornienko (2004, 2005, 2006), Ireland (1994, 2001), Frank (1985, 2005), and Rege (2008). Models of the supply of status goods include Pesendorfer (1995), Bagwell and Bernheim (1996), Díaz et al. (2008), and Rayo (forthcoming). For a survey on the literature of the demand for status goods, see Heffetz and Frank (2008).
    ${ }^{2}$ Prices taken at amazon.com on March 7, 2011.
    ${ }^{3}$ Bagwell and Bernheim (1996), pg. 352.
    ${ }^{4}$ See, for instance, Vickers and Renand (2003).
    ${ }^{5}$ US $\$ 146$ billion were spent globally on jewelry in 2005, and the expected expenditure for 2015 is US $\$ 230$ billion. These figures belong to a KPMG study. See Shor (2007).

[^2]:    ${ }^{6}$ This result is valid under Cobb-Douglas (multiplicative) or Leontief technologies for the joint production.
    ${ }^{7}$ We consider consumption taxes because they are sufficient to generate efficient outcomes and because this is one of the most studied forms of intervention in status good markets. See, for instance, Ireland (2001), Rege (2008) and Hopkins and Kornienko (2004).

[^3]:    ${ }^{8}$ More recent studies, such as Robson (1992), Direr (2001), Diaz et al. (2008), have also used this approach, and focused on the overconsumption (or oversaving) issue and their welfare and distributional implications.

[^4]:    ${ }^{9}$ One can interpret the matching concerns in our model as marriage concerns (Becker, 1973, 1974; Cole et al., 1992, 1995; Pesendorfer, 1995), job market candidate/employer matching (Hosios, 1990; Roth and Peranson, 1999, Bulow and Levin, 2006), client/customer matching (e.g., lawyers, doctors, college admissions), energy rationing (McAfee, 2002), graduate school advisor/advisee matching or any other matching situation. For instance, in the job market, candidates would be Greens, firms would be Reds, and while the quality of firms would be common knowledge, the quality of each particular candidate would be her/his private information. Before observing signals, firms would know only the distribution of candidates' qualities.
    ${ }^{10}$ Rates $\widehat{\tau}$ and $\widehat{\hat{\tau}}_{N^{*}}$ are calculated, respectively, by formulas (20) and (23), in Section 7 , and $N^{*}$ is given by equation (8). More precisely, $\bar{\tau}>(1-\phi) /(1+\phi)$, and $\bar{\tau}>-1+3\left[(1+\phi)^{1 / 3}-(6 c)^{1 / 3}\right] /[(1+\phi)(6 c)]^{2 / 3}$.

[^5]:    ${ }^{11}$ Appendix B has an extension in which joint production exhibits Leontief technology. Like multiplicative production functions, Leontief technologies also display this desirable property.
    ${ }^{12}$ Hopkins and Kornienko (2004) used a model with general utility functions, where status signals wealth to show that the amount spent on status goods grows more than proportionally with income and that welfare may decrease as the economy grows.

[^6]:    ${ }^{13}$ This property of the demand also holds when function $z(i, j)$ is Leontief, as in Appendix B, but it is not valid for a generic supermodular function $z(i, j)$.

[^7]:    ${ }^{14}$ Because $N^{*}$ may fail to be an integer, the optimal number of brands is either $\left[N^{*}\right]$ or $\left[N^{*}\right]+1$, where $[N]$ denotes the largest integer that is smaller or equal to $N$. Although the number of brands is a non-negative integer, we will consider that it is a real number, as an abstract exercise.

[^8]:    ${ }^{15}$ Because $c<1 /[3(1+\tau)]$, it follows from equation (14) that:

    $$
    \frac{d \pi^{M}}{d c}=1-\frac{1}{[3 c(1+\tau)]^{1 / 3}}<0, \quad \text { and } \quad \frac{d \pi^{M}}{d \tau}=\frac{-\left[1+(3 c)^{2 / 3}(1+\tau)^{4 / 3}\right]}{6(1+\tau)^{2}}<0
    $$

[^9]:    ${ }^{17}$ This is not a robust result. Under Leontief technology for $z(i, j)$, studied in Appendix B, the optimal tax rate is not zero when $\phi=1$.

[^10]:    ${ }^{18}$ In the socially optimal taxation of a monopoly, the two cases (with different joint production functions) compare as follows (using the obvious notation): $\frac{1+\widehat{\tau}^{\text {Leontief }}}{1+\hat{\tau}^{C D}}=\frac{3}{4}$.

