HOW INSIDE MONEY MAKES INFLATION COSTLY FOR MOST (BUT GAINFUL FOR SOME)

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Abstract

It is argued that inflation creates private incentives for (socially costly) inside money to supplant (socially costless) outside money. Consequently, the familiar ‘shoe leather cost’ of inflation, that operates through a reduced demand for money under inflation, is supplemented by a separate social cost of inflation that operates through an increased supply of (inside) money under inflation. It is further argued that allowance of the costliness of an inflation-induced expansion of inside money changes the character of the distribution of the costs of inflation. Certain suppliers of inside money may experience a net gain from an inflation. The upshot is that inflation is no longer necessarily a ‘common enemy’, but may be welcomed by some economic interests.
Theories of why inflation is costly are plentiful (see Dowd 1994 for a survey). This paper advances an additional explanation as to why inflation is costly. This paper finds an ‘inflation cost culprit’ in the apparently useful phenomenon of inside money. This paper argues that inflation creates private incentives for socially costly inside money to supplant socially costless outside money.

The paper’s argument turns critically on a certain theory of the supply of inside money. This theory is squarely based on optimisation, is impelled by Hicksian themes of the competition between inside and outside money, and sets out from the question, ‘As outside money has a (private) opportunity cost that a mere promise to pay outside money does not, why is outside money used at all?’.

The paper’s theory of inside money (see Coleman 2007 for a full treatment) identifies the nominal rate of return on physical capital, $\iota$, as the key determinant of the supply of inside money, and implies that the supply of inside money is a positively function of the nominal rate of return on capital.

The paper’s theory of the supply of inside has important implications for the analysis of the costliness of inflation. Since inflation increases the nominal rate of return on capital, $\iota$, the theory implies that inflation will increase the issue of inside money. And that increase in inside money will substitute for a certain amount of outside money. But that substitution amounts to the substitution of a socially costless form of money – outside money- by (it will be argued) a socially costly form of money –

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1 See Hicks on the competition between inside and outside money (Hicks 1939, 1989).
inside money. Inflation has therefore has a social cost on account of the supply response of inside money.

Further, the paper’s theory of the supply of inside has important implications for the analysis of the distribution of the costs of inflation. For while the theory asserts that inflation induces a supply response in inside money that is socially costly, the theory allows that some persons may experience a net gain from an inflation-induced expansion of inside money. These are persons whose costs of supplying inside money are so favourable that they supply a disproportionate amount of the total increase in inside money, and that permits them to acquire capital by the issue of inside money, at a net gain to themselves. Because of this possibility - because some suppliers of inside money may experience a net gain from an inflation-induced expansion of inside money - inflation is no longer necessarily a common enemy but may be welcomed by part of the community.

1. A model of the supply of inside money.

As has been underlined, the thesis of this paper rests on a particular theory of the supply of inside money. This section is, therefore, is devoted to concisely presenting this theory.

The elements

The model assumes that holding money provides a benefit that is represented by the appearance of real money holdings in the utility function.
\[ U = u(C, C_1, ..., C_T; h, h_1, ..., h_T) \]  \hfill (1)

\[ C = \text{consumption} \]

\[ h = \text{holdings of real money balances} \]

\[ T = \text{final period; the current period is indexed as zero} \]

The model assumes that money holdings, \( h \), can consist of either outside money or inside money. Outside money is state money; ‘fiduciary’ notes and coin. Inside money is a (credible) promise to pay outside money. More precisely, inside money is a credible promise to pay outside money to the bearer of the promise, on the demand of the bearer, and at no cost to the bearer.\(^2\) These promises will circulate within that network of people who have been persuaded of these promises credibility.

As inside money is a credible promise to pay the bearer outside money, the benefit of an extra unit of inside money is the same as the benefit of an extra unit of outside money, \( U_h \). Given this perfect substitutability of inside and outside money, there is from the point of view of the money holder, just ‘money’, and the money holder will hold money until the marginal utility of money relative to the marginal utility of consumption – i.e. the implicit yield on money - equals the nominal rate of return on capital.

\[ \frac{U_h}{U_C} = r \]  \hfill (2)

\(^2\) Economic historians have extensively documented cases where the population at large have used as a medium of exchange such promises to pay. See Shann (1938 pp. 52-3), and O’Connell and Reid (2005).
\[
U_h = \text{marginal utility of real balances}
\]
\[
U_c = \text{marginal utility of consumption}
\]
\[
\iota = \text{nominal rate of return on physical capital}^3
\]

The principal cost of the supply of inside money is assumed to comprise the cost of making any promise to pay a *credible* promise. There are costs in making a promise a credible, namely the costs of providing evidence of the solvency and the honesty of the issuer of the promise. Evidence of solvency includes audited accounts, and perhaps investment in ‘conspicuous capital’ (e.g., ostentatious buildings). Evidence of honesty might include demonstrations of the willingness of persons of known honesty to associate with, and speak for, the issuer of the promise. These evidences are costly, and we will call these costs ‘credibility costs’.  

We suppose that there are two reasons one has to spend more on credibility to increase one’s issue of inside money;

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3. It proves convenient to measure the nominal rate of return on capital as the increment in nominal value, between period zero and period one, expressed as a proportion of the nominal value in period one (rather than in period zero). Thus
\[
\iota \equiv \frac{\rho + \pi + \rho \pi}{[1 + \rho][1 + \pi]} = \frac{\rho}{1 + \rho} + \frac{\pi}{1 + \pi} - \frac{\rho}{1 + \rho} \frac{\pi}{1 + \pi} \approx \rho + \pi .
\]

4. There is another cost of the supply of inside money. This cost turns on our assumption that the promise to pay money is a promise to pay money at no cost to the bearer, where ‘cost’ includes inconvenience and time loss to the bearer in being paid. The provision of honouring a promise in a way that is both convenient and timely to the bearer also involves cost to the issuer. We might call these ‘convenience costs’. There is, thirdly, the matter of ‘operational costs’. It will cost money to produce the physical embodiment of promises in a way that is not worth the while of a forger successfully forging them.
1. ‘Credibility Deepening’. As the magnitude of these liabilities rise there must be more scrutiny to establish whether the issuer can and will meet these expanded liabilities (‘John can pay $1,000, but can he pay $10,000? More evidence is needed’).\(^5\)

2. ‘Credibility Widening’. If the issue is to expand, the network amongst which these promises are accepted must expand. More persons must be persuaded that the issuer is solvent and faithful to his promises.

The increasing costs of issuing premises can be represented by letting \( Z \) be the total costs of issuing \( n \) of inside money.

\[
Z = Z(n) \quad Z' > 0 \quad (3)
\]

\[n \equiv \text{issue of inside money in real terms}^6\]

The marginal cost of issue will prove to be a significant variable, and we symbolise it \( \zeta \).

\[
\zeta \equiv \frac{\partial Z}{\partial n} = Z'(n) > 0 \quad (4)
\]

\( \zeta (n) = Z'(n) \) is the cost of establishing the credibility of the nth dollar promised.

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\(^5\) The magnitude of \( n \) is assumed to be known or knowable.

\(^6\) \( N \) is the nominal issue. \( Z \) is related to the real circulation of notes, \( N/P \equiv n \), not \( N \). The same nominal issue may be truly considered either very extensive or very slight, depending on whether \( P \) is large or small.
We will assume $\frac{\partial \zeta}{\partial n} > 0$. That is, the marginal cost of establishing credit worthiness is increasing in $n$ (‘increasing marginal costs’), at least for ‘low’ and ‘high’ magnitudes of $n$. This assumption is required for the existence of a maximum in the households’ inside money issue problem.

*Figure 1: The marginal cost of inside money rises with inside money*

We will make one final assumption about the ‘technology’ of issuing inside money. It concerns how long the credibility of a promise to pay outside money lasts. Once a promise has been made credible, for how long will it be credible? Forever? For the current period? A finite number of periods? We make the extreme assumption that credibility lasts only ‘one period’. Promises made this period are credible for redemption at the opening of the following period, but are otherwise have zero
credibility in the following periods. This is not essential to the paper’s conclusion, but is simplifying.

The optimum

The individual issues an amount of inside money that is utility maximising.

The individual’s maximisation problem is,

Choose $C, C_1, \ldots C_T; h, h_1, \ldots h_T$

\[
\begin{align*}
\text{Max} \quad U &= u(C, C_1, \ldots C_T, h, h_1, \ldots h_T) \\
\text{subject to} \\
\text{Period 0:} \quad P_C + H + K_1P + PZ(n) &= Pw + P\rho K + PK + M + N \\
\text{Period 1:} \quad P_1C_1 + H_1 + P_1K_2 + P_1Z(n_1) &= P_1w_1 + P_1\rho K_1 + P_1K_1 + H + N_1 - N \\
\text{etc} \quad (5)
\end{align*}
\]

$M = \text{endowment of outside money at opening of period zero}$

$K = \text{endowment of physical capital}$

$w = \text{real wage}$

$\rho = \text{rate of profit}$

$H = \text{holdings of nominal money balances}$
The period budget constraints can be consolidated into a single budget constraint,

\[ C + \frac{C_1}{1 + \rho} + \ldots + h \frac{[\rho + \pi + \rho\pi]}{[1 + \rho][1 + \pi]} + h_1 \frac{[\rho_1 + \pi_1 + \rho_1\pi_1]}{[1 + \rho_1][1 + \pi_1]} + \ldots \]

\[ = m + n \frac{\rho + \pi + \rho\pi}{[1 + \rho][1 + \pi]} + \frac{n_1}{[1 + \rho_1][1 + \pi_1]} - Z(n) - \frac{Z(n_1)}{1 + \rho} - \ldots + w + \frac{w_1}{1 + \rho_1} + \ldots + K[1 + \rho_{-1}] \]

(6)

\( \pi \) = rate of inflation

\( m \) = endowment of outside money in real terms

Optimisation with respect to \( n \) implies,

\[ \zeta = Z'(n) = \frac{\rho}{1 + \rho} + \frac{\pi}{1 + \pi} - \frac{\rho}{1 + \rho} \frac{\pi}{1 + \pi} = \iota \]

(7)

The equality says that inside money is issued until the marginal credibility cost, \( \zeta \), equals the nominal rate of return on capital, \( \iota \).

This key result can be rationalised an ‘Argument from Increasing Capital Holdings’.

Suppose that promises to pay are used by the issuer to purchase capital. Suppose also the capital thus acquired is sold next period in order to meet the redemption of the
promise to pay. This is a ‘feasible’ variation in the lifetime ‘plans’ of the decision maker. What is the net payoff to this variation? The addition to costs is, clearly, $\zeta$.

What is the benefit? There is no benefit from increased money holdings, since (by assumption) there is no increase in money holdings. But there is a benefit from the capital that has been acquired. This lies in the income of capital so acquired, which is $\iota$ per dollar. So, matching the cost of issuing money with benefit yields,

$$\zeta = \iota$$

(6) is a restatement of (5), the condition for optimal issue that was derived by utility maximisation. 7

The upshot of the equimarginal condition is that Figure 1’s plotting of the marginal cost of money issue, $\zeta$, against the quantity of inside money, $n_m$ may now be re-

7 A more formal demonstration of ‘The Argument From Capital Holding’ goes this way. The total net benefit from issuing more inside money to acquire capital for one period is;

$$-\zeta U_c + \rho U_{c1} + \frac{P_1 - P}{P_1} U_{c1}$$

This expression allows for the purchased capital to provide one period of profit, $\rho U_{c1}$, as well as for consumption gain from the depreciation of the nominal debt between its issue in period zero, and its redemption in period 1, $\frac{P_1 - P}{P_1} U_{c1}$. If the issuer is optimising the net benefit is zero.

$$-\zeta U_c + \rho U_{c1} + \frac{P_1 - P}{P_1} U_{c1} = 0$$

But $U_{c1}[1 + \rho] = U_c$, total net benefit may be written

$$-\zeta + \rho \frac{1}{1 + \rho} + \frac{1}{1 + \rho} - \frac{P}{P_1} \frac{1}{1 + \rho} = 0$$

or
interpreted as a plotting of the nominal rate of return on capital, against the quantity of inside money.

*Figure 2: The quantity of inside money rises with the nominal rate of return on capital*

$$\begin{align*}
\text{t} \\
\text{n}
\end{align*}$$

2. How inside money makes inflation socially costly.

The model of inside money advanced in section 1 has an implication of considerable significance: there *is* an aspect of ‘printing money’ in issuing inside money. There is a net income to be derived from doing it. The issuer gains a capital asset, and receives it income, less the costs of establishing credibility. Plainly, allowing people to build their own printing presses, and spend the outside money they print, is socially wasteful. And similarly there is waste in persons issuing inside money: resources are
devoted to establishing credibility for the purpose of avoiding the opportunity cost of outside money. But the opportunity cost of outside money is purely a private cost, and involves no social cost. Thus the reduction in the holding of outside money, that the issue of inside money permits, produces no reduction in social costs. But it does involve costs. Inside money is socially wasteful.

This contention can be underlined by an examination of Figure 3. Figure 3 plots the supply schedule of inside money.

*Figure 3: The social cost and private benefit of the issue of inside money*

Let nominal rate of return on capital, $\tau$, equal $aa$. Then the supply of inside money is $ee$. The total variable costs of supplying $ee$ of inside money is $cc$ $bb$ $ee$ $dd$, given that supply curve is interpretable as the marginal cost curve. The gross benefit to suppliers is $aa$ $bb$ $ee$ $dd$; this equals $in$, the income from the capital that can be thought to have been acquired by the issue of inside money. The net benefit to suppliers $= aa$ $bb$ $cc$;
the excess of \( m \) over the costs of issuing \( n \). But the critical point is that there is no social benefit in the supply of \( e \). For if the suppliers, contrary to their private interest, had supplied zero inside money, where would be the social loss? Or – to vary the thought – if inside money was banned, where would be the social loss? For the given nominal rate on return on capital, the same amount of real money would be held, \( h \). The sole difference is that the whole of real money demand, \( h \), would now held in the form of outside money – and the cost of the supply of inside money is therefore saved.

The creation of a social cost by inside money can be elaborated to yield a comparative-static result about the size of that cost and the size of inflation. Since the nominal rate of return on capital rises with the inflation rate, we can easily see that inflation induces an unnecessary and costly expansion of inside money. Inflation induces the replacement of socially cheap outside money with socially costly inside money.

Figure 4 illustrates the costliness of an inflation-induced issue of inside money.

Figure 4 plots two schedules: the supply of inside money, \( n \); and the demand for money, \( h \). The magnitude of real outside money, \( M/P \), can be read as the horizontal distance between these two schedules since equality of money demand with money supply requires that real outside money, \( M/P \), equal the excess of \( h \) over \( n \).

\[
\frac{M}{P} = h(t) - n(t)
\]
Let \( gg \) be the rate of return on capital when inflation is zero. Then \( gg \) is the rate of return on capital when inflation is \( aa \). Then the ‘credibility cost’ of inflation increasing from zero to \( aa \) equals \( cc \), and this is a social cost.

*Figure 4: The social cost of inflation’s expansion of inside money*

How large is the increase in ‘credibility costs’ that results from increased inflation? Let \( \Gamma \) symbolise the total cost (in consumption equivalent) of the ‘credibility costs’ of the supply of inside money. The question is, how large is \( \frac{d\Gamma}{d\pi} \)? We may write,

\[
\frac{d\Gamma}{d\pi} \equiv -\frac{dU}{dn} \frac{dn}{d\pi} \frac{1}{U_c}
\]  

(9)
Given our assumption that credibility of inside money issue lasts only one period, we may write \( dU = -\zeta U_c dn \). But \( \zeta = 1 \) so,

\[
\frac{d\Gamma}{d\pi} = \frac{dn}{d\pi} \tag{10}
\]

Thus,

\[
\frac{d\Gamma}{d\pi} = \epsilon_n' n
\]

where

\[
\epsilon_n' = \frac{dn}{dt} \frac{1}{n}
\]

Evidently, the magnitude of the increment in ‘credibility costs’ from extra inflation is governed, first, by the size of inside money, \( n \), and second by the elasticity of the supply of inside money to the nominal rate of return on capital.

3. How inside money can create an “inflation constituency”.

There is a paradoxical aspect about the analysis of section 2. It was argued that inflation is socially costly, on account of the costly expansion of inside money it
induces; a social cost represented in Figure 4 by ee ff cc dd. Yet, if all persons faced
the same marginal costs of inside money, every person would expect to gain from
inflation; they would expect to gain an equal share of the net benefit cc bb aa dd.

How can each person measure a benefit to themselves from the impact of inflation if
there is a social loss arising from the impact of inflation?

To explore this puzzle, recall that in the judgement of each issuer, issuing inside
money has allowed them to earn an income earning asset (capital) with something
else (inside money) that has no costs, apart from credibility costs. In other words,
each issuer believes that by issuing inside money they acquire more capital. But it is
impossible for each and every person to acquire more capital by issuing inside money:
the capital stock is a given. Or, to be more precise, assuming everyone has the same
supply function of inside money, no one will own any more capital under inflation
than they would if no one issued inside money. The only thing the issue of inside
money does in this circumstance is to raise the current price level more than it would
have. For as monetary equilibrium requires,

\[
\frac{M}{P} = h(t) - n(t)
\]  

(11)

we can infer,

\[
\frac{dP}{P} = \left[ \frac{n' n}{n h - n} - \frac{h' h}{h h - n} \right] dt
\]

(12)

and so,
Clearly, the response of the current price level to an increase in the rate of inflation is enlarged by the presence of the \(\frac{n'}{n} \frac{n'}{h - n} - \frac{h'}{h} \frac{h}{h - n}\) term. The upshot is that the inside money issued goes, not into buying capital, but in restoring depleted holdings of real money balances\(^8\). If, for some reason, no one issued inside money save for one person, that single person would benefit from the issue of inside money in times of inflation, and by means of the purchase of capital that their issue would finance. However, if everyone issues inside money, no one will own more capital than they would have in the absence of issues, and all will have merely wasted some resources in establishing credibility for the expanded issue. We have a ‘paradox of action’; a ‘prisoner’s dilemma’: everybody privately benefits from expanding their issue of inside money in times of inflation, but everyone suffers from everybody expanding their issue of inside money in times of inflation. Everybody would be better off if there was no inflation.

The proposition of the preceding paragraphs that everybody would be better off under zero inflation is dependent on our assumption that everyone has the same supply function of inside money. If costs functions are heterogeneous the benefits and costs of inflation-induced inside money issue are heterogeneous. To illustrate, suppose the population falls into two groups. One group faces prohibitively high credibility costs, and consequently issues zero inside money at all rates of inflation. The second group

\[\frac{dP}{P} \approx \left[\frac{n'}{n} \frac{n'}{h - n} - \frac{h'}{h} \frac{h}{h - n}\right] d\pi\]

\(^8\) The issuer has managed to maintain real balances without selling their capital. That is ‘where’ their issues have gone: not in increasing their capital, but in not having to sell it.
faces manageable credibility costs, and consequently issues some inside money at all rates of inflation. Clearly, an expansion of inflation in this model results in what amounts to a transfer of capital from the High Cost group to the Low Cost group. It appears as if the Low Cost group has imposed an inflation tax on the High Cost group, and that the High Cost group has paid for it by selling their capital to the Low Cost group. And the appearance is not illusory. We have already observed that the responsiveness of inside money to inflation raises the current period price level. This extra jump in P amounts to a tax on the real value of outside money. Everyone who holds outside money ‘pays’ the tax, but - under our present assumptions - not all receive the ‘tax proceeds’. Only the Low Cost group issue inside money, so only they receive the ‘tax proceeds’. If the High Cost group is large enough in its money holdings compared to the Low Cost group, then the tax proceeds will exceed the Low Costs groups own ‘tax burden’ plus the Low Cost group’s increased credibility costs – and the Low Cost group shall have experienced a net benefit from inflation.

4. Can inside money really be that bad?

The theory advanced in sections 1 to 3 has a credibility issue of its own. Can inside money really be a bane? Could it really be desirable to ban it?

The answer to these questions is not controversial. Inside money is not really a bane. It is a bane in the model. But it is not a bane in the real world. It is a bane in the model because the model has allowed for the costliness of inside money, but ignored its
(social) benefits it supplies in the real world. Recall Jevons’ 1875 enumeration of the desirable properties of money:

1. Value  
2. Portability  
3. Indestructibility  
4. Homogeneity  
5. Divisibility  
6. Stability of value  
7. Cognizability  

Clearly inside money has manifest advantages over outside money in the matter of Portability. It also has advantages relative to outside money regarding Divisibility and Indestructibility. In matters of Value, and Stability of Value, inside money has no disadvantage relative to outside money. It is only in Homogeneity and Cognizability that inside money is (usually) inferior to outside money, and the present analysis has given its exclusive attention to that inferiority, to the complete neglect of inside money’s various advantages.

Therefore the implicit recommendation of the present analysis – that inside money be banned – would not survive allowance for the various advantages of inside money. Nevertheless, the conclusion that the issue of inside money is (welfare) inefficiently large would survive. Further, since inflation will not add to inside money’s advantages of Portability etc - but will (according to the model) increase the issue of
inside money - the “excess issue” of inside money (from a welfare criterion) would increase with inflation.
References


