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On endogenous growth with physical capital, human capital and product variety

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Abstract

We set up an endogenous growth model with physical capital, human capital and blueprints for intermediate goods. The model can generate steady-state growth or stagnation. Along the adjustment path for a developing economy we can distinguish different stages of development. The first stage is characterized by physical factor accumulation. At the second stage the economy follows a growth path which is mainly characterized by the accumulation of skills. Growth of the fully developed economy is identified by an increasing variety of goods originating from costly R&D efforts. Transition to a higher stage of development is explained endogenously. Thus, the model provides a high degree of generality by encompassing the standard neoclassical growth model and modern endogenous growth theory. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Modern textbooks on economic growth usually contain three main theoretical parts, each devoted to a 'different' approach in growth theory. The first part

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usually begins with a discussion of Solow's (1956) neoclassical growth model together with the endogenous savings extention of Cass (1965) and Koopmans (1965). The second part introduces endogenous growth through physical capital and human capital accumulation following Uzawa (1965) and the modern formulation of Lucas (1988), Rebelo (1991), or Caballé and Santos (1993). The third part explains the mechanics of R&D-based growth models, where main contributions were developed by Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992). Asked by the layman to identify the 'right' model out of this three-part – set the professionals' most heard answers are: (a) it depends, (b) each model has its own merits. One purpose of the paper is to show that these answers are sound and theoretically well founded.

The paper presents a model where growth is driven by physical capital accumulation, knowledge accumulation and R&D-based technological progress. It combines the Uzawa–Lucas setting with the basic model of Grossman and Helpman (1991, Chapter 3) which introduces endogenous technological change through increasing variety of inputs. The main difference to existing contributions in the literature is that we do not focus solely on the case were individuals invest in human capital and R&D but also consider cases were they do not invest due to 'too small' incentives. It will be shown that the neoclassical growth model as well as the Uzawa–Lucas economy are not only included in the augmented Grossman–Helpman model as special cases, but that each of these models may serve as the best approximation out of this three-type set for a transitional period during the development process. However, a sufficiently high productivity of the educational system together with the permission to invent new products ensures that the economy ends up as an innovative one. For reasonable parameterizations physical capital contributes approximately 50% to steady state growth. The other half are contributions from the 'growth engines' increasing labour quality and technological progress with approximately equal shares. Whereas innovations (or technological progress) are an engine for growth, knowledge formation is the engine for innovations.

Long-run growth is independent of scale effects. It is semi-endogenous in the terminology of Jones (1995), i.e. steady-state growth of the fully developed economy is determined by parameters of preference and technology that are often regarded to be exogenous. Besides the original Jones (1995) contribution these characteristics are also displayed in models developed by Keller (1996) and Arnold (1998). Whereas in Jones (1995) the rate of innovations is driven by population growth (born researchers), in Arnold's, Keller's and our setting innovations are driven by human capital accumulation (skills of researchers). Both Jones (1995) and Arnold (1998) allow a more general production function for new blueprints that includes not only knowledge spillovers but also duplication externalities. All three authors concentrate their analysis on the advanced innovative economy. Our model emphasizes the role of physical capital which allows an analysis that encompasses the development process towards an innovating economy characterized by extensive growth and accumulation of skills.

The paper is organized as follows. Section 2 lays out the theoretical framework and identifies the characteristic features of different stages of development. Section 3 begins with a comparison of steady-state and convergence properties. We then show that the market solution generates the optimal steady-state growth rate. Deliberate policy, however, may induce an increase of per capita income levels. In Section 3.3 we present a numerically calibrated version of the model that combines the development process through physical and human capital accumulation with the R&D-based models. This leads to a general assessment of the 'different' types of endogenous growth models. Section 4 concludes by mentioning extensions and limitations.

2. The model

2.1. Technologies

Production of the homogenous final good Y requires the variable inputs physical and human capital and an index of intermediates. Following Grossman and Helpman (1991) the latter is represented by

$$D = \left[\int_0^n x(i)^\alpha \, \mathrm{d}i \right]^{1/a}, \tag{1}$$

where n denotes the measure (henceforth referenced as 'number') of available varieties, $x(i)$ is the input of intermediate good i, and $0 < \alpha < 1$ controls the elasticity of substitution between intermediates, $\varepsilon = 1/(1-\alpha) > 1$. Consumption goods, C, investment goods, I, and intermediate goods are all produced with the same technology. They can be transformed one to one without further cost from output of the industrial sector (Y) which is produced with Cobb–Douglas technology:

$$Y = A_1 K^\beta D^\eta H_Y^{1-\beta-\eta}, \quad 0 < \beta, \eta, \beta + \eta < 1, \quad A_1 > 0. \tag{2}$$

The existing stocks of human and physical capital are denoted by H and K, and H_Y is the share of human capital employed in the final good sector. Physical capital is only used for the production of goods. For simplicity, we neglect depreciation, which leads to the economy's resource constraint

$$Y = C + \dot{K} + \int_0^n x(i) \, \mathrm{d}i. \tag{3}$$

Production of a new intermediate good requires the invention of a new blueprint. For emphasis of the following exposure, we assume that output of new ideas is solely determined by the aggregate knowledge employed in the R&D-sector and exclude decreasing returns as well as scale effects in n:

$$\dot{n} = \delta H_n \tag{4}$$

with efficiency parameter $\delta > 0$. Additionally, individuals may spend part of their human capital, H_H, on development of skills. This non-market activity is described by a production function of the Uzawa (1965) and Lucas (1988) type:

$$\dot{H} = \xi H_H \tag{5}$$

with efficiency parameter $\xi > 0$.

2.2. Households

The population size is normalized to one so that all aggregate magnitudes can be interpreted as per capita quantities. Labour (human capital) is supplied inelastically and leisure does not enter the utility function. Therefore, full employment requires

$$H = H_Y + H_n + H_H\,. \tag{6}$$

Households earn wages, w, per unit of employed human capital $(H - H_H)$ and returns, r, per unit of aggregate wealth, A, which leads to the budget constraint $\dot{A} = rA + w(H - H_H) - C$. Subject to this constraint and the knowledge formation technology (5) they maximize intertemporal utility $U_t = \int_t^\infty [(C^{1-\theta} - 1)/(1-\theta)e^{-\rho(\tau - t)}]\,d\tau$, where $\rho > 0$ denotes the time preference rate and $0 < 1/\theta < 1$ defines the intertemporal elasticity of substitution. Using the control variables $C > 0$ and $H_H \geq 0$, we obtain from the first-order conditions

$$g_c \equiv \dot{C}/C = (r - \rho)/\theta, \tag{7}$$

$$H_H > 0 \quad \text{and} \quad g_w \equiv \dot{w}/w = r - \xi \tag{8a}$$

or

$$H_H = 0\,. \tag{8b}$$

Eq. (7) is the standard Ramsey rule. Eqs. (8a) and (8b) indicate that the growth rate of wages must be sufficiently high compared to the interest rate to ensure investment in human capital. It will be shown below that

$$\xi > \rho \tag{9}$$

must hold for Eq. (8a) to be the long-run solution.

2.3. Firms and markets

The market for final goods is perfectly competitive and the price of final goods is normalized to one, which implies a rental charge of $\beta Y/K$ for a unit capital. No-arbitrage requires that this rental charge equals the interest rate

$$r = \beta Y/K. \tag{10}$$

Furthermore, equating price and marginal production costs yields

$$p_D = Y/D, \tag{11}$$

$$w = (1-\beta-\eta)Y/H_Y\,, \tag{12}$$

where p_D represents the price index for intermediates. Each firm in the R&D sector owns an infinite patent for selling its variety $x(i)$. Producers act under monopolistic competition and maximize operating profits

$$\pi(i) = (p(i)-1)x(i), \tag{13}$$

where $p(i)$ denotes the price of an intermediate and 1 is the unit cost of Y. Facing the price elasticity of demand $\varepsilon = 1/(1-\alpha)$ each firm charges a price

$$p = p(i) = 1/\alpha. \tag{14}$$

With identical technologies and symmetric demand, the quantity supplied is the same for all goods (but not necessarily at all points in time), $x(i) = x$. Hence, Eq. (1) simplifies to

$$D = n^{1/x}. \tag{15}$$

From $p_D D = pxn$ together with Eqs. (11) and (14) we obtain the total quantity of intermediates employed as

$$xn = \alpha\eta Y. \tag{16}$$

After insertion of Eqs. (16) and (14) into Eq. (13) profits can be rewritten as a function of aggregate industrial output and the number of existing firms:

$$\pi = (1-\alpha)\eta Y/n. \tag{17}$$

Let v denote the value of an innovation. Then free entry into R&D requires $wH_n\,\mathrm{d}t \geqslant v\,\mathrm{d}n$ in a general equilibrium. With Eq. (4) these equilibria are characterized by

$$w = \delta v \quad \text{and} \quad H_n > 0, \dot{n} > 0 \tag{18a}$$

or

$$w > \delta v \quad \text{and} \quad H_n = 0, \dot{n} = 0. \tag{18b}$$

Table 1
Stages of development

	1	2	3
Equivalent in literature	Cass (1965) and Koopmans (1965)	Uzawa (1965) and Lucas (1988)	Grossman and Helpman (1991) augmented with Lucas (1988)
Engine of growth	Expansion of the capital labour ratio	Growth through factor accumulation and improvement in the quality of labour	Economic growth driven by innovations. Innovative growth driven by accumulation of knowledge
Convergence towards	Stagnation if $\xi < \rho$, Stage 2 otherwise	Stage 3	Steady-state growth, independent of scale effects
Long-run model solution	$g_Y = g_K = g_C = 0$	$g_Y = g_K = g_H = g_C$ $= (\xi - \rho)/\theta$	$g_Y = g_K = g_C > (\xi - \rho)/\theta$ $0 < g_n = g_H < (\xi - \rho)/\theta$

Finally, no-arbitrage for investors requires that the interest rate equals the dividend rate π/v plus the rate of capital gain $\dot{v}/v$:

$$g_v \equiv \dot{v}/v = r - \pi/v. \tag{19}$$

With Eqs. (1)–(19) the economy is fully described. In the following sections we will show that this setup encompasses the basic models in growth theory and that the economy evolves according to the stages of development summarized in Table 1.

Before we proceed with the analysis we compute some equations that will be useful at all stages of development. Insertion of Eq. (16) into Eq. (3) simplifies the resource constraint to

$$\dot{K} = (1 - \alpha\eta)Y - C \tag{20}$$

and insertion of Eqs. (15) and (16) into production function (2) gives

$$Y^{1-\eta} = A_2 K^{\beta} n^{(1-\alpha)\eta/\alpha} H_Y^{1-\beta-\eta}, \tag{21}$$

where $A_2 \equiv A_1(\alpha\eta)^{\eta}$. Let u_1 denote the fraction of human capital in production; $u_1 = H_Y/H$. After log-differentiating Eq. (21) with respect to time income growth is then determined by

$$(1-\eta)gY = \beta g_K + [(1-\alpha)\eta/\alpha]g_n + (1-\beta-\eta)(g_{u_1} + g_H), \tag{22}$$

where the growth rate of a variable z is written as g_z. Log-differentiation of Eqs. (10) and (12) provides

$$g_r = g_Y - g_K \tag{23}$$

and

$$g_w = g_Y - (g_{u_1} + g_H)\,. \tag{24}$$

2.4. The standard neoclassical growth model

In this case individuals decide temporarily (or permanently if $\xi < \rho$) *not* to invest in human capital. We assume they do *not* engage in R&D either.[1] With $H_Y = H$ or $u_1 = 1$ we are situated in a neoclassical growth model with endogenous savings (Cass, 1965; Koopmans, 1965). To see this, let $A_3 \equiv \{A_2 n^{(1-\alpha)\eta/\alpha} H^{1-\beta-\eta}\}^{1/(1-\eta)} > 0$. The system's dynamics is then obtained from Eqs. (7), (10), (20) and (21) as

$$\dot{K} = (1 - \alpha\eta) A_3 K^{\beta/(1-\eta)} - C \tag{25}$$

and

$$\dot{C} = C[\beta A_3 K^{-(1-\beta-\eta)/(1-\eta)} - \rho]/\theta. \tag{26}$$

Since the production function exhibits decreasing returns to scale in the variable factor, K, an equilibrium $K^* = (\beta A_3/\rho)^{(1-\eta)/(1-\beta-\eta)}$, $C^* = (1 - \alpha\eta) A_3 K^{*\beta/(1-\eta)}$ with zero growth exists. Analysis of the Jacobian J of Eqs. (25) and (26) in equilibrium gives $\det J = -(1 - \beta - \eta)\rho/[(1 - \eta)K^*] < 0$. Hence, the economy converges with saddlepath dynamics towards the steady state (K^*, C^*). At the steady state we have

$$g_C^* = g_Y^* = g_K^* = 0 \quad \text{and} \quad r^* = \rho. \tag{27}$$

However, from Eqs. (23) and (24) we see that due to physical capital accumulation with decreasing marginal returns, the interest rate decreases with $g_r = g_Y - g_K$, whereas wages per unit human capital rise with $g_w = g_Y$. If $\xi > r^* = \rho$, the economy therefore necessarily arrives at a point from which on households will invest permanently in human capital formation. In other words,

[1] This can be ensured by the specification of a sufficiently high n (cf. Grossman and Helpman, 1991, Chapter 3.1): If the economy expands without knowledge accumulation $H = \bar{H}$, $g_n = \delta H_n/n$ must go to zero for expanding variety. On the path of adjustment we have $w = \delta v$ and hence with Eqs. (17) and (12) $\pi/v = [\delta(1-\alpha)\eta u_1 H]/[(1-\beta-\eta)n]$. The economy converges towards $\dot{v} = 0$, $\rho = r = \pi/v$. At the steady state R&D is not longer profitable, and hence $u_1 \to 1$. This implies that n converges towards $\bar{n} = [\delta(1-\alpha)\eta\bar{H}]/[\rho(1-\beta-\eta)]$.

physical capital is not longer the relatively scarce production factor and the economy enters stage 2.

2.5. The developing economy: Extensive growth and productivity growth through improvements in factor quality

With human capital accumulation ($\dot{H} > 0$) the economy is capable of long-run growth through accumulation of physical capital, expanding quantity of intermediate goods, and improvements in the quality of labour without developing new products. Basically, we are then in the Uzawa–Lucas framework (henceforth referenced as UL). Below we will show that due to the potential to develop new products this type of model describes only another transitory stage of the growth process, which may, however, be valid for a long period of time. This subsection develops the characteristics of such a path towards a fully developed economy. For that purpose we now assume that Eq. (8a) holds. Let $H_H = (1 - u_1)H$. With Eq. (8a) and $g_n = 0$ we can then summarize the dynamics of Eqs. (22)–(24) to

$$g_r = -[(1-\beta-\eta)/\beta](r-\xi). \tag{28}$$

Since $\partial g_r/\partial r < 0$, the interest rate converges towards $r^* = \xi$ independently of the remaining system dynamics. Let $\chi = C/K$ denote the consumption capital ratio. From Eqs. (10) and (20) we obtain

$$g_K = [(1-\alpha\eta)/\beta]r - \chi \tag{29}$$

and with Eq. (7)

$$g_\chi = [1/\theta - (1-\alpha\eta)/\beta]r + \chi - \rho/\theta. \tag{30}$$

Let the ratio between the two types of capital be defined by $\omega \equiv K/H$. Then from Eqs. (5) and (29)

$$g_\omega = [(1-\alpha\eta)/\beta]r - \chi - (1-u_1)\xi, \tag{31}$$

which is independent of ω. From Eqs. (23) and (24) we compute $g_{u_1} = g_r + g_K - g_H - g_\omega = g_\omega + g_r - g_w$ and hence after insertion of Eqs. (8a), (28) and (31)

$$g_{u_1} = [(1-\alpha)\eta/\beta]r - \chi + \xi u_1 + [(1-\beta-\eta)/\beta]\xi. \tag{32}$$

The system dynamics is described by the set of differential equations in r, χ and u_1 given by Eqs. (28), (30) and (32). At the steady state we have

$$r^* = \xi, \quad u_1^* = 1 - (\xi-\rho)/(\theta\xi), \quad \chi^* = (1-\alpha\eta)\xi/\beta - (\xi-\rho)/\theta. \tag{33}$$

The equilibrium is a feasible solution if $u_1^* < 1$, which implies $\xi > \rho$ for $H_H > 0$ at the steady state. Otherwise, the economy converges towards the neoclassical equilibirum of stagnation obtained in the previous section. From Eq. (33) with

Eqs. (28), (7) and (8a), the steady state of the developing economy is characterized by $g_r = g_w = 0$ and $g_C = (\xi - \rho)/\theta$. Eq. (31) implies a constant ratio of physical to human capital at the steady state, $g_K^* = g_H^*$, and from Eq. (23) we have $g_Y^* = g_K^*$. Insertion of $H_H = (1 - u_1^*)H$ into Eq. (4) provides $g_{H^*} = (\xi - \rho)/\theta$, and therefore we summarize

$$g_C^* = g_Y^* = g_K^* = g_H^* = (\xi - \rho)/\theta. \tag{34}$$

The eigenvalues of the characteristic equation for Eqs. (28), (30) and (32) are easily obtained as $\lambda_1 = \xi$, $\lambda_2 = 1$, $\lambda_3 = -(1 - \beta - \eta)/\beta$. The economy exhibits saddlepath dynamics. Since the model characteristics are already well known, we can refer to the literature.[2] However, our system contains one element, which has not yet been discussed, the incentive to innovate new blueprints for intermediate goods. This can best be seen by solving a seemingly contradiction. Contrary to the original UL approach we conclude from Eq. (2) with $\beta + (1 - \beta - \eta) = 1 - \eta < 1$, decreasing returns to scale with respect to physical and human capital. However, the condensed 'production function' Eq. (21) displays linear homogeneity for a constant measure of intermediates n (a constant technology): $\beta/(1 - \eta) + (1 - \beta - \eta)/(1 - \eta) = 1$. The solution is that the economy expands not only through factor accumulation, but also due to the increasing input of already known intermediate goods (see Eq. (16)). From Eq. (17) we then conclude that the potential profit of investment in R&D increases along the adjustment path. With rising factor accumulation the growth rate of wages converges towards zero and the rate of return on investment in physical capital decreases towards ξ. This implies that there must be a point in time from which on the value of an innovation, $v(t) = \int_t^\infty r^{-[R(\tau) - R(t)]}\pi(\tau)\,\mathrm{d}\tau$, with $R(\tau) = \int_0^t r(s)\,\mathrm{d}s$, equals the cost of an innovation, w/δ. Up from this point Eq. (18b) is not longer valid and hence the economy cannot be described with the UL model any longer. The question of calculating this point of transition towards an innovating economy cannot be answered within the UL framework. However, we can solve the problem recursively, i.e. we can compute the equilibrium and adjustment dynamics for an innovative economy with $g_n > 0$. We can then start arbitrarily close to (but of course not exactly in) the equilibrium and move backwards in time. We must then approach at a point, from which on Eq. (18a) is no longer valid. Backward looking, the innovative economy then enters the UL stage of economic growth.

2.6. Growth of the innovative economy

For an innovative economy with knowledge accumulation Eqs. (8a) and (18a) must hold. After log-differentiation with respect to time Eqs. (18a) and (19) can

[2] A detailed discussion is provided in Barro and Sala-i-Martin (1995, 182–194).

be summarized as

$$g_w = r - \delta\pi/w. \tag{35}$$

After subsequent substitution of profits from Eq. (17), wages from Eq. (12) and the growth rate of wages from Eq. (8a) into Eq. (35) we obtain the human capital share in final good production:

$$u_1 = \frac{\xi(1-\beta-\eta)}{\delta(1-\alpha)\eta}\frac{\eta}{H} \tag{36}$$

as determined by the ratio of existing blueprints (firms) to human capital, and therefore

$$g_{u_1} = g_n - g_H\,. \tag{37}$$

From Eqs. (24) and (37) the growth rate of innovations can be written as

$$g_n = g_Y - g_w\,. \tag{38}$$

Insertion of Eqs. (23), (37) and (38) into Eq. (22) provides growth of the interest rate according to

$$(1-A_4)g_r = -\left[A_4 + \frac{1-\beta-\eta}{\beta}\right]g_w + A_4 g_K$$

where

$$A_4 \equiv \frac{(1-\alpha)}{\alpha}\frac{\eta}{\beta}, \tag{39}$$

or, after substiting g_ω and g_K from Eqs. (8a) and (29),

$$(1-A_4)g_r = \{A_4[(1-\alpha\eta)/\beta - 1] - (1-\beta-\eta)/\beta\}r$$
$$- A_4\chi + [A_4 + (1-\beta-\eta)/\beta]\xi. \tag{40}$$

The set of differential Eqs. (30) and (40) determines the system dynamics in the r, χ, time space and we can state the following proposition.[3]

Proposition 1. For the innovating economy there exists a unique positive saddle-point-stable equilibrium

$$r^* = \frac{\theta\xi(1+A_5)-\rho}{(\theta-1)+A_5\theta}, \quad \chi^* = \left(\frac{1-\alpha\eta}{\beta} - \frac{1}{\theta}\right)r^* + \frac{\rho}{\theta},$$

[3] This proposition follows Lutz Arnold's suggestions on an earlier version of the paper.

with

$$A_5 \equiv \frac{\alpha}{1-\alpha}\frac{1-\beta-\eta}{\eta} \tag{41}$$

if (i) $(1-\alpha\eta)/\beta > 1$ *and* (ii) $A_4 = [(1-\alpha)/\alpha](\eta/\beta) < 1$. *From these conditions we can conclude* $\alpha > 1/2 > \beta \geq \eta$ *as sufficient conditions for a better interpretation. At the steady state human capital and innovations grow with the common rate*

$$g_n^* = g_H^* = \frac{(\xi-\rho)A_5}{(\theta-1)+A_5\theta}, \tag{42}$$

and income per capita, consumption, and physical capital grow with the common rate

$$g_Y^* = g_K^* = g_H^* = \frac{(\xi-\rho)(1+A_5)}{(\theta-1)+A_5\theta} = (1+1/A_5)g_n^*. \tag{43}$$

Proof. (1) Condition (i) ensures positivity of χ^*, while r^* is always positive for the innovating economy, characterized by $\xi > \rho$. The determinant of the Jacobian J of Eqs. (30) and (40) evaluated at the steady state is calculated as

$$\det J = \frac{A_4}{1-A_4}\left(\frac{1-\alpha\eta}{\beta} - 1\right) - \frac{1-\beta-\eta}{\beta(1-A_4)} + \frac{A_4}{1-A_4}\left(\frac{1}{\theta} - \frac{1-\alpha\eta}{\beta}\right).$$

Condition (ii) allows to multiply $\det J$ with $(1-A_4)$ without changing sign. Then, we conclude that $\det J$ is negative, since $-(1-\beta-\eta)/\beta < A_4(\theta-1)/\theta$. The equilibrium is a saddlepoint.

(2) In the (r, χ)-plane the $(\dot{r}=0)$-locus will always be flatter than the $(\dot{\chi}=0)$-locus if $(1-\alpha\eta)/\beta - 1 - (1-\beta-\eta)/(\beta A_4) < (1-\alpha\eta)/\beta - 1/\theta$. Since $-(1-\beta-\eta)/(\beta A_4) < (\theta-1)/\theta$, this is true, which ensures uniqueness of the finding in (1).

(3) From $g_\chi^* = g_r^*$ we conclude that C, K, and Y grow with equal rates at the steady state. Inserting r^* into Eq. (7) we obtain Eq. (43) and substitution of Eqs. (43) and (8a) in Eq. (38) provides g_n^* in Eq. (42).

(4) To reveal the dynamics of knowledge formation we define the knowledge-ideas ratio as $\psi \equiv H/n$ and obtain from Eqs. (4), (5) and (36)

$$g_\psi = \xi[1 - (\xi A_5/\alpha + g_n)/(\delta\psi)] - g_n\,. \tag{44}$$

Since g_n^* has already been determined in Eq. (3) as independent of ψ, the $(\dot{g}_n = 0)$-locus is vertical and stable in a (g_n, ψ)-plane. The $(\dot{\psi} = 0)$-locus given by $\psi = (\xi/\delta)(\xi A_5/\alpha + g_n)/(\xi - g_n)$ is increasing with a pole at $g_n = \xi > g_n^*$. Therefore, a unique equilibrium exists. Since g_n converges independently, and $\dot{\psi} > 0$

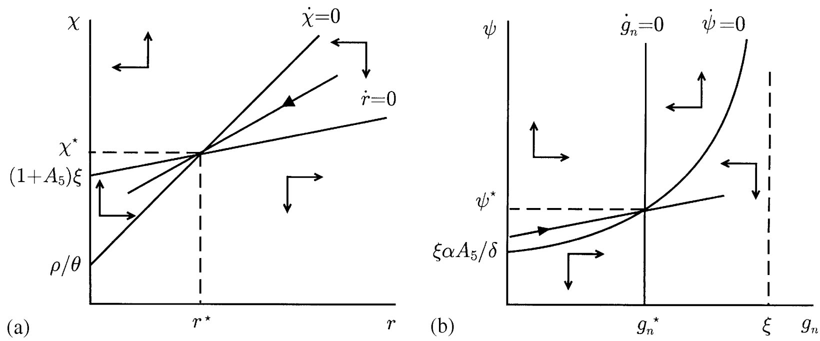

Fig. 1. (a) Convergence of r and χ, (b) convergence of ψ.

above the ($\dot{\psi}=0$)-locus, and $\dot{\psi}<0$ below, there exists a unique saddlepath towards

$$\psi^* = \frac{\xi}{\delta}\frac{(1-\beta-\eta)\xi/[(1-\alpha)\eta] + g_n^*}{\xi - g_n^*}, \tag{45}$$

which implies $g_H = g_n$ in equilibrium. □

Fig. 1 summarizes the results.

3. Discussion

3.1. Steady-state analysis, convergence characteristics

We first compare the steady-state growth paths for both model variants. It should, however, be kept in mind that the potential to innovate ensures that the steady state of the augmented Grossman/Helpman model variant (henceforth GH) is the solution for every knowledge accumulating economy ($\xi > \rho$) in the very long run. Let the superscripts UL and GH denote results obtained from the UL model variant and the GH variant. Then from division of Eqs. (43) and (34) we obtain

$$\frac{g_Y^{GH*}}{g_Y^{UL*}} = \frac{\theta(1+A_5)}{\theta(1+A_5)-1}, \tag{46}$$

hence steady-state growth of an innovating economy surpasses steady-state growth of a UL economy. The factor g_Y^{GH*}/g_Y^{UL*} will be higher the more prepared individuals are to postpone consumption in response to high rates of returns (higher $1/\theta$), and the lower A_5. Inspection of Eq. (41) shows that A_5 will be small if the elasticitiy of substitution between intermediates is low (α is small), and the relative importance of intermediate goods in the production process, measured by $\eta/(1-\beta-\eta)$, is large. We can verify that these factors define favourable conditions for high steady-state growth for the innovating economy by calculating $\partial g_Y^{GH*}/\partial A_5 = -(\xi-\rho)/[(\theta-1)+A_5\theta]^2 < 0$ from Eq. (43). However, we find with $\partial g_n^*/\partial A_5 = (\theta-1)(\xi-\rho)/[(\theta-1)+A_5\theta]^2 > 0$ from Eq. (43) that c.p. promising conditions for growth imply low rates of innovations. The intuition for this result is that if the importance of each intermediate good and the importance of the intermediate goods mix as a whole is high, the economy can spend less resources on R&D (and hence less educational effort) than an otherwise identical economy to achieve the same growth rate. However, the stability criteria (i) and (ii) in Proposition 2.1 require that, loosely speaking, intermediates substitute well for each other, and that the relative importance of the intermediate goods index in production is not too high. We can now get an intuition of what would happen in case of 'too favourable' conditions for growth: The economy will explode with a perpetual rising consumption capital ratio χ and imploding returns on physical capital r. Comparison of Eqs. (34) and (42) shows that human capital growth for both model types would converge for $\theta \to 1$. However, for every $\theta > 1$ human capital growth in the GH steady state will be lower than the corresponding rate in the UL model: $g_H^{GH*} < g_H^{UL*} = (\xi-\rho)/\theta$. Taken together, a lower rate of knowledge growth explains a higher growth rate of income per capita in the GH model.

We follow Arnold (1998) in performing a growth accounting exercise. If a growth accountant would be confronted with our steady-state economy with constant hours worked ($g_{u_1} = 0$), he would calculate a contribution of total factor productivity to growth of $[(1-\alpha)/\alpha][\eta/(1-\eta)]g_n + (1-\beta-\eta)/(1-\eta)g_H$ from Eq. (22). Since $g_H = g_n$ at the steady state we can calculate the contribution of technological progress relative to the contribution of labour quality as $[(1-\alpha)/\alpha][\eta/(1-\beta-\eta)] = 1/A_5$, which is the factor by which income per capita growth surpasses growth of knowledge and ideas (cf. Eq. (43)). It follows from (ii) in Proposition 1 that if the production elasticity of physical capital is not larger than the production elasticity of labour, the factor $1/A_5$ is not larger than one for an economy with a saddlepoint stable steady state. One could argue that reasonable parameterizations would provide a factor $1/A_5$ near to one (e.g. $\beta = 0.4, \eta = 0.3, \alpha = 0.6$ provides $1/A_5 = 1$) with the implication that a growth accountant would assign nearly half of TFP growth to technological progress measured by increasing product variety. However, it should be recalled that the economy would stagnate in the long run if knowledge remains constant. In this sense, human capital is the 'ultimate resource' (Simon, 1981) of the model. Economic growth is fostered by technological progress, whereas technological progress is driven by human capital accumulation. This is the main difference of our GH model variant (and Keller (1996) and Arnold, 1998) compared to the approach of Jones (1995). The intuitive interpretation is that in Jones' model people become researchers by birth (and hence technological progress is driven by population growth), whereas in our model people become researchers by education (and hence technological progress is driven by human capital accumulation). Countries with high rates of productivity growth are not longer necessarily identified as countries with high population growth.

The model provides a rich variety of convergence characteristics. First, we obtain club-convergence (according to Galor, 1996). Depending on initial endowments and the productivity of knowledge formation, the model describes a club of economies converging towards an underdeveloped state of stagnation, a second club of developing (but not innovating) economies with transitory high rates of physical and non-physical factor accumulation converging towards the innovative economy, and finally a third club of fully industrialized economies developing towards a steady state with perpetual increasing ideas. Across the different clubs, we observe different kinds of convergence behaviour. The poorest countries converge towards a state of stagnation. Countries in the second club converge towards a common growth rate, a common rate of physical to human capital, $\omega = K/H$, and a common capital output ratio, K/Y. Along the path countries with identical initial endowment ω will always display the same growth rate. Hence, from two countries endowed with identical ω the one with the absolutely lower initial endowment with K and H will never catch-up the initial gap. Moreover, from two identical countries the one with the absolutely higher initial human capital endowment will reach the point of transition to an innovating economy earlier. This follows from the fact that for economies on the UL path growth of the knowledge–ideas ratio is solely determined by $g_\psi = g_H$. Within the club of innovating countries we observe convergence towards a common income growth rate, and a common knowledge–ideas ratio. However, from two economies the one which has entered the stage of an innovating economy earlier, will always posses an absolutely larger stock of human capital and realize an absolutely larger income level than the otherwise identical economy. Hence, the model predicts catching-up in *rates* for all developing economies but no catching-up in *levels*.

3.2. Fiscal policy and welfare

If we consider the model's parameters as exogenously given, the model displays semi-endogenous growth in the terminology of Jones (1995). However, in contrast to other semi-endogenous growth models like Jones (1995) and Arnold (1998), the steady state growth rate is not only determined by preference parameters and production elasticities (which are usually regarded as exogenous to fiscal policy) but also by market structure via parameter α. Hence, one might believe that long-term growth can be improved by standard fiscal policy measures like taxes and subsidies. Therefore, we provide the following proposition.

Proposition 2. In an innnovating economy a social planner would allocate $(1-\alpha)$ *less resources in final good production and would realize an* $\alpha^{-\eta/(1-\eta)}$ *times higher* *income per capita level than the decentralized solution for every given endowment K, n, and H. He, however, realizes the same long-run growth rates for knowledge, ideas, and output as the decentralized solution.*

A proof is provided in the appendix. An intuition for this result is easily conveyed. A benevolent dictator would abolish mark-up pricing in the intermediate sector and command the sectoral resource allocation. The welfare gain is reflected by the scale effect in resource allocation and income per capita levels. Long-term growth, however, is independent from scale effects but driven by the rate of knowledge growth. Interestingly, the steady-state growth rate is not only independent of scale effects, but also independent of the productivity parameter in research, δ. From $\partial\psi^*/\partial\delta < 0$ in Eq. (45) we get an intuition of this result. An exogenous increase in research productivity (and hence wages) implies also a decreasing importance of human capital, or, differently speaking, the optimal reaction of individuals to an exogenous increase of knowledge productivity in research is to allocate more resources to this activity and to reduce the skill formation activity. At the steady state both effects exactely balance each other.

From Proposition 2 we can directely conclude the invariance of long-term growth with respect to standard fiscal policy instruments. Invariant technology and preference parameters are functioning as an exogenously given 'speed limit' for long-run growth (Blinder, 1997). However, we would like to mention two arguments against the hasty conclusion of the meaninglessness of fiscal policy for growth. First, Proposition 2 states that there is a maximum possible efficiency gain of 'good' public policy. Internalizing the market imperfection may lead to an instantenous increase of income per capita by the factor $\alpha^{-\eta/(1-\eta)}$. For example for $\alpha = 0.5$ and $\eta = 0.25$ (or $\alpha = 0.6$ and $\eta = 0.3$) this factor is about 1.25. Hence, income per capita in an economy with an optimal performing government would exceed the laissez faire economy by about one quarter. Note that the result of non-convergence in levels guarantees that this lead remains forever.

Our second argument is related to the problem of long-run growth and (non-) development. It originates from the observation of two long-run steady states. The model's only answer to the question why growth rates differ in the long run and why countries diverge is that productivity of knowledge formation in some countries is 'too low'. A sufficiently high increase of the crucial parameter ξ induces the stagnating economy to leave the development trap. A higher ξ speeds up the development process in the UL model. Moreover, it explains a higher growth rate for the innovating economy. Efficiency parameters, however, are not just exogenously given Greek symbols for the transformation of different input units. They reflect conditions for production as determined by e.g. institutions and infrastructure. In this sense, differences in institutions and infrastructure do not only explain why growth rates differ, but also provide a meaningfull basis for the introduction of long-term growth oriented development policies. The next section discusses the quantitative effect of a discrete change in the productivity of knowledge formation. We can, however, easily imagine more sophisticated implementations of endogenous governmental policy. One way, for example, is to follow Barro's (1990) approach and introduce tax-financed governmental services. Let G denote educational services. A 'Barro-type' human capital production function would then have the form of $\dot{H} = G^{\gamma} H_H^{1-\gamma}$, $0 < \gamma < 1$. A complete formal analysis of such a model is beyond the scope of this paper. Static efficiency of the educational system, however, would require equalization of marginal products of individual and governmental expenditure for schooling, i.e. $G/H_H = \gamma/(1-\gamma)$. We can easily think of G/H_H as schools or universities per inhabitants. The model's unique explanation for economic stagation, slow catching-up performances and 'too slow' long-run growth would then be a 'too small' share of governmental spending on educational infrastructure.

3.3. Calibration results

This section presents adjustment paths for a calibrated economy and identifies the point of transition from the developing country to the fully industrialized country, i.e. the transition from the UL framework towards the augmented GH economy. When combining the model variants we use the fact that state variables cannot jump ($\dot{n}$ and $\dot{H}$ cannot be infinity for finite H according to Eqs. (4) and (5), $\dot{K}$ cannot be infinity for finite C). We then proceed according to the method of backward integration (see Mulligan and Sala-i-Martin, 1993; Brunner and Strulik, 1997), i.e. we start arbitrary close to the steady state of the innovating economy and move backwards in time by integrating Eqs. (30), (40) and (44). It should be recalled that backward integration – contrary to other methods like e.g. log-linearization around the steady state – enables us to obtain the exact adjustment path besides numerical errors.[4]

Backward looking from the steady-state position the adjustment process for the innovating economy comes to an end at the point where $u_2 = 0$, or $\dot{n} = 0$, respectively. We keep the critical knowledge–ideas ratio ψ_{crit} at this point in mind. We then obtain the whole ajdustment path of the UL-economy by backward integration of Eqs. (28), (30) and (32). Backward looking from the steady state the adjustment now comes to end at the point where $u_1 = 1$, i.e. $g_H = 0$. At this point the economy enters (backward looking) the Cass–Koopmans world. After obtaining the adjustment paths we specify an initial $\psi(0)$ and calculate the corresponding time path using forward integration of $\dot{H}$ implied by the numerical solution of Eqs. (28), (30) and (32), and $g_{\psi} = g_H$ in the UL-model. We then cut the relevant time paths up to ψ_{crit} and link them with the already

[4] We employ a fourth order Runge–Kutta method with variable step control provided in MATLAB. We start with an discretization error tolerance of 10^{-6}. If the state variables do not match at the transition point we decrease the tolerance until they do.

obtained paths for the innovating economy. In principle, we can proceed in the same manner to combine the neoclassical model with the developing economy now using the critical value of ω_{crit} at the point $u_1 = 1$ as the connecting state variable. However, for the sake of brevity, we concentrate our analysis on the most challenging case of connecting the two endogenous growth models.

It is immediately obvious that the duration of transition depends crucially on the specification of the initial endowment with human capital and the number of existing varieties. Therefore, we decide to determine the initial $\psi(0)$ endogenously. For this purpose we imagine an economy stagnating at the neoclassical equilibrium due to a low productivity in the knowledge formation sector. This implies a knowledge–ideas ratio of $\bar{\psi} = \rho(1-\beta-\eta)/(\delta(1-a)\eta)$ (cf. footnote 2). We then imagine an exogenously given increase in productivity of knowledge formation to $\xi > \rho$ that induces to voluntary substitute working time against education. Hence, we enter the Uzawa–Lucas world with $\psi(0) = \bar{\psi}$. This specification has the interesting side effect that the duration of the transition period towards the innovating economy (i.e. the length of time that the economy develops along the UL growth path) will be relatively independent from specification of production elasticities for labour, capital and intermediates, and the elasticity of substitution for intermediate goods. Actually, sensitivity analysis shows that the length of the transition period varies by about 10% for feasible alternative parameters values. To convey an intuition for this result, note that a positive shift of the term $(1-\beta-\eta)/(\delta\eta(1-\alpha))$ will not only increase the initial knowledge–ideas ratio $\psi(0)$ but also the final value ψ^* according to Eq. (45). The finding that the human capital-blueprint ratio varies only slightly along the path of the innovating economy (which will be demonstrated below) then leads to the result of relative invariance of the speed of transition. However, it can be shown that the transition period is heavily influenced by the specification of the individual preference parameters and the productivity of knowledge formation, which directely effects human capital growth in the development process along the UL-path. The assumption $\psi(0) = \bar{\psi}$ may be seen as intuitively plausible, but it is by no means necessary. We can easily imagine an economy in which individuals begin to invest in human capital before the event of stagnation in a neoclassical steady state. Therefore, we will present alternative scenarios with varying values for initial endowment with knowledge and ideas.

The USA are often regarded as the world-wide technological leader. To avoid the inclusion of catching-up processes in calibrating the model, we therefore select US-data. From the dataset compiled by Jorgenson and Fraumeni (1993, p. 16) we exclude the time span of extraordinary high growth rates after World War II and concentrate on the data for the period 1973–1986.[5] In calibrating the

[5] Actually, inclusion of data for the 1950s and 1960s for steady-state calibration leads to an extraordinary low estimate of $\alpha = 0.12$, which implies a very low elasticity of substitution and the indication of explosive growth.

model we try to determine as much parameters from the data as possible. We begin by calculating the production elasticities $\beta = 0.36$ and $(1-\beta-\eta) = 0.28$ from the contributions of capital stock and hours worked to growth. We then equalize the average contribution of TFP-growth (0.00425) with the model's contribution of technology to growth, $[(1-\alpha)/\alpha][\eta/(1-\eta)]g_n^*$. We calculate value added growth less the contribution of hours worked as average income per capita growth of 1.85% and match this result to $g_Y^* = (1 + [(1-\alpha)/\alpha][\eta/(1-\beta-\eta)])g_n^*$. This gives an estimate of $\alpha = 0.54$ and $g_n^* = 0.88\%$. Since the model also predicts $g_Y^* = (r^* - \rho)/\theta$ with r^* specified in Eq. (41), preference parameters and the productivity parameter ξ must be determined simultaneously with the interest rate in order to fulfil g_Y^*. Table 2 provides an overview of feasible solutions (rounded numbers). We choose $\rho = 0.023$ and $\theta = 2$ for our base run scenario. Values in this order of magnitude are frequently used in numerical calibration excercises, so that our results can be compared to other findings. This selection implies a steady-state interest rate of 6% and a productivity parameter value for knowledge formation of $\xi = 0.05$. We finally fix the productivity parameter to $\delta = 0.1$ which implies an initial $\psi = 0.39\%$ and 7.5% of human capital employed in R&D at the steady state. The bold lines in Figs. 2–6 display the computed transitional dynamics.

Due to our initial value specification, the Uzawa–Lucas framework serves as a description of development dynamics for about 90 years of extensive growth in factor quantities and quality. At the beginning the economy is scarcely endowed with both types of capital. Hence, capital growth rates (Fig. 2), interest rates (Fig. 3), and the investment output ratio (Fig. 5) are high and most of the working time is spent in the production process (Fig. 3). During the adjustment process with half-life of approximately 20 years, interest rates decrease, individuals decide to spend an increasing amount of time on human capital accumulation, and the knowledge–ideas ratio increases. The economy has almost approached at its steady-state growth rate (of 1.37%) when human capital is abundant enough (the wage rate for researchers is sufficently low compared to the value of an innovation) for new inventions to be valuable. The economy

Table 2
Model calibration

$r^* = 0.1 \Leftrightarrow \xi = 0.09$		$r^* = 0.08 \Leftrightarrow \xi = 0.07$		$r^* = 0.06 \Leftrightarrow \xi = 0.05$		$r^* = 0.04 \Leftrightarrow \xi = 0.03$	
θ	ρ	θ	ρ	θ	ρ	θ	ρ
1.5	0.072	1.5	0.052	1.5	0.032	1.5	0.012
2	0.063	2	0.043	2	0.023	2	0.003
4	0.026	4	0.006	3.3	0	2.2	0
5.4	0	4.3	0	—	—	—	—

enters the augmented Grossman and Helpman world. The use of new varieties can well be interpreted as technological progress, since it induces an increase in the productivity of all inputs. Consequently, the beginning of the innovation path is characterized by sharp increases in the investment ratio and the time

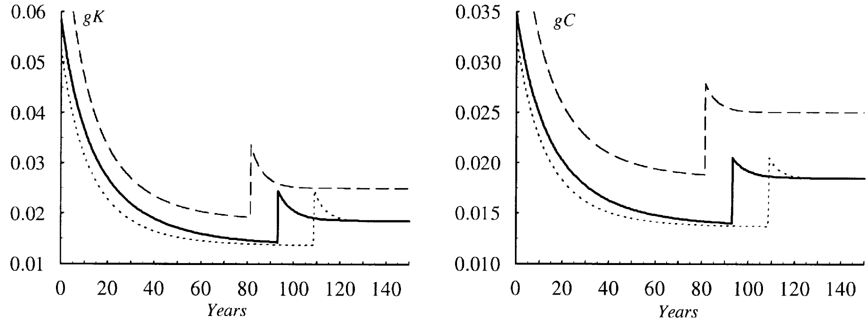

Fig. 2. Transitional dynamics: Capital growth and consumption growth.

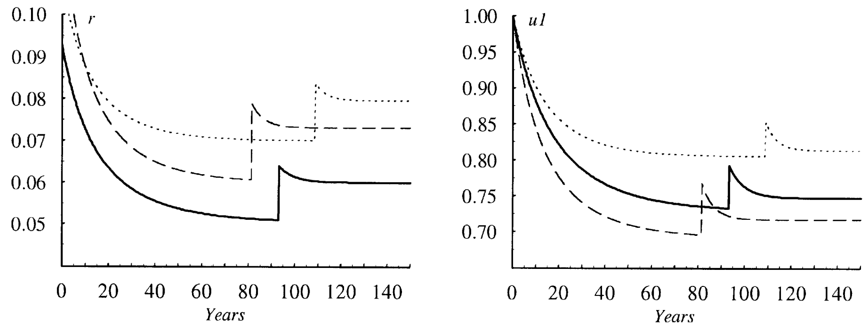

Fig. 3. Transitional dynamics: Interest rate and employment in production.

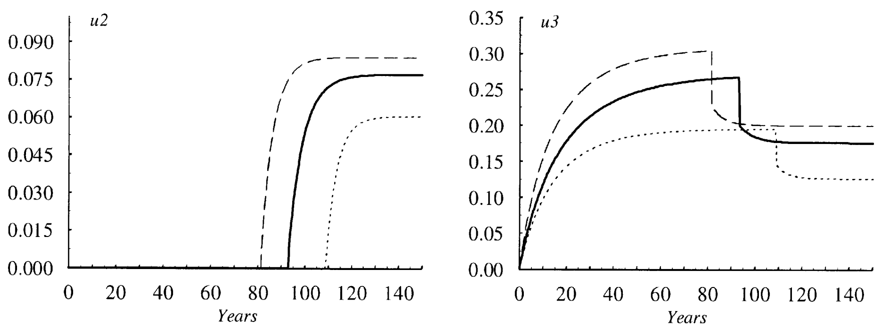

Fig. 4. Transitional dynamics: Employment in R&D and knowledge formation.

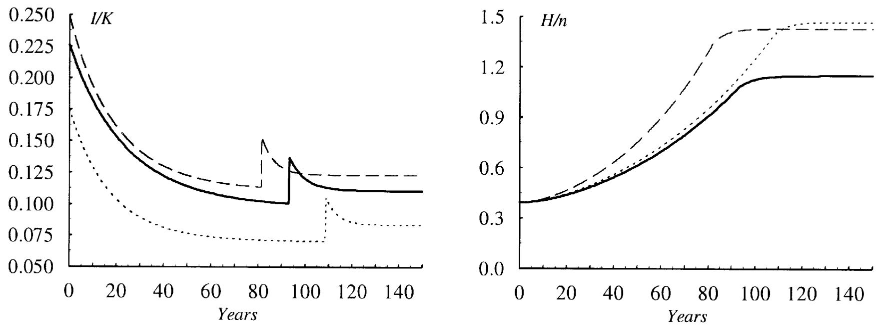

Fig. 5. Transitional dynamics: Investment ratio and human capital blueprint ratio.

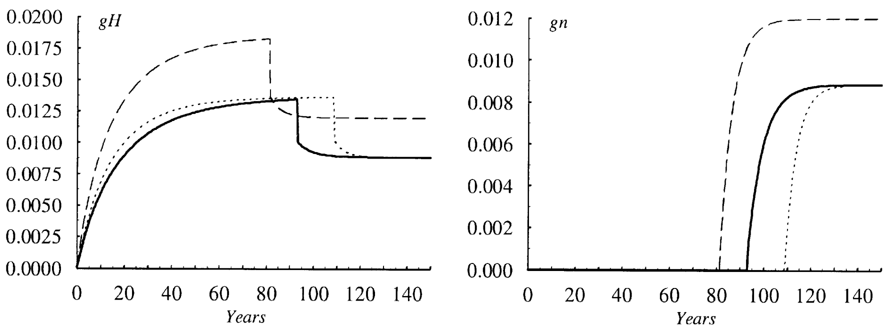

Fig. 6. Transitional dynamics: Growth rates of human capital and technology.

spent in production, which implies a decrease of the human capital share spent on education. Interestingly, adjustment processes for the innovating economy are very fast. Within about 15 years the economy has approximately reached the steady state with a growth rate that surpasses the Uzawa–Lucas growth rate with factor 1.35. Along the adjustment path the knowledge–ideas ratio remains almost constant, 7.5% of the economy's human capital is engaged in R&D and creates new ideas with a rate of 0.88% annually. Seventy-five percent of human capital is employed in production and the remaining part of 17.5% is engaged in the accumulation of knowledge.

We can perform an exercise about what a growth accountant would have calculated confronted with our economy at the beginning, midth and end of the transition path. The results presented in Table 3 fit quite well with several growth accounting exercises.[6] The development process is in great parts

[6] Compare, for example, Young's finding (Young, 1994) that the convergence process of the East Asian NIC's was in large parts 'fueled' by factor accumulation.

Table 3
Growth accounting percentage contribution to growth

Contribution of	Beginning developing economy	Midth developing economy	Beginning innovating economy	End innovating economy
Factor quantity	100	70.3	67.2	55.9
Factor quality	0	29.7	31.3	21.1
Technological progress	0	0	1.5	23.0

characterized by factor accumulation. This process goes hand in hand with the accumulation of skills. The growth process of fully industrialized economies is characterized by large contributions of labour quality growth and technological progress, approximately in equal magnitudes. However, the growth accountant would still assign 56% of income growth to physical capital accumulation.

We can deduce an assessment of the Uzawa–Lucas framework. This kind of model can be identified as inappropriate for the explanation of growth processes in fully industrialized economies, which are characterized in large parts by innovations. But, it serves very well as a description of adjustment processes in developing economies, i.e. 'the mechanics of development'. This, however, is exactly the claim in the title of Lucas' (1988) seminal paper.

Let us now focus on two alternative scenarios. The dotted lines represent a neighbouring case to our basic scenario with $\theta = 2$ and $\rho = 0.043$. The higher rate of time preference implies that we must assume a higher productivity of knowledge formation $\xi = 0.07$ to ensure the same long-run growth rate $g_Y^* = 0.185$. We give up the assumption of an endogenously determined initial endowment. Instead we endow the economy with the same initial H/n as in the basic scenario to guarantee comparability. The higher rate of time preference induces a slower growth of real and physical capital. The investment output ratio is comparatively small, while interest rates and wages per unit H are higher at each point in time. Consequently, R&D becomes profitable later in time. In other words, the 'second' economy enters the stage of innovation about 15 years later. Due to the higher productivity of knowledge formation this economy employs relatively less human capital in education and research but realizes the same steady-state growth rates. About 85% of human capital is employed in the production of goods. Therefore, this economy produces less ideas per unit of human capital, which is reflected by a higher knowledge–ideas ratio.

Our last scenario – displayed by dashed lines – combines the two previous cases by abolishing the requirement of $g_Y^* = 0.0185$. It assumes $\theta = 2$ and $\rho = 0.023$ from the base run but a higher productivity parameter of $\xi = 0.06$. This hypothetical scenario implies a steady-state growth rate of $g_Y = 0.025$ and demonstrates the high explanatory power of the knowledge formation sector for long-run growth. An increase in educational productivity of 20% leads to an increase of long-run growth of about 35%. The effects on the speed of transition, are, however, rather small. The state of an innovating economy is reached approximately 10 years earlier.

4. Conclusions

The paper has presented an approach that encompasses 'different' theories on economic growth in one model. It has been demonstrated that each theory can be assigned to a 'different' state of economic development. It is, therefore, not true that these theories are incoherent complements. On the contrary, for every given set of parameters and initial values, it can be explained endogenously which approach may serve as the best approximation of the economic growth process. Especially, the Uzawa–Lucas approach may serve well to characterize 'mechanics of development' if productivity in the knowledge accumulation sector is sufficiently high. However, the mere permission to invent suffices that every Uzawa–Lucas economy does not stabilize in its steady-state but turns into a Grossman–Helpman economy with rising variety of products and hence endogenous technological progress. In this economy physical capital contributes in large parts to income per capita growth. The incentive to innovate produces long-run growth that surpasses growth through factor accumulation and quality improvements. Perpetual growth of ideas, however, requires the accumulation of knowledge. The paper therefore highlights the importance of education and training.

Finally, we briefly discuss conceivable extensions. Each of them would not only imply an increase in analytical complexity but would most probably alter the quantitative behaviour of the system. Hence, our calibration results should not be taken literally. At first, we can expect a slowdown of the pace of development from the introduction of depreciation for physical and human capital. We expect the same from the allowance of international knowledge spillovers (according to e.g. Grossman and Helpman, 1991, Chapter 9) if human capital remains immobile. The assumption of national knowledge spillovers is not necessary to generate long-run growth. However, from the inclusion of positive spillover effects we can expect a higher contribution of technological progress to long-term growth relative to quality improvements. The same can be expected if we allow negative externalities in R&D like duplication and overlap of research (according to e.g. Jones, 1995). Finally, all results obtained hinge on the crucial assumption of constant returns in the knowledge formation sector. A more sophisticated knowledge production function displaying linear homogeneity with respect to physical and human capital would solve this problem only seemingly. If one believes in decreasing returns in skill accumulation, our models leaves the neoclassical steady state of stagnation as the only feasible long-run solution. Thus, we suggest a refinement of the knowledge formation sector as the most fruitful direction for further research.

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Appendix: The social planner's problem

We concentrate solely on a solution with innovation. Since the intermediate goods sector is symmetric, the social planner will employ the quanity $x(i) = x$ of each good. He maximizes $\int_t^{\infty}[(C^{1-\theta}-1)/(1-\theta)\varepsilon^{-\rho(\tau-t)}]\,\mathrm{d}\tau$ subject to the resource constraint

$$\dot{K} = Y - C - nx, \tag{A.1}$$

where

$$Y = A_1 K^{\beta} n^{\eta/\alpha} x^{\eta} H_Y^{1-\beta-\eta} \tag{A.2}$$

and the technologies (4) and (5) using the control variables C, H_Y, H_n, and x. From the current value Hamiltonian for this problem we obtain the first-order conditions

$$-\theta g_C - \rho = g_{\lambda}, \tag{A.3}$$

$$\eta Y = nx, \tag{A.4}$$

$$\mu\delta = v\xi, \tag{A.5}$$

$$\lambda(1-\beta-\eta)Y/H_Y = v\xi, \tag{A.6}$$

$$\lambda\beta Y/K = -\dot{\lambda}, \tag{A.7}$$

$$v\xi = -\dot{v}, \tag{A.8}$$

$$\lambda(\eta Y/(\alpha n) - x) = -\dot{\mu}, \tag{A.9}$$

where λ, μ and v are denoting the co-state variables corresponding to the states K, n, and H, respectively. From Eqs. (A.3) and (A.7) with Eq. (10) we obtain the Ramsey rule (7). From Eqs. (A.4), (A.6) and (A.9) we get

$$v\xi \frac{H_Y}{n} \frac{\eta}{1-\beta-\eta} \frac{1-\alpha}{\alpha} = -\dot{\mu}. \tag{A.10}$$

Differentiating Eq. (A.5) yields $-\dot{\mu} = -\xi/\delta\dot{v}$, and combination with Eq. (A.8) $-g_v = -g_\mu = \xi$. Insertion into Eq. (A.10) provides, with $H_Y = u_1 H$,

$$u_1 = \frac{\xi}{\delta} \frac{1-\beta-\eta}{\eta} \frac{\alpha}{1-\alpha} \frac{n}{H}. \tag{A.11}$$

From insertion of Eq. (A.4) into Eq. (A.2) we obtain income per capita as

$$Y^{1-\eta} = A_1 \eta^\eta K^\beta n^{[(1-\alpha)/(\alpha\eta)]} (u_1 H)^{1-\beta-\eta}. \tag{A.12}$$

Log-differentiation with respect to time of Eq. (A.12) provides Eq. (22) and imposing the steady-state conditions $g_Y = g_K$, $g_n = g_H$, and $g_{u_1} = 0$ provides

$$g_Y = (1 + 1/A_5) g_n \tag{A.13}$$

which corresponds to the last part of Eq. (43) in Proposition 1. Differentiation of Eq. (A.6) and insertion of Eq. (A.7) and Eq. (A.8) yields

$$g_Y - g_H - g_{u_1} = r - \xi\,, \tag{A.14}$$

which is the social planner's equivalent to Eq. (8a). Imposing the steady-state condition $g_H = g_n$ and $g_{u_1} = 0$ and equating the result with Eq. (A.13) provides Eq. (42) in Proposition 1. Let the superscripts *SP* and *DC* denote the planner's choice and the decentralized solution, respectively. Then, for every given K, H, and n we obtain from dividing Eq. (A.11) by Eq. (36): $u_1^{SP}/u_1^{DC} = \alpha$, and from dividing Eq. (A.12) by Eq. (21): $Y^{SP}/Y^{DC} = \alpha^{-\eta/(1-\eta)}$.

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