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INTERPRETING AND USING EMPIRICAL ESTIMATES OF THE MCF

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Interpreting and Using Empirical Estimates of the MCF

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There is considerable variability in numerical estimates of the marginal social cost of public funds (MCF) due to differences in demand-supply elasticities, differences in the welfare measures of changes in the excess burden of taxation, and differences in the conceptual measure of the MCF used. In a model with standardised parameters, Fullerton [8] finds the different welfare measures of the excess burden make little difference to the numerical estimates, where most of the variance can be explained by conceptual differences. In particular, some studies estimate the conventional Harberger [10] measure of the MCF while others estimate a modified measure. We formally derive the modified MCF in a public good economy and compare it to the conventional measure. Being project specific it must be used and interpreted differently. We also derive the relationship between the actual MCF and the compensated MEB, and extend the conventional MCF to accommodate higher effective marginal tax rates on income due to withdrawal of family and other tax benefits.

Key Words: Marginal social cost of public funds, marginal excess burden of taxation  
JEL Classification: D61; H20; H40.

1. INTRODUCTION

In policy evaluation analysts follow a convention established by Pigou [15] and Harberger [10] by using the marginal social cost of public funds (MCF) to measure the social cost of each dollar of revenue raised by the government to balance its budget. It will in general exceed unity due to the marginal excess burden of taxation (MEB), where MCF = 1+MEB > 1 whenever MEB > 0. And the larger the MEB the lower the optimal level of government spending.\footnote{In economies with no lump-sum taxes the optimal level of provision of pure public goods is lower than it is in the first best economy without restrictions on lump-sum taxes. But Wilson [21] shows how, in the absence of distributional effects, it is possible for the optimal level of public good provision to exceed the first-best level when there are distorting commodity taxes as well as a poll tax.}

Clearly, empirical estimates of the MCF provide policy analysts with important information needed for setting Ramsey optimal taxes and choosing the optimal level of government spending, where Ramsey optimal taxes have the same MCF that makes the social cost of raising government revenue independent of the tax used.

Ideally, analysts in each country would have access to estimates of the MCF for all taxes, which is rarely the case. They normally have access to estimates for one or two of the main taxes levied, in particular, income taxes. Table 1. below summarises empirical estimates of the MCF in five countries - the United States, Canada, Sweden, New Zealand and Australia. While it seems reasonable enough to expect that these estimates would vary across countries due to differences in tax structures, it is problematic when they vary across studies within the same country. While a small amount of this variability can be explained by differences in modelling parameters and the combination of taxes being changed, most of it is due to differences in the conceptual measures of the MCF.
Table 1: Numerical Estimates of the MCF

<table>
<thead>
<tr>
<th>Study</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>United States:</strong></td>
<td></td>
</tr>
<tr>
<td>Ballard and Fullerton (1992) - wage taxes</td>
<td>1.05 - 1.32</td>
</tr>
<tr>
<td>Fullerton (1991) - wage taxes</td>
<td>1 - 1.25</td>
</tr>
<tr>
<td>Ballard, Shoven and Whalley (1985) - all taxes</td>
<td>1.17 - 1.56</td>
</tr>
<tr>
<td>- wage taxes*</td>
<td>1.16 - 1.18</td>
</tr>
<tr>
<td>Browning (1987) - wage taxes</td>
<td>1.32 - 1.47</td>
</tr>
<tr>
<td>Stuart (1984) - wage taxes</td>
<td>1.07 - 1.53</td>
</tr>
<tr>
<td><strong>Canada:</strong></td>
<td></td>
</tr>
<tr>
<td>Campbell (1975) - consumption taxes</td>
<td>1.25</td>
</tr>
<tr>
<td>Dahlby (1994) - - wage taxes*</td>
<td>1.38</td>
</tr>
<tr>
<td><strong>Sweden:</strong></td>
<td></td>
</tr>
<tr>
<td>Hansson and Stuart (1995) - all taxes</td>
<td>1.07 - 1.16</td>
</tr>
<tr>
<td><strong>New Zealand:</strong></td>
<td></td>
</tr>
<tr>
<td>Diewet and Lawrence (1995) - all taxes</td>
<td>1.18</td>
</tr>
<tr>
<td><strong>Australia:</strong></td>
<td></td>
</tr>
<tr>
<td>Campbell and Bond (1997) - wage taxes*</td>
<td>1.19 - 1.24</td>
</tr>
<tr>
<td>Findlay and Jones (1982) - wage taxes*</td>
<td>1.28 - 1.55</td>
</tr>
</tbody>
</table>

* Personal income taxes - taxes on labour paid by firms not included.

This is confirmed by Fullerton [8] who uses a standardised model to show how almost all the variability in the estimates for wage taxes in the U.S. by Stuart [19], Ballard, Shoven and Whalley (BSW) [4] and Browning [5] arise from differences in the conceptual measure of the MEB. Stuart measures it as \((CS - dR)/dR\), with \(CS\) being compensating surplus and \(dR\) the actual (observed) change in tax revenue, BSW as \((EV - dR)/dR\), with \(EV\) being the equivalent variation, and Browning as \((EV - dR^*)/dR\), with \(dR^*\) being the change in tax revenue when utility is held constant at the new level. Since \(CS\) and \(EV\) have similar values for small tax changes any difference in the estimates arises from the measure of the change in tax revenue. Using a standardised model of a public good economy with Stuart’s parameters Fullerton finds the MEB for Browning is 25 cents while it is 7 cents for both Stuart and BSW.

Using a similar GE framework, however, we find that most of this difference arises from the fact that Stuart estimates a modified measure of the MCF, while Browning estimates the conventional measure. Indeed, all the US estimates in Table 1 above, except those by Browning, are for the modified MCF. It is used as a scaling coefficient on government revenue spent on inputs used to produce a public good (or service), and as such, is project specific. In contrast, the conventional MCF is used as a scaling coefficient on net changes in the government budget and is independent of the project undertaken. As a way to demonstrate the difference between them, consider the familiar revised Samuelson [16] condition for the optimal supply of a pure public good using the approach recommended by Pigou [15], where \(\sum MRS = MCF \cdot MRT\), with \(\sum MRS\) being the summed marginal consumption benefits and \(MRT\) the marginal resource cost incurred by the government. If this resource cost is the project’s only impact on the government budget there is a deficit of \(MRT\), and the cost to private surplus of

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2The public good economy has a large number of identical consumers (to rule out distributional concerns) who maximise a utility function that is separable over a private good, leisure and a pure public good that is provided exogenously by the government. In this model there is a single tax on wage income paid to labour.

3When computing the compensated change in tax revenue Stuart estimates tax revenue after the equivalent variation \((EV)\) but before the tax change. But the tax revenue should be measured after the \(EV\) and the tax change because it is the equilibrium outcome that would be realised by making the compensating transfers. Indeed, that is the way the excess burden is measured by Auerbach [2].
funding it is obtained by multiplying $MRT$ by the conventional $MCF$ to account for changes in the excess burden of taxation.

In these circumstances there is no difference between the conventional and modified measures of the MCF. But subsequent work by Diamond and Mirrlees [7] and Stiglitz and Dasgupta [18] identified additional welfare effects when extra output of public goods impact on taxed (and other distorted) activities. Goulder and Williams [9] argue these indirect welfare changes can play an important role in policy evaluation, particularly when projects impact on activities subject to income taxes which are significant in most countries. Any resulting change in tax revenue from extra output of the public good ($dR/dG$) causes the budget deficit for the project to be $MRT - dR/dG$, where the revised Samuelson condition becomes $\sum MRS = MCF \cdot (MRT - dR/dG)$. Atkinson and Stern [1] refer to $dR/dG$ as the ‘spending effect’, and when positive it reduces the budget deficit from the project. Ballard and Fullerton [3] argue the Pigou-Harberger-Browning approach that ‘measures the efficiency effects of taxes, given the level of government spending......seems poorly suited to the cost-benefit problem of whether the level of government spending should increase, given that spending must be financed with additional distortionary taxes’ (Ballard and Fullerton [3]: p.119). They therefore rule out lump-sum transfers to separate the welfare effects of the policy changes by writing the revised Samuelson condition as $\sum MRS = MCF^* \cdot MRT$, where the ‘spending effect’ is included in a modified MCF, with $MCF^* = MCF(1 - \frac{1}{MRT} \frac{dR}{dG})$.

While both approaches lead to the same optimal supply of the public good they use very different measures of the MCF. In particular, the conventional measure is independent of the project being funded, while the modified measure is project specific. Thus, for each tax there is a separate modified MCF for every possible way the government can spend the revenue. In all the US studies reported in Table 1 above the ‘spending effect’ is positive so the modified measure of the MCF is less than the conventional measure. And that is the main reason why estimates of the modified MCF by Ballard and Fullerton, Fullerton, BSW and Stuart are lower on average than the conventional measures estimated by Browning. It is also why they obtain estimates of the MCF for lump-sum taxes that differ from unity when it must always be unity for the conventional measure.

A further source of confusion arises from the way the MCF is estimated by the studies reported in Table 1. Dahlby [6] argues the conventional MCF for a lump-sum tax can differ from unity when income effects impact on taxed activities. But he estimates the social value of endowing an extra dollar of revenue on the economy, which measures the shadow value of government revenue and not the MCF. Triest [20] actually refers to the MCF as a shadow price, but that is not the case. Instead, it is the change in private surplus from transferring a dollar of revenue between the private and public sectors of the economy. No additional real income is endowed on (or taken from) the economy, which is why the conventional MCF must always be unity for a lump-sum tax. Jones [14] shows how these two concepts - the shadow value of government revenue and the conventional MCF - play important but different roles in policy evaluation. In this paper we derive them in a public good economy and obtain empirical estimates to show how much their values can differ. Thus, using one of them in place of the other can lead to inappropriate policy choices.

Finally, Atkinson and Stern [1] find there are circumstances where the income effect from a tax change can completely offset the distortionary effect, where the actual welfare loss is zero, with $MEB = 0$ and $MCF = 1$. This suggests the income effects can play a role in policy evaluation in single (aggregated) consumer economies, but we show they cannot in single (aggregated) consumer economies. Ultimately, income effects can only arise from efficiency effects in this setting.

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4 Snow and Warren [17] formally derive the modified measure of the MCF by writing the revised Samuelson condition in this way.
1.1. The Conventional MCF

Before outlining the public good economy model used in Ballard and Fullerton, and Fullerton we provide a definition of the MCF and then illustrate it for a single tax.

**Definition 1.** The conventional measure of the marginal social cost of public funds (MCF) for any tax \( t \) is the direct cost to private surplus from using it to transfer a dollar of revenue to the government budget.

It is illustrated using a partial equilibrium analysis in Figure 1. below where the pre-tax wage \( w \) equates the demand for labour \( (N_D) \) to its supply \( (N_S) \) at the post-tax wage \( w - t \).

\[ MCF_t = 1 + \frac{MEB_t}{a+b-c} \]

(1)

where \( MEB_t = \frac{c}{a+b-c} \) is the conventional measure of the marginal excess burden of taxation. Following Harberger we isolate the welfare loss in \( c \) in a conventional manner by marginally raising \( t \) and returning the revenue to consumers as a lump-sum transfer. This computes the welfare change in a balanced equilibrium where the private and public sector constraints both bind. Clearly, when the tax is non-distorting, with \( c = 0 \), we have \( MCF_t = 1 \). The same applies for a lump-sum tax because because real income is unchanged when the revenue raised is returned through lump-sum transfers. A general equilibrium analysis extends the analysis by including welfare changes in related markets. For example, if the higher wage tax causes activity to expand in other markets that are subject to taxes the extra tax revenue reduces \( MEB_t \). The reverse applies when the the tax change causes other taxed activities to contract.

2. THE PUBLIC GOOD ECONOMY MODEL

We undertake a cost-benefit analysis in a public good economy where a large number of identical consumers choose private good \( X \) and leisure \( H \) to maximise aggregate utility \( U(X, H, G) \), with \( G \) being a pure public good provided solely by the public sector.\(^5\) Distributional effects are ruled by identical consumers who can be aggregated into a single consumer. Throughout

\(^5\)To simplify the analysis we follow Ballard and Fullerton and assume good \( G \) is not provided privately. It is perfectly non-rivalrous and non-excludable.
the analysis $X$ is chosen as numeraire so that all values and prices are measured in units of the good, where expenditure on $X$ is funded from after tax wage income $(w - t)(T - H)$, with $T$ being an endowment of time, profits from private production $\pi$, and lump-sum transfers $L$ from the government budget. The wage tax $\tau$ captures the classic distortion to the labour-leisure choice.

The structure of the economy is illustrated in Figure 2. below where the time endowment is the sole constraint on consumption expenditure. Some of it is consumed directly as leisure while the remainder is supplied as labour used to produce good $X$ which is consumed and used as an input to production good $G$. Good $X$ is produced by competitive private firms whose profits $\pi$ accrue to consumers as owners, whereas $G$ is produced solely by the government using units of good $X$ as input $C$. Since $X$ is numeraire demand will always be equal to supply $F(N) - C$ at the equilibrium price of unity.

![Figure 2: The Public Good Economy Model](image)

In the absence of the wage tax good $G$ is optimally supplied when the summed marginal consumption benefits $\sum MRS$ are equated to the marginal resource cost $MRT = dC/dG$, where $C$ is the amount of $X$ used to produce another unit of good $G$. The role of the MCF becomes apparent once extra output of the public good is funded using the wage tax instead of lump-sum transfers. In this economy the social planner makes policy choices to maximise social welfare which is a function of the exogenous policy variables $G, t$ and $C$, and the time endowment $T$, with:

$$W(G, t, C, T) = \max \left\{ U(X, H, G) \middle| \begin{array}{l} X = (w - t)(T - H) + \pi + L \\ \pi = F(N) - wN \\ L + C = tN = R \end{array} \right\},$$

where $X = (w - t)(T - H) + \pi + L$ is the private budget constraint, $\pi = F(N) - wN$ profit from private production of good $X$, and $L = tN - C$ the government budget constraint that makes lump-sum transfers $L$ and input demand $C$ equal to wage tax revenue $tN$.

By totally differentiating (2) with respect to the policy variables we obtain the conventional welfare equation:

$$\frac{dW(\cdot)}{\lambda} = \sum MRS \cdot dG + t dN(\cdot) - dC.\footnote{A formal derivation of this equation is provided in Appendix A.1.}$$
with \((\cdot) = (G, t, C, T)\), \(\lambda\) the marginal utility of income, and \(\sum MRS = \frac{1}{\lambda} \frac{\partial U}{\partial G}\), the summed marginal consumption benefits from the public good.\(^7\) We use this equation in the next section to isolate aggregated changes in utility (measured in units of the numeraire good \(X\)), where the first term captures direct welfare changes from extra output of good \(G\), the second term welfare gains from reducing the excess burden of taxation when employment rises, and the third term the direct cost to private surplus of increases in public sector demand for private good \(X\). We follow the convention established by Harberger [11] by using lump-sum transfers to balance the government budget. This allows us to separate the welfare effects of the policy variables in a balanced equilibrium. By combining the private and public sector budget constraints we obtain a virtual constraint for the economy of \(X = F(N) - C\), which is the market clearing condition for the numeraire good.

### 3. POLICY EVALUATION AND THE MCF

As a way to demonstrate the role played by the MCF we first derive an optimality condition for the provision of the public good when the government balances its budget using lump-sum transfers, then we revise it when the government uses the distorting wage tax. The public good is optimally supplied in the presence of wage tax when:

\[
\sum MRS = MRT - \frac{dR(\cdot)}{dG},
\]

where \(MRT = dC/dG\) is the marginal cost to government revenue of producing good \(G\), and \(\frac{dR(\cdot)}{dG} = t \frac{dN(\cdot)}{dG}\), the endogenous impact of the entire project on wage tax revenue.\(^8\) Even though revenue for the project is raised using lump-sum transfers there are additional welfare effects when the change in real income affects employment, where the resulting change in wage tax revenue is referred to by Atkinson and Stern as the "spending effect". It was initially identified in the general equilibrium analysis undertaken by Diamond and Mirrlees [7] and Stiglitz and Dasgupta [18]. A positive "spending effect" reduces the excess burden of taxation and contributes to the production cost \(MRT\). The original Samuelson condition was derived in a first best economy without market distortions, and is obtained from (4) by setting the wage tax to zero where we have \(\sum MRS = MRT\). The welfare changes in (4) are illustrated in Figure 3 with a positive "spending effect".\(^9\)

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\(^7\) \(\lambda\) is the multiplier on the private budget constraint in the Lagrangean function for each consumer. In this setting consumers have the same marginal utility of income because they are identical.

\(^8\) This condition is obtained by marginally raising output of the public good \(G\) holding the wage tax \(\tau\) and the time endowment \(T\) constant, where the welfare change is obtained from (3), as:

\[
\frac{dW(\cdot)}{dG} = \sum MRS + t \frac{\partial N(\cdot)}{\partial G} + \left( t \frac{\partial N(\cdot)}{\partial G} - 1 \right) \frac{dC}{dG} = 0.
\]

After combining the endogenous changes in wage tax revenue for the project as \(\frac{dR(\cdot)}{dG} = t \frac{\partial N(\cdot)}{\partial G} + t \frac{\partial N(\cdot)}{\partial G} MRT\), we obtain the welfare change in (4).

\(^9\) When extra output of good \(G\) causes labour supply to rise the after-tax wage falls by the same amount in the presence of the specific tax \(t\). Later we compute numerical estimates when the tax rate is defined in ad valorem terms as \(\tau = t/w\), where the after-tax wage falls less when the wage rate falls endogenously.
In this setting the budget deficit \( MRT - dR(\cdot)/dG \) is funded by lump-sum transfers from consumers. When this revenue is raised using the wage tax we revise the optimality condition in (4), as:

\[
\sum MRS = MCF_t \left( MRT - \frac{dR(\cdot)}{dG} \right), \quad (5)
\]

with \( MEB_t = \frac{-\partial N}{N+1} \) and \( MCF_t = \frac{N}{N+1} \). Notice how the MCF is a scaling coefficient on the net change in the government budget. If the project drives the budget into deficit the cost to private surplus of funding it is larger due to the excess burden of taxation whenever \( MCF_t = 1 + MEB_t > 1 \). The last term in (5) adds the change in the excess burden of taxation due to endogenous changes in the tax wedge. For example, if the project causes the wage rate to rise the tax wedge expands and transfers private surplus to the government budget as additional tax revenue. If this revenue is returned to the consumer as a lump-sum transfer to balance the government budget it cancels the fall in private surplus and there is no welfare change. But when it is returned by reducing the wage tax welfare rises by the reduction in the excess burden of taxation.

Ballard and Fullerton consider a special case for the revised Samuelson condition in (5) where the project has no impact on employment. It is illustrated below in Figure 4 where the distortionary effect from the tax change is completely offset by the ‘spending effect’. Thus, with no net change in the excess burden of taxation Ballard and Fullerton conclude the MCF for the single distorting wage tax is unity. But to anyone familiar with the conventional measure of the MCF it suggests the wage tax is non-distorting, which is not the case. Ballard and Fullerton, like all the other US studies in Table 1. except Browning, estimate a modified measure of the MCF.

\footnote{A derivation of this welfare change is provided in Appendix A.2.}
3.1. A Modified Measure of the MCF

At times there are claims about the properties of the MCF in the public finance literature that are inconsistent with the conventional Harberger measure. For example, Hansson and Stuart [12] argue "An important complication is that the welfare cost of augmenting public revenue using a given tax instrument is not unique but instead depends on the exact use to which the funds are directed." (pp.331-2.) Based on the analysis in the previous section it is clear that the conventional MCF is independent of the way the revenue is spent because the MEB is isolated using lump-sum transfers that return the revenue to taxpayers. In a similar fashion the welfare effects of government spending are isolated using lump-sum transfers which are scaled by the MCF when the revenue is raised using distorting taxes. As Harberger [11] points out, this approach has important practical implications because it allows the welfare effects of tax changes and government spending to be computed by separate government agencies.

Snow and Warren [17] make it clear why the modified measure of the MCF is project specific where they write the revised Samuelson condition in (5), as:

\[
\sum MRS = MCF^*_t MRT. \tag{6}
\]

By using the conventional analysis in (5) it is possible to solve this modified MCF as \(MCF^*_t = MCF_t \left(1 - \frac{1}{MRT} \frac{dR(t)}{dt}\right)\). If the project has a positive ‘spending effect’ this measure of the MCF is less than than the conventional measure. While the revised conditions in (5) and (6) lead to the same optimal level of \(G\), the two measures of the MCF play different roles. The conventional measure isolates the cost to private surplus of raising a dollar of government revenue to balance the government budget, and is independent of the way it is spent, while the modified measure isolates the cost to private surplus of raising a dollar of revenue spent on inputs used to produce the public good which makes it project specific. Indeed, for each tax there can be as many measures of the modified MCF as ways the government can spend the revenue. If analysts treat it as the conventional MCF they will understate the true welfare cost of balancing the government budget when the ‘spending effect’ is positive. Later in section 4 we compute numerical estimates of both measures of the MCF in a public good economy using standardised parameters and find the modified measure is less than unity and approximately 14 per cent lower than the conventional MCF. Analysts frequently use empirical estimates of the modified MCF, like those presented in Table 1 earlier, to measure the social cost of raising...
a dollar of tax revenue. But, as is clear from (6) that is not what it measures; it is the social cost of spending a dollar on project input costs.

Now we can see one of the reasons why studies reported in Table 1 obtain estimates of the MCF for lump-sum taxes that differ from unity. For example, in the special case where the project has no net impact on employment Ballard and Fullerton report estimates below unity because there is a positive "spending effect". Moreover, they find the MCF for the wage tax is unity in this setting. And we can see from the modified MCF in (6) why this happens, the "spending effect" offsets the MEB and there is no change in the wage rate, with $MCF_i^* = MCF_i \left(1 - \frac{1}{MRT} \frac{dR(\cdot)}{dG} \right) = 1$.

3.2. The Shadow Value of Government Revenue

Another reason why studies obtain empirical estimates of the MCF for lump-sum taxes that differ from unity is that they compute the social cost of public funds as a shadow price. In fact, there is no clear distinction between the conventional MCF and the shadow value of government revenue. But they are different and play different roles in welfare analysis. We saw earlier how the conventional MCF measures the cost to private surplus of transferring a dollar of revenue between the private and public sectors of the economy; it used as a scaling coefficient on revenue transfers the government makes to balance its budget. In contrast, the shadow value of government revenue is the welfare change from endowing another dollar of revenue on the economy; it is used to convert compensated welfare changes into dollar changes in utility. In other words, it isolates any income effects. The difference between them can best be understood by noting the MCF is computed by reallocating existing resources, while the shadow value of government revenue is computed by endowing additional resources on the economy.

We can compare the MCF and the shadow value of government revenue by computing the real income the economy can sacrifice at unchanged utility when the project analysed in the previous section is undertaken. Using the social welfare function we obtain a relationship between the actual and compensated welfare changes, of:

$$\left( \frac{\partial W(\cdot)}{\partial G} \frac{1}{\lambda} \right)_D = (S_R)_D \cdot \left( \frac{d\hat{C}}{dG} \right)_D,$$

where $(S_R)_D = -\left( \frac{\partial W(\cdot)}{\partial C} \frac{1}{\lambda} \right)_D$ is the (revised) shadow value of government revenue, and $\left( \frac{d\hat{C}}{dG} \right)_D$ the compensated welfare change from the project. A compensated welfare gain is surplus revenue the project can generate when the wage tax is used to hold utility constant. As an efficiency gain it is revenue the government can pay as foreign aid at no cost to domestic utility. When it is distributed to consumers by lowering the wage tax the shadow value of government revenue converts it into the actual welfare change. Thus, all the income effects from the project are isolated by the shadow value of government revenue.\(^{12}\) This is an important relationship because it tells us that income effects play no role in finding the optimal level of

\(^{11}\)Consider the project when $C$ changes endogenously to hold utility constant, where we have:

$$\left( \frac{\partial W(\cdot)}{\partial C} \frac{1}{\lambda} \right)_D = \left( \frac{\partial W(\cdot)}{\partial G} \frac{1}{\lambda} \right)_D + \left( \frac{\partial W(\cdot)}{\partial C} \frac{1}{\lambda} \right)_D \left( \frac{d\hat{C}}{dG} \right)_D = 0.$$

The first term isolates the actual welfare change for the project, while the second term isolates the transfers that must be made to offset it. We obtain (7) by rearranging these terms.

\(^{12}\)This is the Hatta [13] decomposition which is generalised by Jones [14] to economies with variable producer prices and heterogeneous consumers. It shows us that income effects play no role in welfare analysis in identical consumer economies, while in heterogeneous consumer economies the shadow value of government revenue conveniently isolates any distributional effects.
government spending in single (aggregate) consumer economies. Using the welfare equation in (3) we can solve the shadow value of government revenue, as:

\[(SR)_D = MCF_t \left(1 - \frac{\partial R(\cdot)}{\partial C}\right)\]  

13

After a dollar of revenue is endowed on the government and transferred to the private sector the increase in real income raises leisure (when it is normal) and causes labour supply to contract. The resulting fall in tax revenue (with \(-\frac{\partial R(\cdot)}{\partial C} < 0\)) causes the government budget to go into surplus by less than a dollar (with \(1 - \frac{\partial R(\cdot)}{\partial C} < 1\)). When this it is transferred to consumers by reducing the wage tax each dollar raises private surplus by \(MCF_t\), which is larger than a dollar due to the reduction in the excess burden of taxation.14 But there is also a welfare loss when the increase in leisure causes the wage rate to fall and reduces the tax wedge. When this gain in private surplus is transferred to the government using the wage tax welfare falls by the increase in the excess burden of taxation. It is clear from (8) that the shadow value of government revenue is not the MCF, and it cannot be used in the same way in policy evaluation work. As a scaling coefficient the MCF converts private surplus into government revenue. No additional resources are endowed on the economy. In contrast, the shadow value of government revenue converts extra real income into private surplus, which is why it converts compensated welfare changes into actual welfare changes in (7). In section 4 below where we obtain numerical estimates for the welfare changes using standardised parameters the shadow value of government revenue is approximately 6 per cent lower than the conventional MCF.

### 3.3. The Relationship Between the Compensated MEB and the Actual MCF

As noted earlier in the introduction, most empirical studies estimate the compensated MEB, where some use the EV and others the CV. But analysts normally compute and report actual welfare changes when evaluating public projects, where they use the conventional measure of the actual MCF to scale changes in the government budget. It is rare indeed for them to use these estimates of the MEB to isolate efficiency effects for projects, because as hypothetical welfare changes they are much harder to explain. Indeed, Fullerton argues that 1 plus the compensated MEB computed by Browning “is not generally enough information to evaluate the public project, because the decision rule should be modified by both the "distortionary effect" and "revenue effect".”(p. 305). Since the ‘revenue effect’ isolates income effects in the actual MCF this statement clearly indicates that project evaluation is based on actual welfare changes.

But that raises the obvious question, what is the relationship between the compensated MEB and the actual MCF? In other words, how do we use estimates of the compensated MEB to compute the actual MCF? As suggested by Fullerton, most analysts add the compensated MEB to unity and use that as the MCF. But that ignores any income effects from the ‘distortionary effect’ of the tax change. The correct relationship is obtained by separating the income effects in the actual MCF in (5), as:

\[MCF_t = \frac{1}{1 - \frac{MEB_t}{1 - \theta}},\]  

where \(\text{MEB}_t\) is the compensated MEB and \(\frac{1}{1 - \theta}\) the shadow value of government revenue, with

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13 A derivation of this welfare change is provided in Appendix A.3.

14 When taxed activities are normal an extra dollar of income raises private surplus by more than a dollar due to an endogenous increase in tax revenue. And this gain in surplus is further enhanced when the the transfers are made by lowering distorting taxes. In the current setting the wage tax works to subsidise leisure, where extra real income raises the subsidy cost. Thus, the budget surplus by less than the initial dollar. It is possible, however, for private surplus to rise by more than a dollar, with \((SR)_D > 1\), when the wage tax is used to balance the government budget and \(MCF_t > 1\).
\[ \theta = \frac{\partial H(w_A, G, I)}{\partial t} \]

\( \overline{MEB}_t \) is the fall in real income that causes utility to fall by \( MEB_t \), where the shadow value of government revenue isolates any income effects. \( \frac{1}{\theta} \) measures the change in utility from endowing another unit of real income on the government who transfers it to the consumer using lump-sum transfers. When the surplus revenue is transferred using tax \( t \) the shadow value of government revenue becomes \( \overline{MCF}_t \), where the welfare loss is \( \overline{MCF}_t \cdot \overline{MEB}_t \). Using this decomposition we obtain the familiar relationship between the actual MEB and the actual MCF, of \( MCF_t = 1 + \overline{MEB}_t \).

### 3.4. Welfare Benefit Withdrawal and the MCF

Most estimates of the MCF consider the explicit tax rates on activity but ignore implicit tax rates arising from the withdrawal of welfare benefits as assessable income rises. In Australia these implicit tax rates are quite significant because most households receive some form of family tax benefit when the primary earner has taxable income at or below $150,000. The payments depend on family size and the combined incomes of all members of the household.

<table>
<thead>
<tr>
<th>Taxable Income</th>
<th>Marginal Cash Tax Rate</th>
<th>Family Tax Benefit B (^1)</th>
<th>Rate of Benefit B Withdrawal per $ of Taxable Income</th>
<th>Pre-Tax and Pre-Family Benefit A Family Income</th>
<th>Family Tax Benefit A</th>
<th>Rate of Benefit A Withdrawal per $ of Taxable Income</th>
<th>Effective Marginal Tax Rate</th>
<th>Average Tax Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0.00 $ 3,693.80 0.00 $ 3,693.80 $ 4,631.85 0.00 0.00 0.00</td>
<td>6,000 0.00 $ 3,693.80 0.00 $ 9,693.80 $ 4,631.85 0.00 0.00 0.00</td>
<td>34,000 0.17 $ 3,693.80 0.00 $ 9,694.80 $ 4,631.85 0.00 0.17 0.11</td>
<td>34,001 0.17 $ 3,693.80 0.00 $ 9,694.80 $ 4,631.85 0.00 0.32 0.11</td>
<td>38,865 0.32 $ 3,693.80 0.00 $ 42,559.00 $ 4,631.85 0.00 0.32 0.13</td>
<td>38,866 0.32 $ 3,693.80 0.00 $ 42,560.00 $ 4,631.65 0.20 0.52 0.13</td>
<td>52,297 0.32 $ 3,693.80 0.00 $ 55,991.00 $ 4,631.85 0.00 0.32 0.18</td>
<td>52,298 0.32 $ 3,693.80 0.00 $ 55,992.00 $ 4,631.65 0.00 0.32 0.18</td>
<td>80,000 0.32 $ 3,693.80 0.00 $ 83,693.80 $ 4,631.85 0.00 0.32 0.22</td>
</tr>
<tr>
<td>90,622 0.42 $ 3,693.80 0.00 $ 94,316.00 $ 4,631.85 0.00 0.42 0.24</td>
<td>90,623 0.42 $ 3,693.80 0.00 $ 94,317.00 $ 4,631.15 0.30 0.72 0.24</td>
<td>97,107 0.42 $ 3,693.80 0.00 $ 100,801.00 $ 4,631.85 0.00 0.30 0.72 0.26</td>
<td>97,108 0.42 $ 3,693.80 0.00 $ 100,802.00 $ 4,631.85 0.00 0.42 0.26</td>
<td>150,000 0.42 $ 3,693.80 0.00 $ 153,693.80 $ 4,631.85 0.00 0.42 0.31</td>
<td>150,001 0.42 $ 0.00 1.00 $ 150,001.00 $ 4,631.85 0.00 1.42 0.32</td>
<td>150,002 0.42 $ 0.00 0.00 $ 150,002.00 $ 4,631.85 0.00 0.42 0.32</td>
<td>180,000 0.42 $ 0.00 0.00 $ 180,000.00 $ 4,631.85 0.00 0.42 0.34</td>
<td>180,001 0.47 $ 0.00 0.00 $ 180,001.00 $ 4,631.65 0.00 0.47 0.34</td>
</tr>
</tbody>
</table>

Note: 1. Combined cash tax rate and 1.5 per cent medicare levy. 2. Primary earner has taxable income < $150,000 p.a. and the secondary earner has no taxable income. 3. Includes a supplement of $335.80 p.a. 4. Taxable income plus family tax benefit B (and any other pensions and allowances received). 5. Sum of combined cash tax rate and rate of benefit withdrawal for both A and B.

An example of the way benefit drawback impacts on the effective marginal tax rates is summarised in Table 2. for a two parent household with one child under 5 years of age. There are three regions where the effective rate rises significantly above the cash rate - when pre-tax income is in the range $44,166 to $58,090 it is 20 percentage points higher, when it is in the

---

\(^{15}\)This decomposition is provided in Appendix A.4. In a compensated welfare analysis the MEB isolates the efficiency loss from marginally raising \( t \) dollar of compensating transfers made to hold utility constant, with \( \overline{MEB}_t = -\frac{\partial H(w_A, I)}{\partial t} \).
range $94,317 to $101,004 is 30 percentage points higher and at $150,001 where the family tax benefit B of $3,829 is withdrawn entirely it jumps to 3,829 percentage points.

The effective marginal, marginal cash and average tax rates are illustrated in Figure 5. below where the jumps in the effective rates can be seen more clearly. The effective rate at $150,000 was capped at unity to stop it distorting the scaling in the diagram.

![Tax Rates (%) - 2009/10]

Family tax benefits change with the size and composition of the household, where more dependent children raise them while additional working adults reduce them. Even though households receiving these benefits actually pay less total tax, which makes them politically attractive, they can face quite high marginal effective tax rates. Clearly, it reduces the incentive for households to supply more labour thereby raising the excess burden of taxation. When policy changes impact on labour supplied by high tax individuals the MCF increases.

To demonstrate how welfare benefit withdrawal affects the MCF we transfer an amount $B$ (measured in units of good $X$) to consumers in the public good economy. They are paid as a lump-sum transfer and then withdrawn at a constant rate $\delta$ as income rises above threshold income $\bar{Y}$, where the private budget constraint becomes $X = B + (w-t-\delta)(T-H) + \pi + L$, with $\delta > 0$ when $w(T-H) > \bar{Y}$ and $B > 0$. Once the benefit has been completely withdrawn $\delta = 0$. These transfers mean the government budget constraint becomes $L = (t+\delta)N - B - C$. After making these changes to the social planner’s problem in (2) we obtain a revised Samuelson condition, of:

$$\sum MRS = MCF_t \left( MRT - (t+\delta) \frac{dN(\cdot)}{dG} \right), \quad (10)$$

with $MCF_t = \frac{N}{N+(t+\delta)w}$.\(^{16}\) In effect, the marginal tax rate on income has now risen to $t+\delta$, where in practice the tax rate on family benefits ranges from 20 to 30 percentage points for taxable income levels below $150,001 when family tax benefit A is withdrawn. Once income reaches $150,001 the tax rate $\delta$ spikes at 3,829 percentage points when family tax benefit B is completely withdrawn. For higher income levels only tax rate $t$ applies. Clearly, the impact benefit withdrawal has on the MCF depends on the initial level of taxable income.

\(^{16}\)This obtained using \( MEB_t = \frac{-w(T-H)}{\Delta G} \frac{\partial R(\cdot)}{\partial G} \), with \( \frac{\partial R(\cdot)}{\partial G} = wN + (\tau + \delta)w \frac{\partial N(\cdot)}{\partial G} + (\tau + \delta)N \frac{\partial w(\cdot)}{\partial G} \).
In heterogenous consumer economies the effective tax rate \( t + \delta \) will in general vary across taxpayers. In the following section we examine a range of effective tax rates to account for the effect of this variance impact on the MCF.

4. NUMERICAL ESTIMATES OF THE MCF

We use the public good economy in (2) to compute the different welfare measures obtained in previous sections. Numerical values are obtained by using a log utility function, with \( U = \ln X + \alpha \ln H + \ln G \), and a production technology for producing good \( X \) of \( F(N) = \beta \ln N \). We set the time constraint \((T)\) to 24 and \( \beta = 8 \), while the demand coefficient \( \alpha \) is chosen to make the compensated labour supply elasticity \( \eta = 0.2 \) when the tax rate is 40 per cent. These are the values of the tax rate and labour elasticity for estimates of the MCF reported in Table 1 for the wage tax in the US. Since tax rates are normally defined in ad valorem terms we write the specific tax as \( t = \tau w \), where additional welfare effects arise when changes in the wage rate affect the size of the tax wedge endogenously. After solving the equilibrium values of the endogenously determined variables they are used to compute the welfare changes isolated in previous sections when output \( G \) is optimally supplied. The welfare changes are reported in Table 3 below for a variable and a fixed wage rate, where the fixed wage results from making the marginal product of labour constant. A number of analysts frequently hold producer prices fixed because it simplifies the analysis.\(^{17}\)

\[
\begin{array}{ccc}
\text{Numerical Estimates} & \text{Variable Producer Prices} & \text{Fixed Producer Prices} \\
\hline
\text{Wage Tax } \tau = 0.4 & & \\
\text{Conventional MCF} & 1.12 & 1.13 \\
\text{Modified MCF} & 1.05 & 0.97 \\
\text{Spending effect} & 0.07 & 0.14 \\
\text{Shadow Value of Government Revenue} & 1.05 & 0.97 \\
\text{Lump-Sum Tax} & & \\
\text{Conventional MCF} & 1.00 & 1.00 \\
\text{Modified MCF} & 0.94 & 0.86 \\
\text{Shadow Value of Government Revenue} & 0.94 & 0.86 \\
\text{Wage Tax } \tau = 0.6 & & \\
\text{Conventional MCF} & 1.44 & 1.57 \\
\text{Modified MCF} & 1.29 & 1.14 \\
\text{Spending effect} & 0.13 & 0.27 \\
\text{Shadow Value of Government Revenue} & 1.28 & 1.14 \\
\text{Lump-Sum Tax} & & \\
\text{Conventional MCF} & 1.00 & 1.00 \\
\text{Modified MCF} & 0.87 & 0.73 \\
\text{Shadow Value of Government Revenue} & 0.87 & 0.73 \\
\end{array}
\]

Notice how the positive ‘spending effect’ makes all the estimates of the modified MCF lower than the conventional MCF for both the wage and lump-sum taxes. And the difference is even larger at the higher tax rate and with fixed producer prices. At the tax rate of 40 per cent the modified MCF is 7 percentage points lower when producer prices are variable, while it is 43 percentage points lower for the 60 per cent tax rate when producer prices are fixed. Clearly, when analysts treat these estimates of the modified MCF as the conventional measure they will underestimate the true social cost of balancing the government budget and recommend

\(^{17}\)When the relative wage is endogenous we use the labour market clearing condition to solve changes in the wage rate. Examples of this are provided in Appendix A.5.
‘over-provision’ of the public good. The positive ‘spending effect’ makes the modified MCF for
the lump-sum tax less than unity whereas for the conventional MCF it is always unity.

Based on the preferences we have used the shadow value of government revenue is lower
than the conventional MCF. When an extra dollar of real income is endowed on the economy
consumers increase leisure (because it is normal) and reduce their labour supply. The resulting
fall in employment raises the excess burden of taxation and partially offsets the initial increase
in real income. In effect, the government is forced to raise more tax revenue to balance its
budget and that absorbs some of the initial increase in real income for the economy. All four
estimates of the shadow value of government revenue are the same as the modified MCF in
Table 2 above. This occurs here because the extra output of the public good has no direct
impact on tax revenue. Rather, the change in tax revenue is due to the marginal resource cost
of producing the public good. In other words, the ‘spending effect’ arises from direct changes
in $C$ and not $G$. It is a property of the separable utility function we have used.

As noted earlier in the introduction, a number of analysts compute the marginal excess
burden of taxation by normalising compensated changes in the excess burden of taxation over
actual changes in tax revenue. In section 3 we isolated the optimal supply of good $G$ using
actual welfare changes. That is, as changes in utility measured in units of the numeraire good
$X$. As a consequence, the marginal excess burden of taxation in (5) solves as the actual change
in the excess burden divided by the actual change in tax revenue. In section 3.2 the generalised
Hatta decomposition was used to show why income effects played no role in the welfare analysis
in single (aggregated) consumer economies. Thus, we obtain the same optimal supply of good $G$
by using a compensated welfare analysis where the marginal excess burden of taxation solves
as the compensated change in the excess burden of taxation (referred to as the efficiency loss)
divided by the compensated revenue transfers made by the government to hold utility constant.
Both these measures of the MEB are defined by the welfare analysis used. An actual welfare
analysis isolates changes in the excess burden and revenue transfers as observed equilibrium
outcomes, whereas a compensated welfare analysis measures them as outcomes when utility is
held constant to isolate changes in real income that cause the actual welfare changes. None of
the hybrid measures of the MEB which combine actual and compensated welfare changes arise
naturally as general equilibrium outcomes. If we adopt the approach used by Browning and
derive the MEB for the wage tax as the compensated change in the excess burden of taxation
divided by the actual change in tax revenue it has the following values in our model.

<table>
<thead>
<tr>
<th>Numerical Estimates</th>
<th>Variable Producer Prices</th>
<th>Fixed Producer Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Wage Tax $\tau = 0.4$</strong></td>
<td>$\eta = 0.20$</td>
<td>$\eta = 0.20$</td>
</tr>
<tr>
<td>Conventional MCF</td>
<td>1.12</td>
<td>1.13</td>
</tr>
<tr>
<td>Browning's MCF</td>
<td>1.13</td>
<td>1.15</td>
</tr>
<tr>
<td><strong>Wage Tax $\tau = 0.6$</strong></td>
<td>$\eta = 0.32$</td>
<td>$\eta = 0.33$</td>
</tr>
<tr>
<td>Conventional MCF</td>
<td>1.44</td>
<td>1.57</td>
</tr>
<tr>
<td>Browning's MCF</td>
<td>1.51</td>
<td>1.78</td>
</tr>
</tbody>
</table>

There are only small differences between the estimates of the MCF by Browning and the
conventional measure when the tax rate is 40 per cent and the compensated labour supply
elasticity is 0.2. Thus, almost all of the difference between the estimates by Browning and
the other US studies in Table 1, is that Browning is based on the conventional analysis which
returns tax revenue to consumers as lump-sum transfers while the others estimate the modified
MCF with a ‘positive spending’ which does not isolate the welfare effects of the tax change in
a conventional manner.
5. CONCLUSION

Empirical estimates of the MCF provide policy analysts with important information about the social cost of government spending. Most of the variability in these estimates can be explained by the choice between the conventional and modified measures of the MCF, where the modified measure is project specific. Using a conventional welfare analysis we formally derive the relationship between them, as \( MCF^*_t = MCF_t \left( 1 - \frac{1}{\text{MEB}_t} \frac{dR(\cdot)}{dt} \right) \), where a positive ‘spending effect’ \( (dR(\cdot)dG > 0) \) makes the modified measure lower. By performing this decomposition we showed how a conventional welfare analysis is possible when governments use distorting taxes to make revenue transfers, contrary to assertions by Fullerton, and Ballard and Fullerton. Using a standard public good economy model we find a positive ‘spending effect’ that reduces the modified MCF by 7 to 27 percentage points depending on the compensated elasticity of labour supply and the initial tax rate. Based on these findings, it is important for analysts to know what measure of the MCF is being reported. If they use estimates of the modified MCF as the conventional measure they will understate the social cost of balancing the government budget, while using the conventional MCF as the modified measure overstates the project input costs.

A number of studies report empirical estimates of the compensated MEB, but there is some confusion about the way to use them in project evaluation based on actual welfare changes. We demonstrate the correct way to use them by decomposing the income and substitution effects in the actual MCF, as \( MCF_t = \frac{1}{1 - \text{MEB}_t} \). This decomposition conveniently separates the ‘distortionary’ and ‘revenue’ effects identified by Atkinson and Stern, and, consistent with the conventional measure of the MCF, it is unity for lump-sum taxes.

6. APPENDIX

A.1 After combining the public and private sector constraints in (2) and using the profit function we obtain a virtual budget constraint for the economy of \( X = F(N) - C \). It is the market clearing condition for the numeraire good which allows us to write the utility function as \( W(\cdot) = U(F - C, H, G) \). After totally differentiating it with respect to the exogenous policy variables we find the aggregated dollar change in utility solves as:

\[
dW(\cdot) = \frac{\partial U}{\partial X} [F_N dN(\cdot) - dC] + \frac{\partial U}{\partial H} dH(\cdot) + \frac{\partial U}{\partial G} dG.
\]

Using the first order conditions for optimally chosen consumption of private good \( X \) and leisure, with \( \partial U/\partial X = \lambda \) and \( \partial U/\partial X = \lambda(w - t) \), respectively, and the first order condition for profit maximising private firms, with \( F_N = w \), this welfare change becomes:

\[
dW(\cdot) = -\lambda [wdN(\cdot) - dC] - \lambda(w - t)dN(\cdot) + \lambda \sum MRS dG,
\]

where \( \lambda \) is the marginal utility of income (measured in units of the numeraire good \( X \)), \( \sum MRS = \frac{1}{\lambda} \frac{\partial U}{\partial X} \) the summed marginal consumption benefits from the public good measured in units of good \( X \), and \( dN(\cdot) = \frac{\partial N(\cdot)}{\partial G} dG + \frac{\partial N(\cdot)}{\partial t} dt + \frac{\partial N(\cdot)}{\partial C} dC \) the total change in employment from marginal increases in the policy variables \( G, t \) and \( C \). After rearranging terms we obtain the welfare equation in (3).

A.2 This revised condition is obtained by marginally raising output of the public good \( G \) and allowing wage tax \( t \) to change endogenously to balance the government budget with \( T \) held constant. From (3), the revised \((D)\) welfare change for the optimally supply of the public good, is:

\[
\left( \frac{dW(\cdot)}{dG} \frac{1}{\lambda} \right)_D = \sum MRS + \frac{dR(\cdot)}{dG} - MRT + \frac{\partial N(\cdot)}{\partial t} \left( \frac{dt}{dG} \right)_D = 0.
\]
Using the government budget constraint the tax change solves:

$$\left( \frac{dL}{dC} \right)_D = t \frac{dN(\cdot)}{dC} - MRT + \left( N + t \frac{\partial N(\cdot)}{\partial t} \right) \left( \frac{dt}{dC} \right)_D = 0,$$

with $\frac{dN(\cdot)}{dC} = \frac{\partial N(\cdot)}{\partial C} + \frac{\partial N(\cdot)}{\partial t} MRT$, where:

$$\left( \frac{dt}{dC} \right)_D = - \frac{dR(\cdot)}{dC} - MRT + N + t \frac{\partial N(\cdot)}{\partial t} \left( \frac{dt}{dC} \right)_D = 0,$$

with:

$$\left( \frac{dt}{dC} \right)_D = \frac{1 - t \frac{\partial N(\cdot)}{\partial C}}{N + t \frac{\partial N(\cdot)}{\partial t}}.$$

The revised welfare change in (5) is obtained by substituting this expression for the tax change into the welfare change above.

**A.3** The revised shadow value of government revenue is obtained by exogenously reducing input $C$ holding output of the public good $G$ constant, where from (3) we have:

$$\frac{dN(\cdot)}{dG} = \frac{\partial N(\cdot)}{\partial G} + \frac{\partial N(\cdot)}{\partial C} MRT,$$

where:

$$\frac{dN(\cdot)}{dG} = \left( \frac{dt}{dC} \right)_D = \frac{1 - t \frac{\partial N(\cdot)}{\partial C}}{N + t \frac{\partial N(\cdot)}{\partial t}}.$$

The revised welfare change in (7) is obtained by substituting this expression for the tax change into the welfare change above.

**A.4** We can rewrite the MCF in (5), as:

$$MCF_t = \frac{1}{1 - \frac{t \partial H(\cdot)}{\partial t}}.$$

When the consumer chooses $H$ prices and income are taken as given, but they are functions of the exogenous variables in a general equilibrium analysis. Thus, it is a Bailey demand function, with $H(G, t, C, T) = H[w_A(G, t, C, T), G, I(G, t, C, T)]$ and $w_A = w - t$. Using this function we can separate the income and substitution effects in the tax weighted change in leisure, as:

$$t \frac{\partial H(\cdot)}{\partial t} = t \frac{\partial H(\cdot)}{\partial t} \left( \frac{\partial w(\cdot)}{\partial t} - 1 \right) + \frac{t \partial H(\cdot)}{\partial I} \frac{\partial I(\cdot)}{\partial t},$$

where $(\cdot) = (G, t, C, T)$ identifies partial derivatives of the Bailey demand function, and $(\cdot) = (w_A, G, I)$ partial derivatives of the Marshallian demand function. Using income defined by the aggregated budget constraint in (2), as $I = w_A H + F(N) - C$, we find the income effect solves as $\frac{\partial I(\cdot)}{\partial t} = H \left( \frac{\partial w(\cdot)}{\partial t} - 1 \right) - t \frac{\partial I(\cdot)}{\partial t}$, while the change in the after tax wage is solved using the market clearing condition of the labour market, as $\frac{\partial w_A(\cdot)}{\partial t} = \frac{\partial w(\cdot)}{\partial t} - 1$.

After substitution, and using the Slutsky decomposition, we can write the tax weighted changes in activity, as:

$$t \frac{\partial H(\cdot)}{\partial t} = t \frac{\partial H(\cdot)}{\partial t} \left( \frac{\partial w(\cdot)}{\partial t} - 1 \right) - \frac{t \partial H(\cdot)}{\partial I} \frac{\partial I(\cdot)}{\partial t},$$

18 When deriving this income effect we use the first order condition for competitive private firms, with $F_N = w$. 
with \( \frac{\partial H}{\partial w_A} = \frac{\partial H(w_A, G, U_t)}{\partial w_A} \) being the compensated change leisure. We solve the change in the wage rate by using the labour market clearing condition, where:

\[
- \frac{\partial H(\cdot)}{\partial w_A} \left( \frac{\partial w(\cdot)}{\partial t} - 1 \right) - \frac{\partial H(\cdot)}{\partial I} \frac{\partial I}{\partial t} = \frac{\partial N_D(w)}{\partial w} \frac{\partial w(\cdot)}{\partial t},
\]

where \( N_D(w) \) is the labour demand function. After substituting the change in income, and using the Slutsky decomposition, we have:

\[
\frac{\partial w(\cdot)}{\partial t} = \frac{\frac{\partial H}{\partial w_A} + \frac{\partial H(\cdot)}{\partial I} \frac{\partial I}{\partial t}}{\frac{\partial H}{\partial w_A} + \frac{\partial N_D(w)}{\partial w}}. \tag{19}
\]

This allows us to write the tax weighted change in activity above, as:

\[
\frac{t}{N} \frac{\partial H(\cdot)}{\partial t} = - \frac{MEBt}{1 - \theta},
\]

where \( MEB_t = - \frac{\frac{\partial H}{\partial w_A} + \frac{\partial N_D(w)}{\partial w}}{\frac{\partial H}{\partial w_A} + \frac{\partial N_D(w)}{\partial w}} \) and \( \theta = - \frac{\frac{\partial H(\cdot)}{\partial I} \frac{\partial I}{\partial t}}{\frac{\partial H}{\partial w_A} + \frac{\partial N_D(w)}{\partial w}} \). After substituting this into the MCF above we obtain the decomposition in (9).

**A.5** When the tax rate is defined in ad valorem terms, with \( \tau = t/w \), there are additional terms in the optimality conditions in (5), (6) and (9), and the revised shadow value of government revenue in (8). Once the project causes the wage rate to change endogenously it affects the size of the tax wedge, where the optimality condition in (5), becomes:

\[
\sum MRS = MCF_\tau \left( MRT - \frac{dR(\cdot)}{dG} \right) - MEB_\tau N \frac{dw(\cdot)}{dG}. \tag{5'}
\]

with \( MCF_\tau = \frac{w N + \tau w}{w N + \tau w + \tau N \frac{dw(\cdot)}{dG}} \) and \( \frac{dw(\cdot)}{dG} = \frac{\partial w(\cdot)}{\partial S} + \frac{\partial w(\cdot)}{\partial C} MRT \). The last term in (5’) is an additional welfare gain when the project causes the wage to increase. The larger tax wedge causes a net loss in private surplus which is collected as additional tax revenue. Once the government lowers the tax rate balance its budget there is a welfare gain from reducing the excess burden of taxation, where the modified MCF in (6), becomes:

\[
MCF^*_\tau = MCF_\tau \left( 1 - \frac{1}{MRT} \frac{dR(\cdot)}{dG} - \frac{MEB_\tau \tau N}{MRT} \frac{dw(\cdot)}{dG} \right).
\]

With an ad valorem tax the shadow value of government revenue in (8) solves as:

\[
(S_R)_D = MCF_\tau \left( 1 - \frac{\partial R(\cdot)}{\partial C} \right) - MEB_\tau \tau N \frac{\partial w(\cdot)}{\partial C}, \tag{8'}
\]

while the optimality condition with welfare benefit drawback in (9) will be:

\[
\sum MRS = MCF_\tau \left( MRT - (\tau + \delta) w \frac{dN(\cdot)}{dG} \right) - MEB_\tau (\tau + \delta) N \frac{dw(\cdot)}{dG}. \tag{9'}
\]

Changes in the wage rate are obtained by using the market clearing condition. For the marginal increase in output of the public good, we have

\[
- \frac{\partial H(\cdot)}{\partial w_A} (1 - \tau) \frac{\partial w(\cdot)}{\partial G} - \frac{\partial H(\cdot)}{\partial I} \frac{\partial I}{\partial G} - \frac{\partial H(\cdot)}{\partial G} - \frac{\partial N_D(w)}{\partial w} \frac{\partial w(\cdot)}{\partial G} = \frac{\partial H(\cdot)}{\partial w} \frac{\partial w(\cdot)}{\partial G}.
\]

\(^{19}\text{We use the Slutsky decomposition} \frac{\partial H}{\partial w_A} = \frac{\partial H(\cdot)}{\partial w_A} + \frac{\partial H(\cdot)}{\partial I} H.\)
Using the income effect \( \frac{\partial I}{\partial G} = (1 - \tau)H \frac{\partial \omega}{\partial G} + \tau w \frac{\partial N}{\partial G} \), and the Slutsky decomposition, we have:
\[
\frac{\partial \omega}{\partial G} = - \frac{\partial H}{\partial I} \tau w \frac{\partial N}{\partial w} \frac{\partial w}{\partial G} + \frac{\partial H}{\partial G} - \frac{1}{1 - \theta} \frac{\partial H}{\partial \omega} \frac{\partial N}{\partial w} \frac{\partial w}{\partial G},
\]
with \( \theta = - \frac{\partial H}{\partial G(1 - \tau) + \frac{\partial N}{\partial w}} \). Undertaking a similar decomposition for the marginal increase in \( C \) leads to:
\[
\frac{\partial \omega}{\partial C} = \frac{1}{1 - \theta} \frac{\partial H}{\partial I} \frac{\partial N}{\partial w} \frac{\partial w}{\partial G}.
\]
Once these these partial changes are combined the change in the wage rate for the project, becomes:
\[
\frac{dw}{dG} = \frac{1}{1 - \theta} \frac{\partial H}{\partial I} - \frac{\partial H}{\partial \omega} \frac{MRT}{\frac{\partial N}{\partial w}}.
\]
Numerical estimates are obtained by solving the Marshallian and Hicksian demand functions for leisure, as well as the labour demand function.

REFERENCES


