The course offers an introduction to modern stochastic processes, including Brownian motion, continuous-time martingales, stochastic integration and Ito's calculus, Markov processes, stochastic differential equations, point processes and their applications. The course will include some applications but will emphasise setting up a solid theoretical foundation for the subject.

The course will provide a sound basis for progression to other honours and post-graduate courses including mathematical finance, stochastic analysis and statistics, and actuarial sciences.

The course aims to round off the rigorous introduction to probabilistic reasoning initiated in STAT3004, as well as to substantially enhance students' depth of knowledge in the mathematical underpinning of stochastic process theory.

<table>
<thead>
<tr>
<th>Mode of Delivery</th>
<th>On campus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prerequisites</td>
<td>STAT3004</td>
</tr>
<tr>
<td>Incompatible Courses</td>
<td>STAT7006</td>
</tr>
<tr>
<td>Co-taught Courses</td>
<td>STAT7006</td>
</tr>
<tr>
<td>Course Convener &amp; Lecturer:</td>
<td>Dr. Boris Buchmann</td>
</tr>
<tr>
<td>Phone:</td>
<td>57296</td>
</tr>
<tr>
<td>Email:</td>
<td><a href="mailto:boris.buchmann@anu.edu">boris.buchmann@anu.edu</a></td>
</tr>
<tr>
<td>Office hours for student consultation:</td>
<td>TBA on WATTLE</td>
</tr>
<tr>
<td>Research Interests</td>
<td>Probability and Stochastic Processes</td>
</tr>
<tr>
<td>Relevant administrator</td>
<td>Patricia Penm</td>
</tr>
<tr>
<td>Phone:</td>
<td>51526</td>
</tr>
<tr>
<td>Email:</td>
<td><a href="mailto:patricia.penm@anu.edu">patricia.penm@anu.edu</a></td>
</tr>
<tr>
<td>Tutor:</td>
<td>Adam Nie</td>
</tr>
<tr>
<td>Phone(s):</td>
<td>NA</td>
</tr>
<tr>
<td>Email(s):</td>
<td><a href="mailto:adam.nie@anu.edu">adam.nie@anu.edu</a></td>
</tr>
<tr>
<td>Office hours for student consultation:</td>
<td>TBA on WATTLE</td>
</tr>
</tbody>
</table>
COURSE OVERVIEW

Learning Outcomes

On satisfying the requirements of this course, students will have the knowledge and skills to:

1. Explain the fundamental concepts of stochastic processes in continuous time and their position in modern statistical and mathematical sciences and applied contexts;
2. Demonstrate accurate and efficient use of stochastic calculus techniques;
3. Demonstrate capacity for mathematical reasoning through analysing, proving and explaining concepts from stochastic analysis;

Assessment Summary

<table>
<thead>
<tr>
<th>Assessment Task</th>
<th>Value</th>
<th>Due Date</th>
<th>Date for Return of Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Assignment 1</td>
<td>12%</td>
<td>Fri, 4pm, Week 4</td>
<td>Week 5</td>
</tr>
<tr>
<td>2. Assignment 2</td>
<td>16%</td>
<td>Fri, 4pm, Week 8</td>
<td>Week 9</td>
</tr>
<tr>
<td>3. Assignment 3</td>
<td>12%</td>
<td>Fri, 4pm, Week 11</td>
<td>Week 12</td>
</tr>
<tr>
<td>4. Final Exam</td>
<td>60%</td>
<td>Exam period</td>
<td></td>
</tr>
</tbody>
</table>

Research-Led Teaching

This course provides a concise treatment of stochastic calculus and its applications. It gives a simple but rigorous treatment of one of the cornerstones of modern probability theory. Contrived in the 20th century, it is the foundation of any current research in the area of probability and stochastic processes. Apart from this, it provides useful tools in any area of research dealing with such processes as mathematics, statistics, economics, finance, computer science, engineering and biology.

Feedback

Staff Feedback

Students will be given feedback in the following forms in this course:

1. Written comments, both individually and to the whole class.
2. Verbal comments to the whole class.

Student Feedback

ANU is committed to the demonstration of educational excellence and regularly seeks feedback from students. One of the key formal ways students have to provide feedback is through Student Experience of Learning Support (SELS) surveys. The feedback given in these surveys is anonymous and provides the Colleges, University Education Committee and Academic Board with opportunities to recognise excellent teaching, and opportunities for improvement.

For more information on student surveys at ANU and reports on the feedback provided on ANU courses, go to

http://unistats.anu.edu.au/surveys/selt/students/ and
http://unistats.anu.edu.au/surveys/selt/results/learning/
Policies

ANU has educational policies, procedures and guidelines, which are designed to ensure that staff and students are aware of the University's academic standards, and implement them. You can find the University's education policies and an explanatory glossary at: http://policies.anu.edu.au/

Students are expected to have read the Academic Misconduct Rule before the commencement of their course.

Other key policies include:

- Student Assessment (Coursework)
- Student Surveys and Evaluations

Examination material or equipment

Two sheets of A4 paper with personal annotations on both sides; paper-based dictionary, no approval required (must be clear OF ALL annotations); calculator (Any - programmable or not).

Recommended Resources


<table>
<thead>
<tr>
<th>Session</th>
<th>Summary of Activities</th>
<th>Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Preliminaries from calculus and probability theory</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Brownian motion: existence and paths properties, Brownian motion as martingale and Markov process, hitting times and exit times, maximum and minimum of Brownian motion, distribution of hitting times, reflection principle, zeros of Brownian motion, arcsine law, size of increments, multivariate Brownian motion, random walk stochastic integral in discrete time, Poisson processes.</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Brownian motion: existence and paths properties, Brownian motion as martingale and Markov process, hitting times and exit times, maximum and minimum of Brownian motion, distribution of hitting times, reflection principle, zeros of Brownian motion, arcsine law, size of increments, multivariate Brownian motion, random walk stochastic integral in discrete time, Poisson processes.</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Brownian Motion Calculus: Ito integral, Ito integral process, Ito integral and Gaussian processes, Ito's formula, Ito processes and stochastic differentials, Ito's formula for Ito processes, multivariate Ito processes.</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Stochastic Differential Equations: definition, stochastic exponential and logarithm, linear SDEs, existence and uniqueness of strong solutions, Markov property, weak solutions, backward and forward equations, Stratonovich stochastic calculus.</td>
<td>Assignment 1</td>
</tr>
<tr>
<td>5</td>
<td>Diffusions Processes: martingales and Dynkin's formula, calculations of expectations and PDEs, time-homogenous diffusions, exit times from an interval, representations of ODES, explosion, recurrence and transience, diffusion on an interval, stationary distributions, multidimensional SDEs.</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Martingales: definitions, uniform integrability, martingale convergence, optional stopping, localisation and local martingales, quadratic</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Calculus for Semimartingales: definition, predictable process, Doob-Meyer decomposition, integrals with respect semimartingales, quadratic variation and covariation, Ito's formula for continuous semimartingales, local times, stochastic exponential and logarithm, martingale representation, elements of the general theory, random measures and canonical decomposition.</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Pure Jump Process: definition, pure jump filtration, Ito's formula for processes with finite variation, counting processes, Markov jump processes, stochastic equations for jump processes, generator and Dynkin's formula, explosion in Markov jump processes</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Change of Probability Measure: Change of measure for random variables, for measures, for processes, for Wiener measure, for point processes.</td>
<td></td>
</tr>
<tr>
<td>10-12</td>
<td>Optional Topics: applications to finance, biology or engineering, Levy processes.</td>
<td></td>
</tr>
</tbody>
</table>

**ASSESSMENT REQUIREMENTS**

As a further academic integrity control, students may be selected for a 15 minute individual oral examination of their written assessment submissions.

Any student identified, either during the current semester or in retrospect, as having used ghost writing services will be investigated under the University’s Academic Misconduct Rule.

**Assessment Tasks**

**Assessment Tasks 1-3: Assignments**

**Details of task:** Assignments serve as a research-type assessment, so that ideas are reinforced on a regular basis by problem solving.

As designated on the Course Schedule, three assignments will be made available each week through WATTLE. Due date is Friday, 4pm, in Week 4, 8 and 12, through WATTLE.

Although verbal discussion with others (fellow students, tutors, lecturer) are encouraged, the content of the assignment must be produced by you as an individual and must comply with ANU academic integrity policies.

**Value:** 12% +16% +12%=40% of the final raw mark

**Estimated return date:** 1-2 weeks
Assessment Task 2: Compulsory Final Exam

Details of task: Closed book exam. Further details will be provided closer to the exam date.

Value: 60% of the final raw mark

Results of the Assessment Tasks determine the final raw mark according to the proposed scheme.

Final Exam is compulsory. Students who meet the requirements for a special exam (eg medical certificate) will be provided with one. Students who do not sit the final exam and do not meet the necessary requirements for a special exam will fail.

Examination(s)
See assessment tasks. Further details will be provided closer to the exam date.

Assignment submission
Online Submission: An Assignment Cover Sheet is provided on WATTLE. You have to fill in your details, sign it and attach it to your solution. Both have to be submitted as one file through WATTLE. Please keep a copy of the assignment for your records.

Extensions and penalties
Extensions and late submission of assessment pieces are covered by the Student Assessment (Coursework) Policy and Procedure.

The Course Convener may grant extensions for assessment pieces that are not examinations or take-home examinations. If you need an extension, you must request it in writing on or before the due date. If you have documented and appropriate medical evidence that demonstrates you were not able to request an extension on or before the due date, you may be able to request it after the due date.

No submission of assessment tasks without an extension after the due date will be permitted. If an assessment task is not submitted by the due date, a mark of 0 will be awarded.

Returning assignments
Marked assignments is to be returned via email.

Resubmission of assignments
Resubmission is not allowed under any circumstance.

Referencing requirements
Although formal scholarly referencing may not be necessary for the assignments, you must adhere to academic integrity policies, see Details of Assessment Task 1.

Scaling
Your final mark for the course will be based on the raw marks allocated for each of your assessment items. However, your final mark may not be the same number as produced by that formula, as marks may be scaled. Any scaling applied will preserve the rank order of raw marks (i.e. if your raw mark exceeds that of another student, then your scaled mark will exceed the scaled mark of that student), and may be either up or down.

Privacy Notice
The ANU has made a number of third party, online, databases available for students to use. Use of each online database is conditional on student end users first agreeing to the
database licensor’s terms of service and/or privacy policy. Students should read these carefully.

In some cases student end users will be required to register an account with the database licensor and submit personal information, including their: first name; last name; ANU email address; and other information.

In cases where student end users are asked to submit ‘content’ to a database, such as an assignment or short answers, the database licensor may only use the student’s ‘content’ in accordance with the terms of service – including any (copyright) licence the student grants to the database licensor.

Any personal information or content a student submits may be stored by the licensor, potentially offshore, and will be used to process the database service in accordance with the licensor’s terms of service and/or privacy policy.

If any student chooses not to agree to the database licensor’s terms of service or privacy policy, the student will not be able to access and use the database. In these circumstances students should contact their lecturer to enquire about alternative arrangements that are available.

**Tutorial Seminar Registration**

Tutorial signup for this course will be done via the Wattle website. Detailed information about signup times will be provided on Wattle or during your first lecture. When tutorials are available for enrolment, follow these steps:

1. Log on to Wattle, and go to the course site
2. Click on the link “Tutorial enrolment”
3. On the right of the screen, click on the tab “Become Member of…..” for the tutorial class you wish to enter
4. Confirm your choice

If you need to change your enrolment, you will be able to do so by clicking on the tab “Leave group…..” and then re-enrol in another group. You will not be able to enrol in groups that have reached their maximum number. Please note that enrolment in ISIS must be finalised for you to have access to Wattle.

**SUPPORT FOR STUDENTS**

The University offers a number of support services for students. Information on these is available online from [http://students.anu.edu.au/studentlife/](http://students.anu.edu.au/studentlife/)