The Australian National University
Centre for Economic Policy Research
DISCUSSION PAPER

Why Investors Prefer Nominal Bonds: a Hypothesis

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DISCUSSION PAPER NO. 552
May 2007

ISSN: 1442-8636
ISBN: 1 921262 23 0

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Acknowledgements
The author is indebted to the comments of the participants of the seminar of the Reserve Bank of New Zealand; and the workshop of the School of Economics of the Australian National University. The usual disavowals apply.

ABSTRACT

The paper advances an answer to a puzzle: Why is any lending or borrowing done in terms of money, when such money debt exposes the lenders' wealth to inflation risk? The 'received' answer to this question is that money bonds are just proxies for real bonds, proxies born of insufficient appreciation, or a benign neglect, of inflation risk. As mere 'proxies', this answer implies that money bonds are redundant: anything a money bond could do, a real bond could do. The thesis of the paper is that money bonds are not redundant. Money bonds have a social benefit. That benefit lies in the reduction that money bonds secure in the unpredictability of consumption that arises from the operation of real balance effects in an environment of unpredictable money shocks. It is the very vulnerability of money bonds to inflation makes them useful in immunising the economy against unpredictable redistributions of purchasing power caused by real balance effects.

JEL codes: E44 E52

Keywords: Real balance effect, inflation risk, indexed bonds

This paper is concerned with the question: 'Why is any lending or borrowing done in terms of money, when such money debt exposes the lenders' wealth to inflation risk?'. The 'received' answer to this question might go as follows:

> All money bonds are 'really' real bonds; they just take the form of money bonds. In fact, all money incomes and outlays are 'really' real, but are merely expressed in the form of money. An offer of a wage income, for example, is an offer of a real wage, but will be expressed in the form of an offer of a money wage. Similarly, a loan of a certain amount real resources is expressed in the form of a loan of money.[1] Wages or loans are in the form of money on account of the convenience of using the medium of exchange – money- as the unit of account, rather than (say) the consumer price index. The pursuit of this convenience does create the chance of some unanticipated inflation changing the real value of a debt, but that chance is neglected, or negligible.

The conclusion of this logic is that money bonds are just proxies for real bonds, proxies born of insufficient appreciation, or a benign neglect, of inflation risk. The implication is that money bonds are redundant. Money bonds are without social function. Everything could be done and done better with inflation indexed bonds.

[1] 'If you lent me so much labour and so many commodities; by receiving five *per cent* you always receive proportionate labour and commodities, however represented, whether by yellow or white coin, whether by pound or by ounce'.

David Hume, *Of Interest*, 1754.

The thesis of this paper (drawn from Coleman 2007, pp138-159) is that money bonds are not redundant. Money bonds do have a social function. If money bonds did not exist, it would be necessary to invent them. Or, it would, at least, be advantageous to invent them. The advantage of money bonds lies in the reduction they secure in the unpredictability of consumption that arises from the operation of real balance effects in an environment of unpredictable money shocks. It is the very vulnerability of money bonds to inflation makes them useful in immunising the economy against unpredictable redistributions of purchasing power caused by real balance effects. For it is the very dependence of the real value of money debt to the price level that allows money bonds to operate as a counter-weight to the dependence of the real value of money balances to price level. It is a case of taking a non-neutrality to beat a non-neutrality.

The upshot of the analysis is that, rather than money bonds being born of a negligence of inflation risk, it is the very consciousness of inflation risk that will create a demand and a supply of money bonds.

The argument is based on the analysis of a standard Ramsey-Solow macro model, with the usual suite of assumptions.

The paper proceeds by first demonstrating that money debt is functionless in such a model without monetary risk, and then showing the functionality of money debt in such models with money risk.

The model

Consider the familiar Ramsey-Solow with the standard array of assumptions: identical homothetic preferences, exogenous labour supply, two-factor constant returns to scale production function, perfectly competitive factor and product markets, general equilibrium … In the absence of risk, the model's foundations may be characterised by this suite of equations,

$$U = \frac{C^{\alpha}}{\alpha} + \frac{C_1^{\alpha}}{\alpha[1+\delta]} + \ldots \qquad Y = C + I \qquad K_1 = K + I \qquad Y = y(\frac{K}{L})L$$

When money debt is redundant debt

Let the standard Ramsey-Solow model be augmented by inserting real money into the utility function, and supposing the supply of nominal money is given. Then price level will be determined, Quantity Theory style, by an equality between the demand and supply of money.

In such a monetary economy there may be said to be two types of risk: the risk of a shock to technology (that will impact on the rate of profit), and the risk of a shock to money supplies (that will impact on the value of money). The presence or absence of these two types of risk creates a matrix of four possibilities:

Both technological and monetary risk absent;

Technological risk present, but monetary risk absent;

Monetary risk present, but technological risk absent;

Both technological and monetary risk present.

This section demonstrates that money bonds are redundant in the first two types of economy: economies without monetary risk.

Both technological and monetary risk absent;

If both technological or monetary risk are absent then the optimisation conditions are,

$$Uc = [1+\rho]Uc_1 \qquad \text{capital}$$

$$Uc = [1+r]Uc_1 \qquad \text{real bonds}$$

$$Uc = \frac{1+i}{1+\pi}Uc_1 \qquad \text{money bonds}$$

Inspection indicates that the equimarginal conditions for *both* real bonds and money bonds are redundant. They simply ‘repeat’ the relation between current and future marginal utility that capital secures. The markets for real and money bonds do not do anything that the capital market does not do.

Technological risk present, but monetary risk absent

Suppose now that the future path of the price level is known, but the profit rate is risky. The optimisation conditions are now,

$$Uc = E[Uc_1[1+\rho]] \qquad \text{capital}$$

$$Uc = [1+r]EUc_1 \qquad \text{real bonds}$$

$$Uc = \frac{1+i}{1+\pi}EUc_1 \qquad \text{money bonds}$$

Inspection reveals that real bonds now do not merely repeat the relation between future and current marginal utility secured by capital. This is significant, and is a sign of the functionality that real bonds now possess. In an economy characterised by technological risk the conditions for capital and real bonds will secure - in fairly wide circumstances - the socially efficient sharing of risk borne of technology shocks. More specifically, the conjunction of markets for capital and real bonds will secure the welfare efficient ‘synchronisation’ of the consumption of all interests in the face of these shocks; the consumption of all interests rise and fall in tandem with the social supply of consumption.

This proposition may be easily illustrated using a special case.

The Two Group: Two State: Two Period Model

Suppose there are just two periods; Period 0 and Period 1.

Suppose also there are just two possible states of the world in Period 1: State 0 and State 1. State 1 is the high output state of the world. State 0 is the low output state of the world. We also assume, for the time being, that the high output state of the world is also the high profit rate state of the world, and the low output state of the world is also the lower profit rate state of the world. (This assumption can be relaxed).The probability of State 0 is p.

We additionally suppose that there are two groups or ‘classes’ that make up the population, F and G. The two groups are completely homogeneous in composition, but differ from each other in their relative factor endowments. Group F has a relatively high amount of capital, and group G has a low amount of capital. The two groups have identical preferences.

Finally, we suppose, for the moment, that saving of the two groups in period 0 is exogenous. Consequently, the magnitude of the stock of capital in period 1 is exogenous. Further, total saving in period 1 simply equals the negative of the capital stock, as there is no wish to have capital in period 2. Thus the consumption endowment of each group in period 1 equals their income from the factors they own, *plus* their endowment of capital.

The consumption endowment can be represented in an Edgeworth Box Diagram.

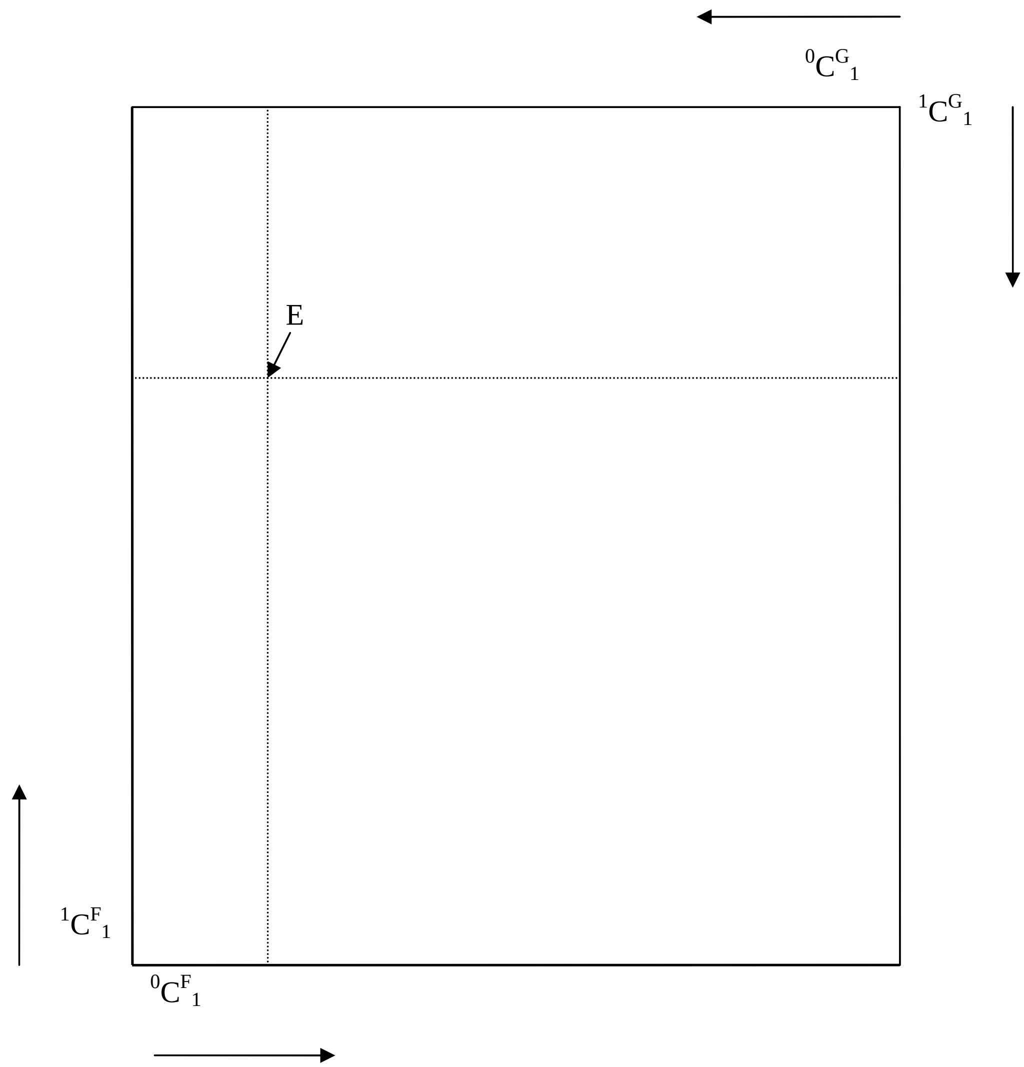

Figure 1: Social and private endowments in a two state model of risk

${}^0C_1^F$ = consumption of F in state of the world 1 in period 1 etc

As F and G are expected utility maximisers, it helps to plot on the Box the 'expected utility indifference curves' of F and G. The expected utility curves of F have a slope of p/[1-p] at the 45 degree ray from the south-west origin.[2]

It also proves helpful to plot on the Box 'iso expected consumption' loci, each locus indicating the combinations of State 1 consumption and State 0 consumption which yield a certain level of expected consumption. These 'iso expected consumption' have a slope of p/[1-p].[3] Thus the expected utility indifference curves and 'iso expected consumption curves' are tangential at the 45 degree line.

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[2] Given $EU^F = pU({}^0C_1^F) + [1-p]U({}^1C_1^F)$ then $dEU^F = p\,{}^0U'd\,{}^0C_1^F + [1-p]{}^1U'dC_1^F$. As ${}^0U' = {}^1U'$ along the 45 degree ray, if $dEU^F = 0$ along the 45 degree ray then $d\,{}^0C_1^F = -\frac{[1-p]}{p}dC_1^F$.

[3] Given $EC_1^F = p\,{}^0C_1^F + [1-p]{}^1C_1^F$ then $dEC_1^F = pd\,{}^0C_1^F + [1-p]d\,{}^1C_1^F$ .If $dEC_1^F = 0$ then $d\,{}^0C_1^F = -\frac{[1-p]}{p}d\,{}^1C_1^F$.

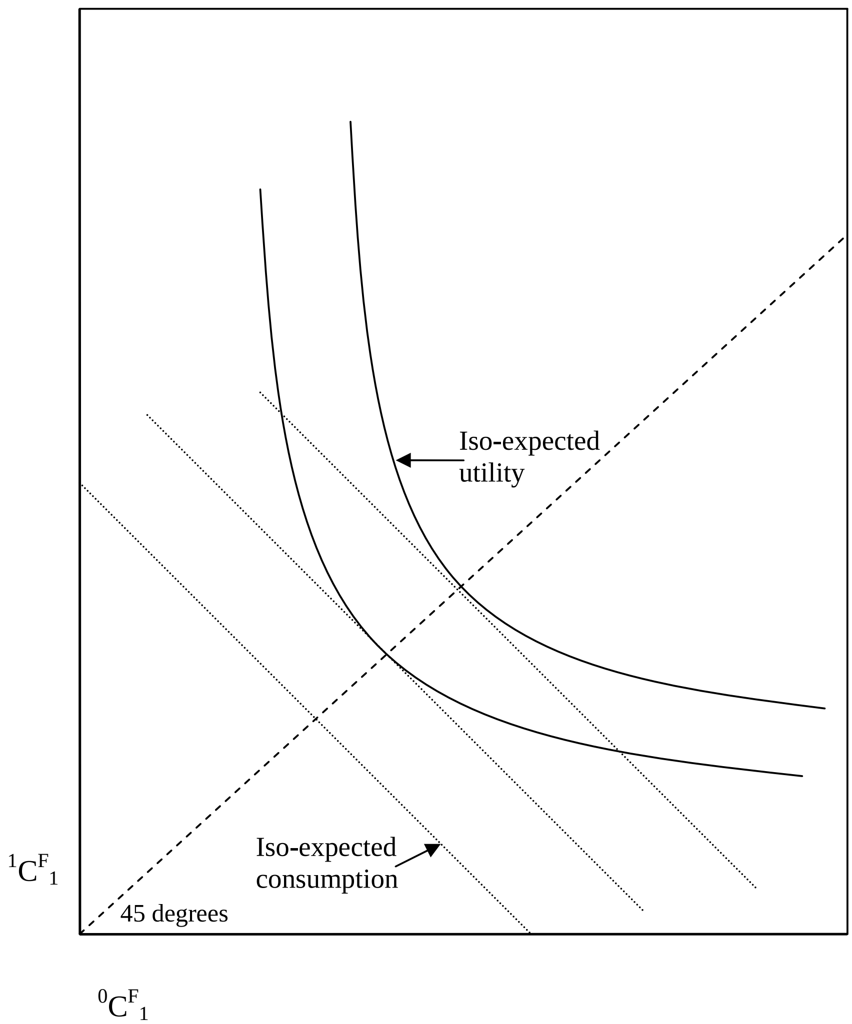

Figure 2: Preferences and expected consumption in a two state model of risk

G's expected utility indifference curves can also be plotted on the Edgeworth box. The expected utility curves of G have a slope of p/[1-p] at the 45 degree ray from the north-east origin.

Plainly, most distributions of consumption between the F and G are pareto inefficient.

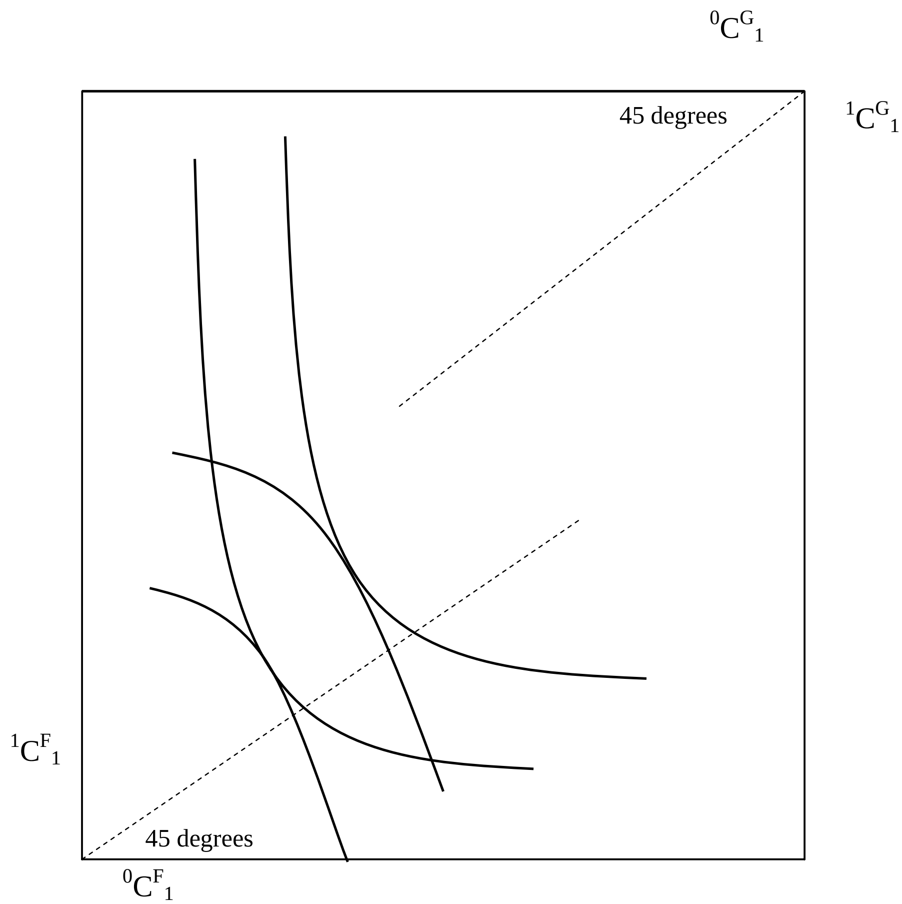

Figure 3: Most consumption points are pareto inefficient

But it is also plain that there exist efficient distributions, that are located at the tangency of the indifference curves of F and G.

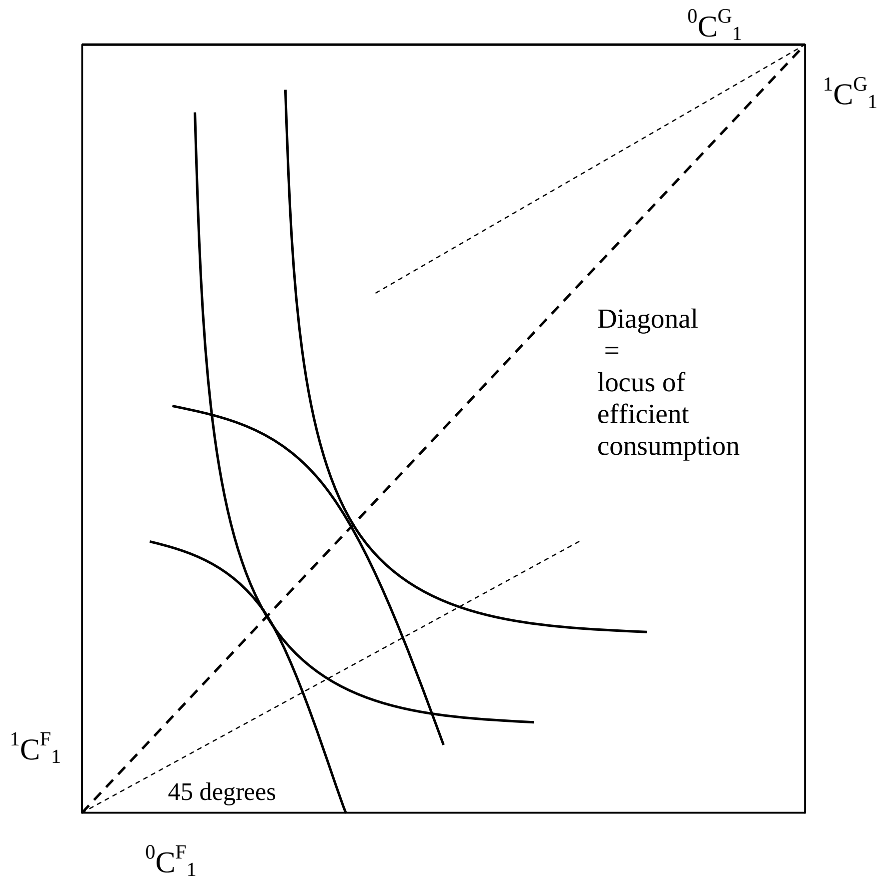

Figure 4: The locus of efficient distributions in consumption

$$\frac{{}^0C_1^F}{{}^1C_1^F} = \frac{{}^0C_1^G}{{}^1C_1^G} = \frac{{}^0C_1}{{}^1C_1}$$

Under our assumptions these efficient distributions lie upon the diagonal of the Box. Thus, under identical and homothetic preferences, efficient management of social risk entails that if society's consumption is x percent higher in the favourable state compared to the unfavourable state, then every individual's consumption is x percent higher in the favourable state compared to the unfavourable state. All boats must rise and fall with the tide, equally. That is the welfare efficient way of dealing with risk under identical and homothetic preferences.

But what is the relation between the efficient allocation of consumption and the market allocation of consumption?

The analysis implies that the market for real bonds will, with some exceptions noted below, secure this welfare efficient allocation. The critical observation to sustain this contention is that F can shift their consumption point from their endowment point by selling bonds in period 0 in order to buy capital in period 0. If bonds are not to dominate capital, or *visa versa*, it must be that ${}^{1}\rho > r > {}^{0}\rho$. This implies that F's purchase of bonds, financed by the sale of equal value of capital, will *reduce* F's consumption by ${}^{1}\rho - r$ in state of the world 1, and *increase* F's by $r - {}^{0}\rho$ in state of the world zero. In other words, F's consumption point is sent 'south east' by F's purchase of bonds. Greater purchases of bonds by F will send F's consumption further 'south-east' along a line with a slope in absolute terms of $\frac{{}^{1}\rho - r}{r - {}^{0}\rho}$

Conversely, F's purchase of capital, financed by the sale of bonds, will shift the F's consumption point 'north west'.

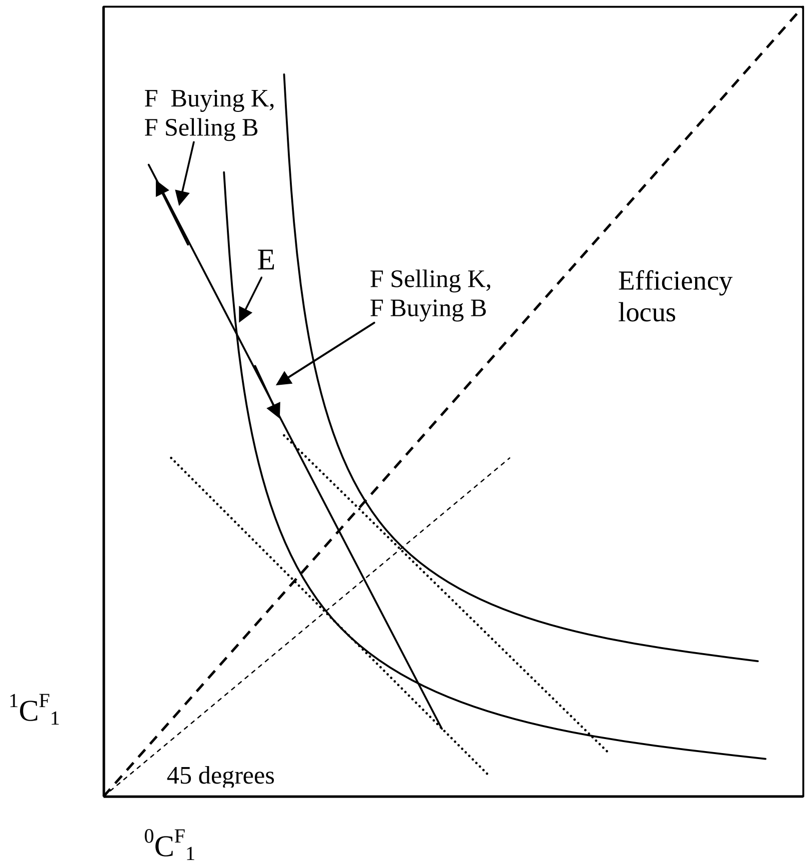

Figure.5: The consumption point is shifted by the exchange of bonds and capital

In the same way, G can shift themselves along the same frontier by either buying capital and selling bonds, or, selling capital and buying bonds.

The Box Diagram shows the general equilibrium, where F wants to buy the same number of bonds that G wants to sell.

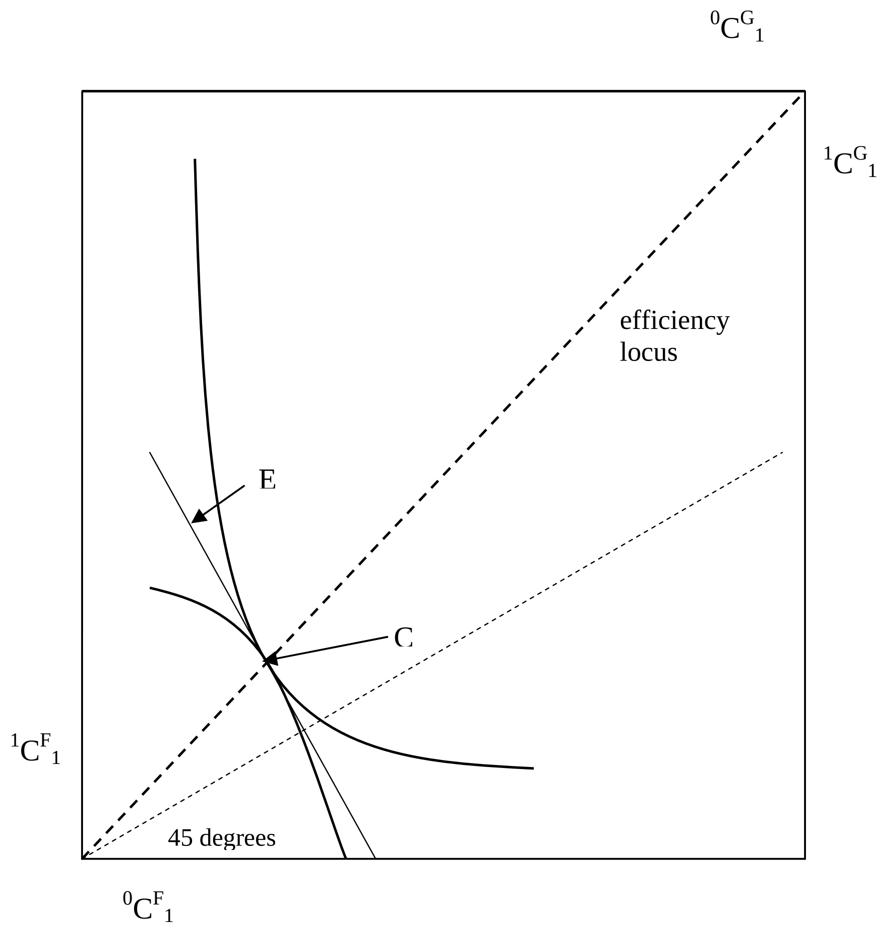

Figure 6: Equilibrium

Evidently, F and G at the equilibrium point are on a higher level of expected utility than at the autarkic point E. There are social gains from trading bonds for capital. Essentially, those burdened with risk have paid others to relieve them part of their risk. By moving south-east of their endowment, F has reduced their expected consumption; but, in approaching the 45 degree line, F has increased the security of their consumption. F in this situation is a 'hedger'. At the same time, G, by shifting their consumption 'south-east' of their endowment; *increases* their expected consumption, but, by withdrawing further from their 45 degree line, *reduces* the security of their consumption. Group G are speculators.

In the general equilibrium, one group will be Hedgers and the other Speculators: one group will be increasing their expected income at the cost to its security, and the other will be reducing their expected income, but to the benefit of their security. Hedgers pay Speculators to assume part of their risk.

The social function of real debt is established. *Real debt allows the relatively risk insulated to supply 'insurance' to the relatively risk exposed over unpredictable fluctuations in factor prices caused by technologisation.*

A 'three state' exposition with an Edgeworth 'Cube'.

Before advancing to money bonds, it is advisable to deal with a query: Is the analysis of the previous section simply a 'wonder' of its two state assumption? We do not think so. Diagrammatically it can be extended to the three state case.

Suppose there are as before two interests, F and G, but there are now three states of the world; 0, 1 and 2. State 0 is 'low profit, low output'; State 1 is 'high profit, high output'; and State 2 is 'very high profit, very high output'. As before, F owns a large amount of capital relative to G, so its endowment rises relative to G in States 1 and 2.

The social, and their distribution, can be represented in an Edgeworth 'Cube'.

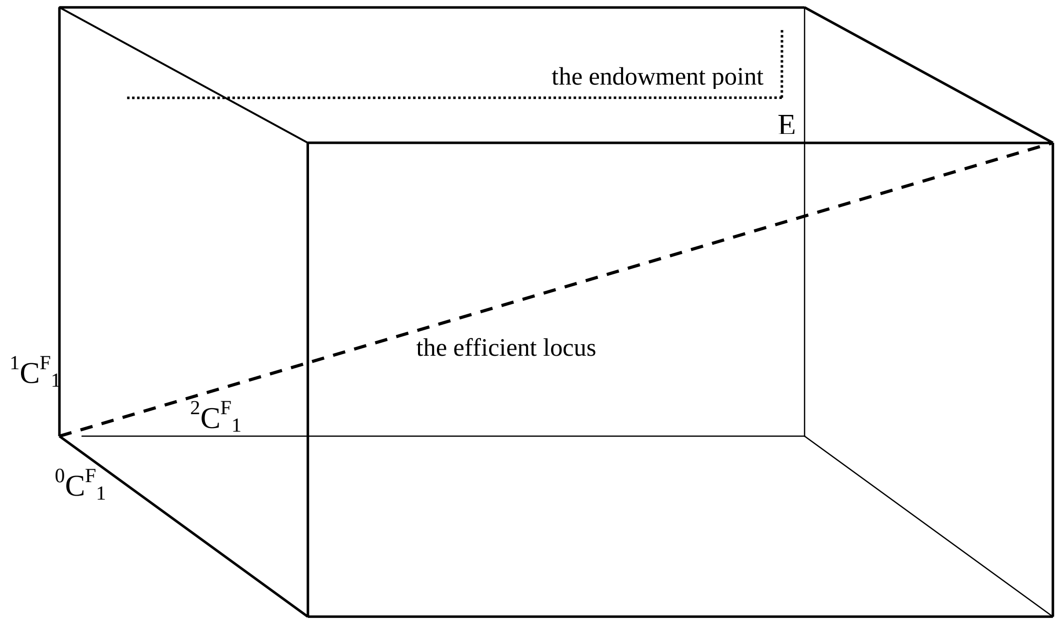

Figure 7: Endowments in a three state world.

The welfare efficient locus of consumption can be represented by a diagonal extending from the hindmost 'South West' corner to the foremost 'North East' corner. Evidently, the Endowment point, E, will (flukes aside) not lie on that diagonal, and not be welfare efficient.

An efficient consumption point can, nevertheless, be reached by means of F's sale of capital to G, and purchase of bonds from G. The magnitude of the real rate of interest on these bonds can be chosen so that it is lower than the rate of profit in state 1 and 2, but higher than the profit rate in state 0. Given this, every such capital-for bonds swap reduces F's consumption in states 1 and 2, but increases it in state 0. As can be seen from Figure 9, this drives the actual consumption point from E and towards the diagonal. A sufficiently large capital-for bonds swap will drive the consumption point to the diagonal.

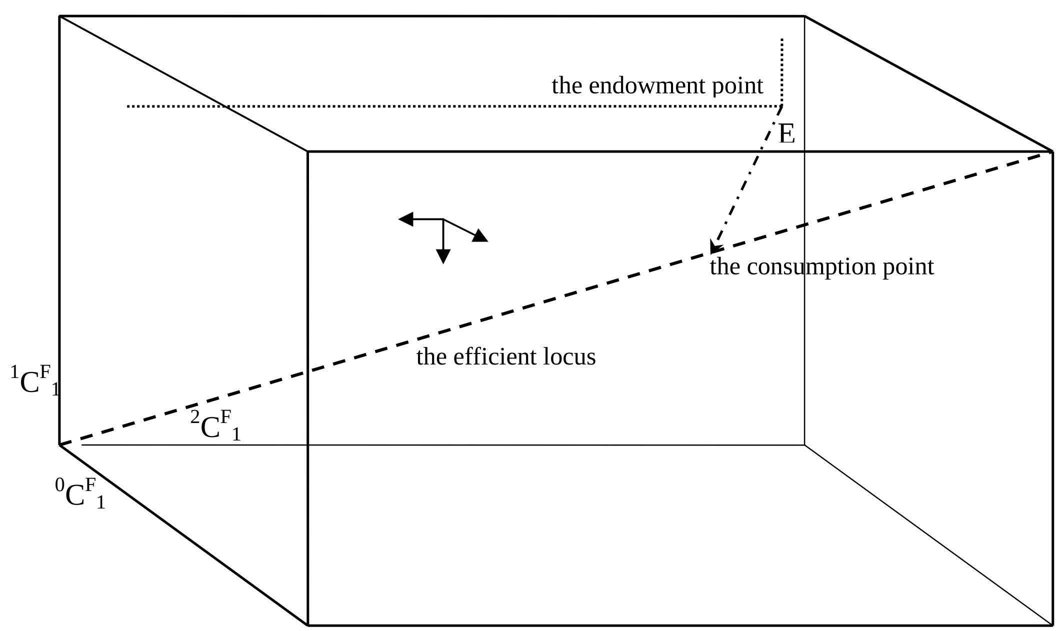

Figure 8: Efficient consumption in a three state world.

Monetary debt as an antidote to monetary risk

What is the significance of the social functionality of real bonds for the social functionality of money bonds? None of the face it.[4] But it is suggestive of a hypothesis: if real bonds are useful in managing technology shocks (= shocks that affect the profit rate), might money bonds be useful in managing money shocks (= shocks that affect the price-level)?

The paper contends this is so. To argue this we now introduce unpredictability in the price-level.

Monetary risk but no technology risk

To demonstrate the implications of price-level risk we first turn to a peculiar, but analytically useful, possibility: where the future path of the profit rate is known but, the price level is risky on account of money supply shocks.

The optimisation conditions will now be,

[4] Under an assumption of perfect price predictability, the synchronisation of consumption secured by real bonds *could* instead be achieved by money bonds; as money bonds 'repeat' the relation between current and future marginal utility that real bonds secure. But the key point is that money bonds not required for the synchronisation consumption. Thus, under environment of perfect price level predictability, money bonds remain a 5th wheel.

$$Uc = [1+\rho]EUc_1 \qquad \text{capital}$$

$$Uc = [1+r]EUc_1 \qquad \text{real bonds}$$

$$Uc = E[Uc_1 \frac{1+i}{1+\pi}] \qquad \text{money bonds}$$

Evidently, it is the condition for real bonds that is now redundant, and that simply 'repeats' the equimargial condition for capital. Real bonds simply replicate capital market under the present assumptions; real bonds cannot do anything that capital does not do.

By contrast, the money bonds equimarginal condition does not merely 'repeat' the equimarginal condition for capital. And that, we will argue, that money bonds secure the socially efficient sharing of consumption in the face of risk in the money supply.

To introduce this contention, it will be useful to first consider how unpredictability in the money supply may impart unpredictability to the consumption of individuals. This possibility can be traced to the 'real balance effects', which redistribute consumption from one individual to another whenever there is a differential in the growth rates of the nominal endowments of different persons.[5]

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[5] The real balance effect (or the windfall in purchasing power) = $\frac{dM^j}{P} - \frac{M^j}{P}\frac{dP}{P} - dh^j$

Assuming no change in real demand for money, the real balance effect of a permanent increase in money is

$$\frac{dM^j}{P} - \frac{M}{P}\frac{dP}{P} = \frac{M^j}{P}[\frac{dM^j}{M^j} - \frac{dP}{P}] = \frac{M^j}{P}[\frac{dM^j}{M^j} - \frac{dM}{M}] = \frac{M^j}{P}[\mu^j - \mu]$$

Conclusion: it is a differential between the growth in j's money endowment and the aggregate money growth that creates a real balance effect for j. But as

$$\mu^j - \mu^k \neq 0$$

Any person who experiences a growth rate in their nominal endowments in excess of the growth rate of some other person will experience an increase in purchasing power relative to that other person. There will be a redistribution of consumption to the person with the larger rate of growth, and from the person with a smaller rate of growth.

But redistribution by way of real balance effects does not itself cause unpredictability in the consumption. If the differential does not vary with the ‘state of the world’,

$$^s\mu^j - {}^s\mu^k = \psi \qquad \text{for all states of the world s}$$

; that is, if it is perfectly predictable; then the redistribution is the same in all ‘states of the world’, and no unpredictability has been imparted to the consumption of any individual.

It is if the differential varies from state to state,

$$^s\mu^j - {}^s\mu^k \neq \psi \text{ for all s}$$

$s^j\mu^j + s^k\mu^k = \mu$ or $[1 - s^k]\mu^j + s^k\mu^k = \mu$ or $\mu^j - \mu = s^k[\mu^j - \mu^k]$
we could equally say that in a two person economy, the real balance effect of a permanent increase in money $= \frac{M^j}{P}s^k[\mu^j - \mu^k]$, and it is a differential between the growth in j’s money endowment and the growth in k’s money endowment that creates a real balance effect for j.

that then the size of the redistribution varies, from state to state, and an unpredictability will been imparted to consumption.

The welfare significance of the creation of unpredictability in consumption by unpredictability in money can be better appreciated by registering once more our earlier conclusion that welfare efficiency requires the consumption of all individuals to be ‘synchronised’ with aggregate consumption. That is, under identical and homothetic preferences, the consumption of all individuals should rise and fall in proportion to aggregate consumption. In other words each individual’s consumption should have exactly the same riskiness as aggregate consumption. But when technology is perfectly predictable (as we are presently assuming) then there are no shocks to aggregate consumption, and so no unpredictibility (or riskiness) in aggregate consumption. Therefore, efficiency requires there be no unpredictibility (or riskiness) in the consumption of any individual. The variance of each individual’s consumption should be zero. But a less than perfectly predictable differential in money endowment growth rates, will (it would seem) impart some unpredictability to consumption, and thereby violate welfare efficiency.

An Edgeworth Box illustration

The notion that an imperfectly predictable differential in money endowment growth rates poses a threat to welfare efficiency can be illustrated by a simple case that can be represented on an Edgeworth Box. The same Box can demonstrate that money bonds will remedy this problem.

Again, let the economy consists of two persons – F and G. Let there be two possible money supply states, 0 and 1. In state zero there is no change in the money supply. In state 1 there is a shock increase in the money supply. Thus ${}^{1}\pi_1 > {}^{0}\pi_1$. Technology, however, is completely invariant, with the consequence that the total consumption endowments are the same in states 0 and 1, and the Box is a 'square'. The action takes place in the change in the distribution of endowments within the square.

Suppose, as the baseline case, that F and G's nominal endowment grow at the same rate in state 1.

State of the world	aggregate money growth	μ^F	μ^G	$\mu^F - \mu^G$
0	0	0	0	0
1	μ	μ	μ	0

Table 1 F and G' nominal endowments growing at same rate

In this case the differential is zero in both states. There is no redistribution of purchasing power in state 1. In terms of the Edgeworth Box the purchasing power of F and G lies on the diagonal, which under our present assumptions is identical to the 45 degree line.

But suppose now that all of the increase in money in state of the world 1 is received by F.

State of the world	μ	μ^F	μ^G	$\mu^F - \mu^G$
0	0	0	0	0
1	μ	$\mu \frac{M}{M^F}$	0	$\mu \frac{M}{M^F}$

Table 2: F's nominal endowment growing faster than Gs in state 1

In state 1 there is a redistribution of purchasing power from F to G. Thus, in terms of the Box, the endowment point E is pushed off the 45 degree line, and to E'.

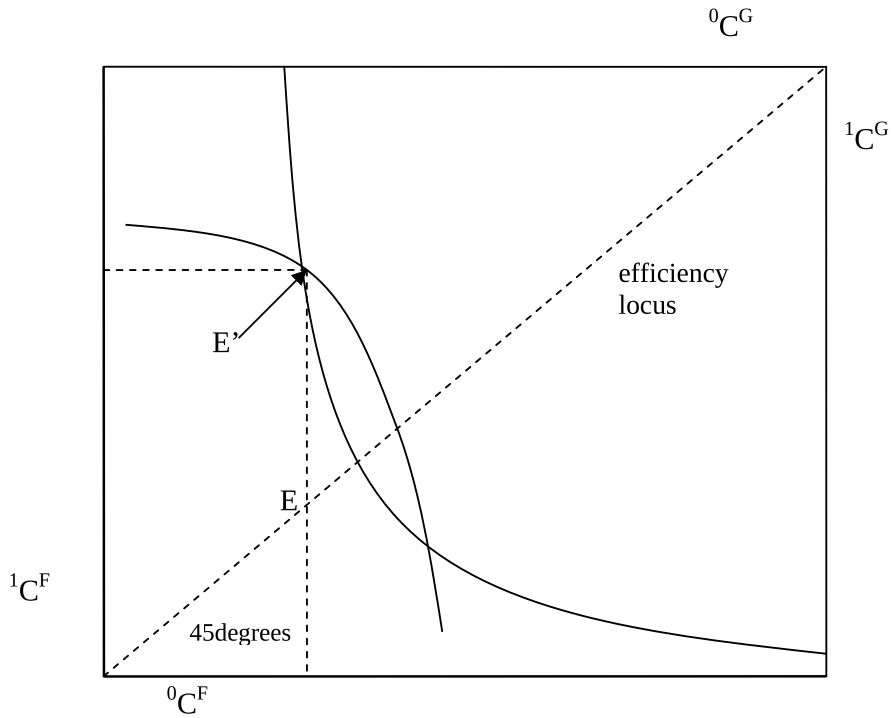

Figure 9: Unpredictibility in the differential in growth in money endowments creates unpredictability in consumption

At E' the expected utility indifference curves intersect, and this would seem to spell a welfare inefficiency.

However, the existence of money bonds will secure efficiency.

The critical observation to sustain this contention is that F can shift their consumption point from their endowment point, E', by (in period 0) selling capital in order to buy money bonds. Assuming $\rho + {}^1\pi > i > \rho + {}^0\pi$ (so that neither bonds nor capital dominate each other), a purchase of money bonds by F, financed by the sale of F's capital, will *reduce* F's consumption by in state of the world 1 (when real interest rates are lower than the profit rate), and *increase* F's consumption in state zero (when real interest rates are higher than the profit rate). More precisely F' consumption falls by $\rho+{}^{1}\pi - i$ in state of the world 1, and rises by $i-[\rho+{}^{0}\pi]$ in state zero. F's purchase of bonds (from G), financed by the sale of capital (to G) sends F's consumption point south east. Greater purchases will send F's consumption further 'south-east' along a line with a slope in absolute terms of $[\rho+{}^{1}\pi - i]/[i-[\rho+{}^{0}\pi]]$.

Conversely, F's purchase of capital, financed by the sale of bonds, will shift the consumption point north west.

This frontier for F also constitutes a frontier for G, who will have their own optimising sale and purchases of bonds and capital.

The market equilibrium occurs where G and F's iso-utility curves are tangential. This occurs when the consumption point is on the 45 degree line.[6] In this equilibrium F sells capital to G, and buys bonds from G. G borrows from F to buy some of F's capital.

[6] Equilibrium requires the slope of the frontier $[\rho+{}^{1}\pi - i]/[i-[\rho+{}^{0}\pi]]$ equal that common slope of the 'expected utility indifference curves'. At the 45 degree line the slope of the 'expected utility indifference curves' slope $\frac{p}{1-p}$, $[1-p][\rho+{}^{1}\pi] + p[\rho + {}^{0}\pi]$ $= i =$ the expectation of the nominal rate on capital. This makes sense; there is zero risk premium on capital due to riskless technology.

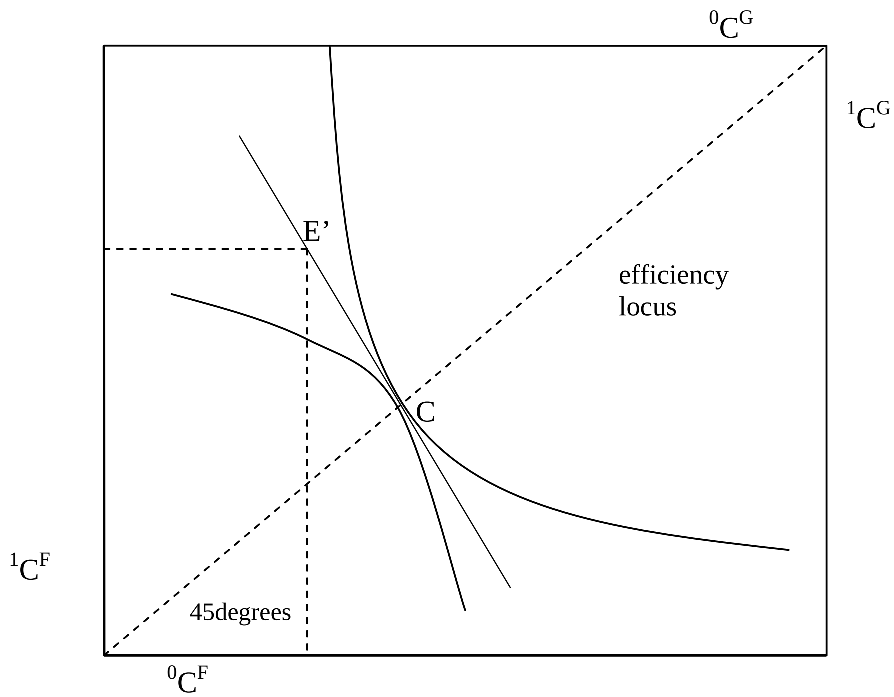

Figure 10: Trading bonds and capital to an efficient point

The critical point is that at the equilibrium consumption point, C, both F and G have zero variance in their consumption. The consumption of each in both states of the world is the same. F's consumption is no higher in state 1 than state 0. The very increase in the money supply that F receives in state 1 creates an inflation that reduces the real value of the bonds F has purchased. Thus a non-neutrality favourable to F (receiving the money shock) has been cancelled by a non-neutrality unfavourable to F (the erosion in the real value of the money principal F is owed by G). Similarly, G's consumption is no lower in state 1 than state 0. The very increase in the money supply that F receives in state 1 creates in state 1 an inflation that reduces the real debt G owes to F. A unfavourable non-neutrality (the 'inflation tax' of the money shock) has been cancelled by an favourable non-neutrality (the erosion in the real value of the money principal they owed.)

Notice that the equilibrium does not eliminate F's 'good fortune' in receiving a disproportionate amount of the monetary shock of state 1: F's expected utility is still greater than it would be in the absence of the monetary shock in state 1. Indeed F's consumption outcome in both states of the world is higher than it would be in the absence of the monetary shock. F's benefit is spread out across states. Correspondingly, G's consumption outcome is lower in both states of the world than it would be in the absence of the monetary shock. G's loss is spread out across states. So relative to no shock, F benefits in both states, and G loses. (But relative to the shock, F's is down in state 1 and G is up, while F is up in state 0 and G is down).

S states and J persons: a successful extension

But we have over stepped ourselves. The following considerations establish that the existence of money bonds does not necessarily ensure that consumption of each individual is state invariant. Recall that we have previously dealt with the two state of the world situation. In Table 2, F's indebtedness to G solves the potential inefficiency, by transferring to state of the world 0 part of F's good fortune in state of the world 1 , and transferring part of Gs ill fortune from state of the world 1 to state of the world 0. The ups and downs in F and G are straightened out.

But now suppose that there are three states of the world. In two of these there is a money shock. In one of these two F gets the whole of the money shock, but in the second G gets the whole of it.

state	aggregate money growth	μ^F	μ^G	$\mu^F - \mu^G$
0	0	0	0	0
1	μ	$\mu \frac{M}{M^F}$	0	$\mu \frac{M}{M^F}$
2	μ	0	$\mu \frac{M}{M^G}$	$-\mu \frac{M}{M^G}$

Table 3: F's nominal endowment grows faster than G's in one state, and G's grows faster than F's in the other

Is there a level of money debt that can 'straighten out' the consumption profiles, so that consumption is invariant to the state? No. There is no magnitude of money debt between F and G that can do that. If F was to lend to G, then F's ill fortune in state 2 is made even worse by the fact that their loan has been eroded in real value, and Gs good fortune is made even better. Thus we do not get zero variance in both person's consumption. Conversely, if G was to lend to F then G's ill fortune in state 1 is made even worse by the fact of their loan, and Fs good fortune is made even better. We do not get zero variance.

Do we therefore conclude that money debt cannot immunise an economy against money risk in an S state economy? No. There exists a monetary risk environment which is 'amenable' to money debt functioning to secure efficiency. Under this plausible modelling monetary risk environment money debt can function in S states as 2.

We can introduce the ‘amenable risk environment’ by considering that money loans worked in the two state world because there was a perfectly coincidence between F’s ‘fortunate state’ and the high inflation state (and, correspondingly, a perfect coincidence between G’s ‘unfortunate state’ and the high inflation state). However, in the three state counter-example, there was no longer this coincidence: the high inflation state was sometimes a fortunate state for F, and sometimes an unfortunate state for F.

It would seem that money lending works if the direction and magnitude of redistribution caused by the money shock can be mapped uniquely into the inflation state. One modelling of money shocks that conforms to this requirement supposes that each person’s money growth is a linear function of growth in aggregate money,

$$ {}^{s}\mu^{j} = \alpha^{j}\, {}^{s}\mu \qquad \text{for all j and all s} $$

This modelling can be interpreted as supposing that the share any person receives of the aggregate money shock is invariant to the size of the aggregate money shock.[7] To double the aggregate shock is to double the size of each individual’s shock. This formulation does not require that all receive the same share, or a share proportionate to one’s initial holdings: there is no requirement that the distribution be “unbiased”. It

[7] The modelling implies $\dfrac{\Delta M^{j}}{\Delta M} = \alpha^{j}\dfrac{M^{j}}{M}$.

could be grossly biased: a given person's share of the total could be could be 90 percent, or one billionth of a percent. The only restriction on bias is that the bias be 'neutral to the scale'; so that if you receive 90 percent of the shock in one state, you receive 90 percent of the shock in all states.

The assumption implies the differential between F's and G's money growth is uniquely relatable to money growth.

$$ {}^{s}\mu^{F} - {}^{s}\mu^{G} = [\alpha^{F} - \alpha^{G}]\ {}^{s}\mu $$

It can be shown that under this assumed modelling, and in spite of any variation across states of the world of the differential in the growth of money endowments, there exists a degree of monetary indebtedness between F and G such that the difference between the growth in monetary endowments of F and G, after allowing for debt flows, is the same in all states. It is proved in an Appendix that, assuming there is only F and G, this degree of indebtedness is,

$$ D_F = \frac{\alpha_F - \alpha_G}{\frac{1}{M^F} + \frac{1}{M^G}} $$

$D_F =$ F's ownership of money bonds

This quantity of debt will make for a uniform excess differential between state of the world 1 and state of the world s. The actual differential of F over G can be calculated to be $[\alpha_F - \alpha_G][i - \rho]$. Assuming that there is some expectation of inflation, then F will have a positive differential in each state if F's growth in money endowment varies more strongly than G's with the total money growth .

Notice the implication that the person who has the disproportionate share of the money shock will lend (ie positive D). They wish to spread out their gain, and they do so by lending. When inflation is high (when they receive the money shock) they are burnt by inflation; but when inflation is low (when they don't receive the money shock) they benefit. Those who are likely to get a disproportionate (ie above average) share of 'inflation' even out the benefit by lending. On the other side, those who will get a below average share of inflation, even out the pain by borrowing. They protect themselves against the possibility of inflation by getting into debt. When inflation comes and they get burnt, they are rewarded by being debtors.

The uniformity in the differential across states can be generalised to n persons. If we assume all persons have the same initial endowment $\bar{M}$, the debt for F is

$$\frac{D_F}{\bar{M}} = \frac{n-1}{n}\alpha_F - \frac{\alpha_G + \alpha_H + \alpha_I + ...}{n} \approx \alpha_F \text{-average share of others}$$

The Algebraic Analysis

Given that we have advanced beyond two states, the diagrammatic analysis of the welfare functionality of money bonds needs to be reinforced by a mathematical analysis.

The equimarginal conditions are

$$U_C^j = [1+\rho]EU_{C1}^j \qquad \text{capital}$$

$$U_{C1}^j = [1+i]EU_{C1}^j[E[\frac{1}{1+\pi}] + \text{cov}(-\pi, \upsilon^j)] \qquad \text{money bonds}$$

where

$$\upsilon^j \equiv \frac{U_{C1}^j}{EU_{C1}^j}$$

Thus,[8]

$$E[\frac{1}{1+\pi}] - \frac{1+\rho}{1+i} = -\text{cov}(\upsilon^j, -\pi)$$

Thus,

$$\text{cov}(\upsilon^j, -\pi) = \text{cov}(\upsilon^k, -\pi) \text{ for all k}$$

This equality establishes a relation between the normalised marginal utilities,

$${}^s\upsilon^j - {}^s\upsilon^k = \varepsilon \qquad \text{all j and k,}$$

ε = disturbance uncorrelated with -π

If ε = 0 then

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8 $$[1+\rho]EU_{C1} = [1+i]EU_{C1}[E[\frac{1}{1+\pi}] + \text{cov}(-\pi, \upsilon\ )]$$

$$1+\rho = [1+i]E[\frac{1}{1+\pi}] + [1+i]\text{cov}(-\pi, \upsilon\ )$$

$${}^{s}\upsilon^{j} = {}^{s}\upsilon^{k}$$

that implies $\frac{C^F}{EC^F} = \frac{C^G}{EC^G}$, which is equivalent to the earlier stated condition of welfare efficiency. [9] This is the situation of the 'amenable risk environment' analysed in the preceding section.

If $\varepsilon \neq 0$ then

$${}^{s}\upsilon^{j} \neq {}^{s}\upsilon^{k}$$

that implies $\frac{C^F}{EC^F} \neq \frac{C^G}{EC^G}$, which is a violation of the earlier stated condition of welfare efficiency. This is the situation where each agent's money shock cannot be perfectly predicted by the size of the aggregate money shock; this is the situation outside the 'amenable risk environment'.

This is not to say there is no risk immunisation at all by money bonds outside of that amenable environment. Recall Table 3. We have argued that in this context perfect

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[9] Proof:

$${}^{s}\upsilon^{F} = \frac{[{}^{s}C^{F}]^{\alpha-1}}{\sum {}^{s}p[{}^{s}C^{F}]^{\alpha-1}}$$

If $\frac{{}^{0}C_1^F}{{}^{1}C_1^F} = \frac{{}^{0}C_1^G}{{}^{1}C_1^G} = \frac{{}^{0}C_1}{{}^{1}C_1}$ then $\frac{C^F}{EC^F} = \frac{C^G}{EC^G}$. But if $\frac{C^F}{EC^F} = \frac{C^G}{EC^G}$ then

$${}^{s}\upsilon^{F} = \frac{[{}^{s}C^{G}\frac{EC^F}{EC^G}]^{\alpha-1}}{\sum {}^{s}p[{}^{s}C^{G}\frac{EC^F}{EC^G}]^{\alpha-1}} = {}^{s}\upsilon^{G}$$

immunisation is not possible. Is therefore, none? What is the probability of states 0, 1 and 2 were 0.5, 0.49 and 0.01 respectively. Would not F lending to G cause some improving dimunition in the variance of consumption?

Money bonds retain a function outside the 'amenable risk environment'. They remove the inefficiencies in consumption allocation that, in their absence, are systematically or predictably related to the size of the aggregate money shock. They leave only ('only') inefficiencies in consumption allocation that are not predictably related to the size of the aggregate money shock

Technological and monetary risk concurrently

Thus far we have concentrated on the case where there is monetary risk, but no technological risk. What happens when there is simultaneously monetary risk and technological risk?

Suppose the money shocks are perfectly correlated with technology shocks, <u>and</u> there is a two person, two state world. So there is a high output and high money state, and a low output and low money state. This situation can be represented in a Box diagram, and its inspection reveals there is no difficulty in reaching efficiency.[10]

But what if we don't make those assumptions?

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[10] The $\mathrm{cov}(-\pi, \upsilon^j)$ are the same for F and G, but now non-zero, since the υ now vary across states.

Both Technology Shocks and Monetary Shocks Present

There are now three optimising conditions,

$$Uc^{j} = E[[1+\rho]Uc_{1}^{j}] \qquad \text{capital}$$

$$Uc^{j} = [1+r]EUc_{1}^{j} \qquad \text{real bonds}$$

$$Uc^{j} = [1+i]E[\frac{Uc_{1}^{j}}{1+\pi}] \qquad \text{money bonds}$$

The condition for capital and real bonds secure an efficiency in consumption in allocation in the face of technological shocks

The condition for capital and money bonds secure a welfare efficient distribution in consumption in the face of monetary shocks, at least in the face of an appropriate risk environment.

$$E[\frac{1}{1+\pi}] - \frac{1+\rho}{1+i} = -\mathrm{cov}(\upsilon^{j}, -\pi)$$

Thus,

$$\mathrm{cov}(\upsilon^{j}, -\pi) = \mathrm{cov}(\upsilon^{k}, -\pi) \text{ for all k}$$

So, just as in the previous section under a certain environment of monetary risk, welfare efficiency will be secured. Certainly, welfare inefficiencies will not be predictable on the size of the aggregate money shock.[11]

[11] In order to eliminate any welfare inefficiency caused by money shocks any redistribution from the monetary side must be state-invariant, i.e. perfectly predictable. The real balance effect for F:

$$\frac{dM_F}{P} - \frac{M_F}{P}\frac{dP}{P} - dh_F = \frac{M_F}{P}[\frac{dM_F}{M_F} - \frac{dP}{P} - \frac{dh_F}{h_F}]$$

Assuming the monetary shock is permanent this can be rewritten,

$$= \frac{M_F}{P}[\frac{dM_F}{M_F} - [\frac{dM}{M} - \frac{dh}{h}] - \frac{dh_F}{h_F}] = \frac{M_F}{P}[\mu_F - \mu - \dot{h} - \dot{h}_F]$$

But the immunisation against real shocks by real bonds side will ensure $\dot{h} - \dot{h}_F = 0$, as everybody's real consumption grows by the same rate in all states. So, the real balance effect for F:

$$= \frac{M_F}{P}[\mu_F - \mu]$$

So we want the cross-person differentials in the growth in nominal endowments to be the same in all states. Let s index monetary shocks, and prime index technology shocks, then state-invariant differential in the growth in nominal endowments between F and G requires the satisfaction of,

$$[\alpha_F - \alpha_G]\frac{[{}^1\Delta - {}^s\Delta]}{M}M = [D_F - D_G][{}^s\pi - {}^1\pi] \qquad \text{all s}$$

$$[\alpha_F - \alpha_G]\frac{[{}^1\Delta - {}^s\Delta]}{M}M = [D_F - D_G][{}^{s'}\pi - {}^{1'}\pi]$$

$$[\alpha_F - \alpha_G]\frac{[{}^1\Delta - {}^s\Delta]}{M}M = [D_F - D_G][{}^{s''}\pi - {}^{1''}\pi]$$

etc

But we know (see Appendix) that D_F and D_G can be chosen to satisfy one of these. But to satisfy one is to satisfy all since $[{}^s\pi - {}^1\pi] = [{}^{s'}\pi - {}^{1'}\pi] = [{}^{s''}\pi - {}^{1''}\pi]$ etc

Thus we have a dual functionality: real bonds cope with real (technological shocks) and money bonds cope with monetary shocks. There is no suggestion, therefore, that money bonds ‘dominate’ real bonds. Yes money bonds can do something that real bonds cannot do. But, conversely, real bonds can do something money bonds cannot do.

Some Remarks

How real is the ‘amenable monetary risk environment’ ?

It has been argued that there is a certain environment of monetary risk that can be immunised against by money bonds. This environment of monetary risk is that where aggregate money growth is unpredictable but each person’s money growth is a linear function of aggregate money growth. Equivalently, where the differential in the growth rate of nominal endowments of different persons is perfectly correlated with aggregate money growth.

Is this kind of risk environment a freak occurrence? Are differentials completely uncorrelated with the aggregate growth in the money supply? Or can this kind of risk environment be given are plausible rationalisation? There are several scenarios for rationalising a correlation between the growth in aggregate money and differentials in the growth in the nominal endowments of persons.

Spending vs Revenue biases in the disposal of seignoirage

Siegnoirage may be entirely devoted to cutting taxes; or entirely devoted to increasing transfers; or it may be devoted some more neutral combination off the two. Suppose that seigniorage is not disposed of neutrally between those whose contribution to government revenue is a net positive, and those whose contribution to government revenue is a net negative. Suppose it is instead entirely devoted to increasing transfers such that net negative contributions is still more negative. Under this scenario, we have a situation where each person's money growth is a linear function of aggregate money growth. The coefficient on this linear function is zero for those who make a net positive contribution to government revenue. The coefficient on this linear function is positive for those who make a net negative contribution to government revenue. Under this schema, the theory implies that those who make a net negative contribution to government revenue will borrow in money terms from those who make a net positive contribution.

Bond holders and tax payers

Increases in the money supply commonly involve purchases of government debt by central banks from private holders of government debt. It can be argued here that these purchases amount to a gift of money to tax payers, (and not bond owners).

One may argue that Central Bank purchases of government debt are equivalent to money grants to tax payers from a scrutiny of simply granting money to tax payers.

Suppose individuals are divided into two groups: owners of (inflation indexed) government debt, and tax payers. Suppose also, for the sake of argument, that government debt owners pay no tax, and tax payers own no government debt. And suppose in this situation that the government unpredictably prints money, and simply gives it to tax payers. In this story, high money growth states redistribute consumption away from bond holders and towards tax payers. The price level, and the nominal demand for money, will rise in accordance with the growth rate of aggregate money endowment, but as the money endowment of tax payers has grown at a greater rate than this, tax payers have excess money balances that they partly spend on bonds. Bond holders, by contrast, have received no increase in their money endowment, and in the face of rising prices and their rising money demand, sell some of their bonds to tax payers. In this story tax payers will hedge their upside by lending money to government bond holders.

The key question is: how can the purchase of government debt by the central bank be any different from a grant of money of to tax payers? In the case of the grant, money is given to tax payers who buy bonds, and thereby obtain relief against taxes. In the case of the purchase, the money is granted to the central bank who buys government bonds, and thereby also obtain relief for the tax payer against taxes.[12] It makes no difference whether the new money is given to tax payers to purchase bonds, or given to central banks to purchase bonds on tax payers' behalf. In either case the increase in

[12] This case for the equivalence of an open market purchase of bonds with a gift of money to tax payers does assume indexed government debt. But introducing nominal money debt simply introduces an element of debt cancellation. The central bank's purchase of nominal debt is equivalent to a transfer of money to tax payers, combined with a cancellation of a certain amount of the debt. This does create complications: debt holders will want a higher rate of interest to compensate to the probability of cancellation by way of inflation.

the money supply goes wholly to one group, and in the context of unpredictable inflation, a welfare-inefficient unpredictability consumption threatens. That is removed by tax payers lending in nominal terms to holders of government debt.

How important quantitatively is monetary risk immunisation?

Paper only presents an argument why the demand for money bonds might be non-zero. No argument that the demand for money bonds might be ‘large’ or even ‘significant’. No attempt to show that the demand for money bonds, thus rationalised, might be comparable observed preponderance of money bonds can be explained.

The welfare functionality of money bonds and the cost of inflation.

The analysis of the welfare functionality has implications for the theory of the costliness of inflation. On the face of it, the analysis is an argument against the costliness of unpredictable inflation. The argument implies that money shocks, such that each individuals allocation is given proportion of the aggregate money shock, will have no welfare costs. Thus in the analysis, the randomness of aggregate shocks is not welfare costly. Neither is the biasedness in the distribution of the shock. It is the randomness of the bias of the distribution that money bonds cannot immunise against and is welfare costly.

The welfare functionality of monetary equilibrium.

In advanced a social functionality to money bonds, it has also advanced a social functionality to monetary equilibrium, and the instability that it implies. In models without monetary risk, it is not clear what social function is served by 'monetary equilibrium'; that is, by an equality between the demand for money and its supply. On the contrary, it would seem advantageous to make the real money supply as large as possible this period, presumably by keeping P way below the equilibrium P, and creating an excess supply of money. By contrast, the present analysis confers a the welfare functionality on the existence of monetary equilibrium. It is, to illustrate, a good thing if P increases on response to an increase in current M. Because it is the increase in P, combined with money bonds, that will stop the money increase from causing welfare inefficient disturbances to the allocation in consumption.

Appendix 1

This appendix demonstrates that if $\frac{{}^{s}\Delta^{j}}{M^{j}} = \alpha^{j} \frac{{}^{s}\Delta}{M}$ then the equality of the differentials corresponds with certain distribution of money debt.

The equality of the differential in the nominal endowments of F and G, in state s and state 1, may be stated as,

$$\frac{{}^{1}\Delta^{F} + D^{F}[i-[{}^{1}\pi+\rho]]}{M^{F}} - \frac{[{}^{1}\Delta^{G} + D^{G}[i-[{}^{1}\pi+\rho]]}{M^{G}}$$
$$= \frac{{}^{s}\Delta^{F} + D^{F}[i-[{}^{s}\pi+\rho]]}{M^{F}} - \frac{[{}^{s}\Delta^{G} + D^{G}[i-[{}^{s}\pi+\rho]]]}{M^{G}}$$

where D_F = the amount of bonds owned by F.

Thus,

$$\frac{{}^{1}\Delta_{F} - D_{F}\,{}^{1}\pi}{M^{F}} - \frac{[{}^{1}\Delta_{G} - D_{G}\,{}^{1}\pi]}{M^{G}} = \frac{{}^{s}\Delta_{F} - D_{F}\,{}^{s}\pi}{M^{F}} - \frac{[{}^{s}\Delta_{G} - D_{G}\,{}^{s}\pi]}{M^{G}}$$

$$\frac{{}^{1}\Delta_{F}}{M^{F}} - \frac{{}^{1}\Delta_{G}}{M^{G}} - [\frac{{}^{s}\Delta_{F}}{M^{F}} - \frac{{}^{s}\Delta_{G}}{M^{G}}] = [\frac{D_{F}}{M^{F}} - \frac{D_{G}}{M^{G}}][{}^{1}\pi - {}^{s}\pi]$$

But assuming

$$\frac{{}^{s}\Delta^{j}}{M^{j}} = \alpha^{j}\,\frac{{}^{s}\Delta}{M}$$

then equality across states requires

$$[\alpha_F - \alpha_G][\frac{{}^{1}\Delta}{M} - \frac{{}^{s}\Delta}{M}] = [\frac{D_F}{M^F} - \frac{D_G}{M^G}][{}^{1}\pi - {}^{s}\pi]$$

$$[\alpha_F - \alpha_G][{}^{1}\mu - {}^{s}\mu] = [\frac{D_F}{M^F} - \frac{D_G}{M^G}][{}^{1}\pi - {}^{s}\pi]$$

This by the Quantity Theory reduces to

$$\frac{D_F}{M^F} - \frac{D_G}{M^G} = \alpha_F - \alpha_G$$ [13]

This pattern of debt will make the differential in the growth in nominal endowments invariant across states.

This expression of the indebtedness between a pair that secures an invariant differential in the growth in nominal endowments across states allows us to obtain the expression when there are three persons;

[13] We have used an approximation which ignores cross products. The exact formulation would be,

$$D_F - D_G = -\frac{1+i}{1+\rho}[\alpha_F - \alpha_G]M$$

$D_F + D_G + D_H = 0$ Market Equilibrium

$\frac{D_F}{M_F} - \frac{D_G}{M_G} = \alpha_F - \alpha_G$ F and G synchronised (see above)

$\frac{D_F}{M_F} - \frac{D_H}{M_H} = \alpha_F - \alpha_H$ F and H synchronised

This is an expression in three equations and three unknowns: D_F, D_G, D_H.

Appendix 2. The supply and demand of money bonds.

The capability of money bond markets to immunise the economy against monetary shocks would be more probingly analysed by a supply and demand of money bonds apparatus that, unlike Box analysis, is not restricted to two states.

Supply and demand schedules can be derived from two optimisation conditions.

$$U_C = [1+\rho]EU_{C1} \qquad \text{capital}$$

$$U_{C1} = [1+i]EU_{C1}[E[\frac{1}{1+\pi}] + \text{cov}(-\pi, \upsilon\ )] \qquad \text{money bonds}$$

where

$$\upsilon^j \equiv \frac{U^j_{C1}}{EU^j_{C1}}$$

Thus,[14]

$$E[\frac{1}{1+\pi}] - \frac{1+\rho}{1+i} = -\text{cov}(\upsilon^j, -\pi)$$

But as

$$E[\frac{1}{1+\pi}] - \frac{1+\rho}{1+i} \approx i - E\pi - \rho$$

we may write

$$i - E\pi - \rho \approx -\text{cov}(\upsilon^j, -\pi)$$

The LHS of the equality, $i - E\pi - r$, is the expected excess real rate of return a money bond over riskless capital. Thus if we take this excess to be positive - if we assume the real rate of return on a money bond will on average be greater than the rate of return

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14

$$[1+\rho]EU_{C1} = [1+i]EU_{C1}[E[\frac{1}{1+\pi}] + \text{cov}(-\pi, \upsilon\ )]$$

$$1+\rho = [1+i]E[\frac{1}{1+\pi}] + [1+i]\text{cov}(-\pi, \upsilon\ )$$

on capital owing to the riskiness of the real return on money bonds – then the RHS of the equality (that is, negative of the covariance of normalised marginal utility with the growth rate of purchasing power negative $\text{cov}(\upsilon^j, -\pi)$) can be identified as 'the inflation risk premium'.

$$\omega_{-\Pi}^{j} \equiv -\text{cov}(\upsilon^j, -\pi)$$

Thus,

$$i - E\pi - \rho \approx -\text{cov}(\upsilon^j, -\pi) \equiv \omega_{-\Pi}^{j}$$

Equivalently,

$$\rho - [i - E\pi] \approx -\omega_{-\Pi}^{j}$$

In terms of F and G,

$$\rho - [i - E\pi] \approx -\omega_{-\Pi}^{F}$$

$$\rho - [i - E\pi] \approx -\omega_{-\Pi}^{G}$$

The LHS may be considered as the 'price' of money bonds; it is their expected opportunity cost. The RHS is a magnitude that will change with the amount of money bonds owned, or owed. Thus for F and G we can trace out a relation between the 'price of bonds' and the quantity of bonds demanded, or supplied.

At D = 0 (ie zero money bonds) the magnitudes of $-\omega_{-\Pi}^{F}$ and $-\omega_{-\Pi}^{G}$ are determined by the co-movement of each groups consumption endowment with the negative inflation rate; the rate of growth of purchasing power. The party with the greater growth of nominal endowment during high inflation, F, will have

> a positive co-movement in consumption and inflation ,
>
> a positive co–movement in marginal utility and the growth of purchasing power,
>
> a negative $\omega_{-\Pi}$ $(\equiv -\text{cov}(\upsilon^{j}, -\pi))$,
>
> and so a positive $-\omega_{-\Pi}$

We can treat $-\omega_{-\pi}^{F}$ as the vertical axis plotting of F' demand for bonds in a figure that plots $\rho - [i - E\pi]$ on the vertical axis, and money bond issues on the horizontal axis. As F's ownership of bonds becomes positive (D > 0), the magnitude of $-\omega_{-\pi}^{F}$ falls, by the logic explained earlier: the bond holder, as 'hedger', is reducing the comovement of their consumption opportunity with the deflation rate. A 'downward' sloping demand curve is traced out for bonds.

We can also trace out a supply curve for bonds. The party with the lesser growth of nominal endowment during high inflation, G, will have,

> a negative co-movement in consumption and inflation ,
>
> a negative co–movement in marginal utility and the growth of purchasing power,
>
> a positive $\omega_{-\Pi}$ $(\equiv -\text{cov}(\upsilon^{j}, -\pi))$,
>
> and so a negative $-\omega_{-\Pi}$

We can treat $-\omega^G_{-\Pi}$ as the vertical axis plotting of G's supply of bonds in a figure that lots $\rho - [i - E\pi]$ on the vertical axis, and money bond issues on the horizontal axis. As G's money debt (liability in money bonds) becomes positive, the magnitude of $-\omega^G_{-\Pi}$ rises, by the logic explained earlier Box analysis. The money debt owed by G makes a high inflation state less of a bad state for G; the correlation between growth in purchasing power gets smaller in absolute terms. $-\omega^G_{-\Pi}$ gets less negative. An 'upward' sloping supply curve for bonds is traced out.

Because total consumption is invariant to the inflation state –and so F only gains at the expense of G - the supply and demand schedules are symmetric.[15]

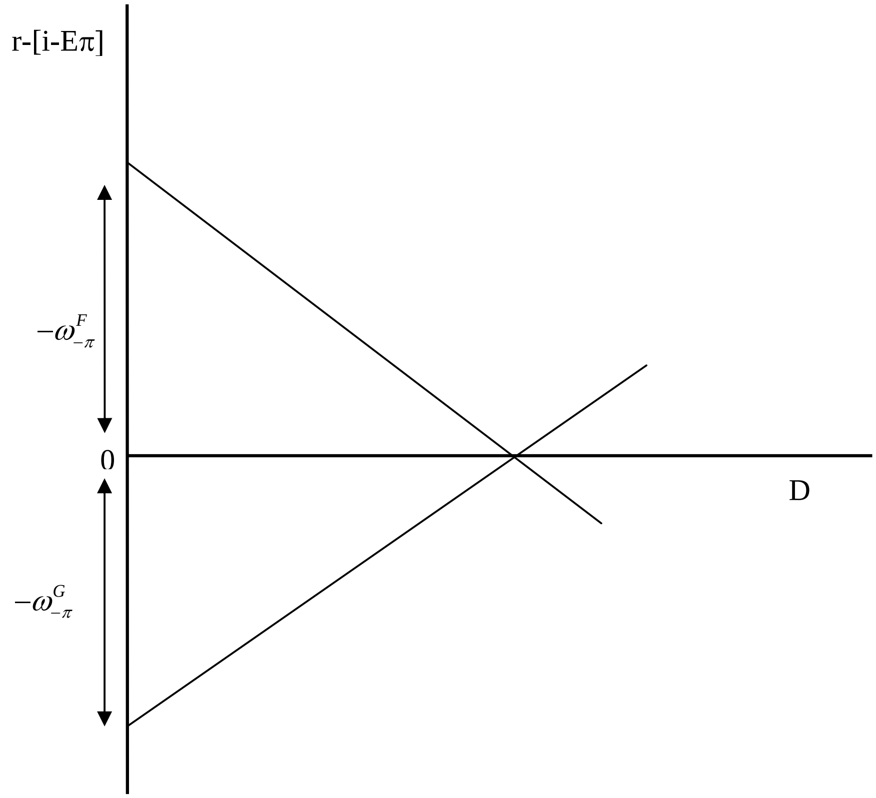
[15] In the two state case

$$\operatorname{cov}(\frac{C^F}{EC^F}, \pi) = \operatorname{cov}(\frac{C^G}{EC^G}, \pi)$$

$$\operatorname{cov}(\frac{C^F}{EC^F}, \pi) = p[[C^F + \mu\frac{M}{P} - EC^F][\pi - E\pi] + [1-p][[C^F - EC^F][0 - E\pi]$$

$$\operatorname{cov}(\frac{C^F}{EC^F}, \pi) = [C^F - EC^F][p\pi + [1-p]0] - E\pi[C^F - EC^F] + p\frac{M}{P}\mu$$

$$\operatorname{cov}(\frac{C^F}{EC^F}, \pi) = p\frac{M}{P}\mu \qquad \text{Similarly } \operatorname{cov}(\frac{C^G}{EC^G}, \pi) = -p\frac{M}{P}\mu$$



Money bond market equilibrium

At the intersection, the

$$\omega^F_{-\pi} = \omega^G_{-\pi} = 0$$

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